Equalization for OFDM Over Doubly Selective Channels

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***Abstract*—In this paper, we propose a time-domain as well as a frequency-domain per-tone equalization for orthogonal frequency-division multiplexing (OFDM) over doubly selective channels. We consider the most general case, where the channel delay spread is larger than the cyclic prefix (CP), which results in interblock interference (IBI). IBI in conjunction with the Doppler effect destroys the orthogonality between subcarriers and, hence, results in severe intercarrier interference (ICI). In this paper, we propose a time-varying finite-impulse-response (TV-FIR) time-domain equalizer (TEQ) to restore the orthogonality between subcarriers, and hence to eliminate ICI/IBI. Due to the fact that the TEQ optimizes the performance over all subcarriers in a joint fashion, it has a poor performance. An optimal frequency-domain per-tone equalizer (PTEQ) is then obtained by transferring the TEQ operation to the frequency domain. Through computer simulations, we demonstrate the performance of the proposed equalization techniques.**

***Index Terms*—Basis expansion model (BEM), doubly selective channels, orthogonal frequency-division multiplexing (OFDM), per-tone equalization (PTEQ), time-domain equalization (TEQ).**

I. Introduction

ORTHOGONAL frequency-division multiplexing (OFDM) has attracted a lot of attention, due to its simple implementation and robustness against *frequency-selective* channels. In this paper, we consider OFDM transmission over *doubly selective* (time- and frequency-selective) channels. In doubly selective channels, the time variation of the channel over an OFDM block destroys the orthogonality between subcarriers and so induces intercarrier interference (ICI). In addition to this, interblock interference (IBI) arises when the channel delay spread is larger than the cyclic prefix (CP), which again results in ICI. Hence, equalization techniques are required to restore the orthogonality and so to eliminate ICI/IBI. In this paper, we propose time-domain as well as frequency-domain per-tone equalization techniques (PTEQ) to combat these channel effects. An emerging application that uses OFDM as a transmission technique is digital video broadcasting (DVB). DVB encounters long-delay multipath channels. Using a CP of length equal to the channel order, results in a significant decrease in throughput. On the other hand, applying DVB over mobile channels for high speed terminals (motor way speeds) induces ICI which has been shown to decrease performance significantly. The motivation of this paper is to combat these channel effects for such applications.

Different approaches for reducing ICI have been proposed, including frequency-domain equalization and/or time-domain windowing. In [1], [2] the authors propose matched-filter, least-squares (LS) and minimum mean-square error (MMSE) receivers incorporating all subcarriers. Receivers considering the dominant adjacent subcarriers have been presented in [3] and [4]. For multiple-input multiple-output (MIMO) OFDM over doubly selective channels, a frequency-domain ICI mitigation technique is proposed in [5]. A time-domain windowing (linear preprocessing) approach to restrict ICI support in conjunction with iterative MMSE estimation is presented in [6]. ICI self-cancellation schemes are proposed in [7] and [8]. There, redundancy is added to enable self-cancellation, which implies a substantial reduction in bandwidth efficiency. To avoid this rate loss, partial response encoding in conjunction with maximum-likelihood sequence detection to mitigate ICI in OFDM systems is studied in [9]. *However, all of the above-mentioned literature assume the channel delay spread fits within the CP, and hence, no IBI is present*. Moreover, in this literature, the time-varying (TV) channel matrix (or an estimated version of it) is required to design the equalizer. This, in return, requires a large number of parameters to be identified (tracked).

In this paper, we assume the channel delay spread is larger than the CP, and moreover, we approximate the TV channel by the basis expansion model (BEM). In this BEM, we assume that only the BEM coefficients are known at the receiver which is easier to obtain [10]. The BEM coefficients are then used to design the equalizer to equalize the true channel. In [11] and [12], a frequency-domain ICI/IBI cancellation scheme is proposed for discrete multitone (DMT) systems without guard interval. The approach in the aforementioned literature depends on utilizing the nulled (unused) subcarriers to eliminate ICI/IBI, which is different than what we propose in this paper.

In [13], a time-invariant finite-impulse-response (TI-FIR) filter is applied to the time-domain received samples for OFDM transmission over frequency-selective channels whose delay

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spread is larger than the CP. The purpose of this time-domain equalizer (TEQ) is to shorten the channel delay-spread to fit within the CP. For the same problem, an optimum frequency-domain per-tone equalizer (PTEQ) is proposed in [14]. The PTEQ is then obtained by transferring the TEQ operation to the frequency domain.

Similarly, in this paper, we apply a TV-FIR TEQ to convert the doubly selective channel whose delay-spread is larger than the CP into a purely frequency-selective channel with a delay spread that fits within the CP. By doing this, we restore the orthogonality between subcarriers (eliminate IBI and ICI). Hence, an additional one-tap frequency-domain equalizer (FEQ) then allows us to estimate the QAM transmitted symbols. The proposed TV-FIR TEQ optimizes the performance on all subcarriers in a joint fashion. An optimum PTEQ is then obtained by transferring the TEQ operation to the frequency domain. The proposed PTEQ optimizes the performance on each subcarrier separately.

The proposed techniques in this paper are different from those proposed in our earlier literature: [15] for OFDM transmission over doubly selective channels and [16] for single-carrier (SC) transmission over doubly selective channels.

- With respect to [15]:
  1) in this paper, unlike [15], we assume the most general case, where the channel delay spread does not necessarily fit within the CP;
  2) the time-domain equalizer is assumed to be a multitap TV-FIR filter instead of a one-tap TV filter as in [15], which leads to a more general architecture, especially in the frequency domain; this new architecture outperforms the one proposed in [15] [a gain of 3 dB is obtained for the single-input single-output (SISO) case]; moreover, this architecture allows us to approach the performance of the block MMSE equalizer (see Fig. 6).
- With respect to [16]:
  1) SC transmission is considered in [16], whereas the current paper assumes OFDM;
  2) in [16], the channel delay spread fits within the CP;
  3) only time-domain equalization is considered in [16], whereas in this paper, we consider time- and frequency-domain equalization techniques;
  4) in [16], the BEM resolution is assumed to be equal to the window length; in the current paper, we consider a BEM resolution that is an integer multiple of the window size; increasing the BEM resolution (which corresponds to frequency oversampling) results in a better fit when realistic fading channels are considered; note that in [16], applying the proposed equalizer on a Jakes' channel results in a high error floor; this error floor is significantly reduced by frequency oversampling as proposed in this paper; the fact that the BEM resolution is considered to be an integer multiple of the block size also leads to a more general architecture.
  5) in [16], the time-domain equalizer is considered to completely equalize the channel, whereas in this paper, it requires us to shorten the delay spread in order to fit within the CP and to remove the channel time variation; the latter design criterion gives more degrees of freedom.

This paper is organized as follows. In Section II, we present the system model along with the basis expansion channel model. The proposed TEQ is presented in Section III. In Section IV, we introduce the PTEQ. An efficient implementation of the proposed PTEQ is discussed in Section V. In Section VI, we show through computer simulations the performance of the proposed equalizer. Finally, our conclusions are drawn in Section VII.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. We denote the Kronecker delta as $\delta[n]$ and $\mathcal{E}\{\cdot\}$ denotes expectation. We use $\star$ to denote convolution. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$, the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$ and the all ones vector of length M as $\mathbf{1}_M$. The kth element of vector $\mathbf{x}$ is denoted by $[\mathbf{x}]_k$. We denote $\mathbb{Z}^+$ the set of positive integers without zero. Finally, $\mathrm{diag}\{\mathbf{x}\}$ denotes the diagonal matrix with vector $\mathbf{x}$ on the diagonal, and $\mathrm{diag}\{\mathbf{A}_0,\ldots,\mathbf{A}_{M-1}\}$ denotes the block diagonal matrix with the submatrices $\mathbf{A}_0,\ldots,\mathbf{A}_{M-1}$ on the diagonal.

II. System Model

We assume a single-input multiple-output (SIMO) OFDM system (see Fig. 1) with N_r receive antennas, but the results can be easily extended to MIMO systems. At the transmitter, the conventional OFDM modulation is applied, i.e., the incoming bit sequence is parsed into blocks of N frequency-domain QAM symbols. Each block is then transformed into a time-domain sequence using an N-point inverse discrete Fourier transform (IDFT). A CP of length ν is inserted at the head of each block. The time-domain blocks are then serially transmitted over a multipath fading channel. The channel is assumed to be TV. Focusing only on the baseband-equivalent description, the received signal $y^{(r)}(t)$ at the rth receive antenna at time t is given by

$$y^{(r)}(t) = \sum_{n=-\infty}^{\infty} g^{(r)}(t;t-nT)x[n] + \eta^{(r)}(t)$$

where $g^{(r)}(t;\tau)$ is the baseband-equivalent of the doubly selective channel from the transmitter to the rth receive antenna, $\eta^{(r)}(t)$ is the baseband-equivalent filtered additive noise at the rth receive antenna and $x[n]$ is the discrete time-domain sequence transmitted at a rate of $1/T$ symbols per second. Assuming $S_k[i]$ is the QAM symbol transmitted on the kth subcarrier ($k \in \{0,\ldots,N-1\}$, N is the total number of subcarriers in the OFDM block) of the ith OFDM block, $x[n]$ can be written as

$$x[n] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k[i] e^{j2\pi(m-\nu)k/N}$$

where $i = \lfloor n/(N+\nu) \rfloor$ and $m = n - i(N+\nu)$. Note that this description includes the transmission of a CP of length ν.

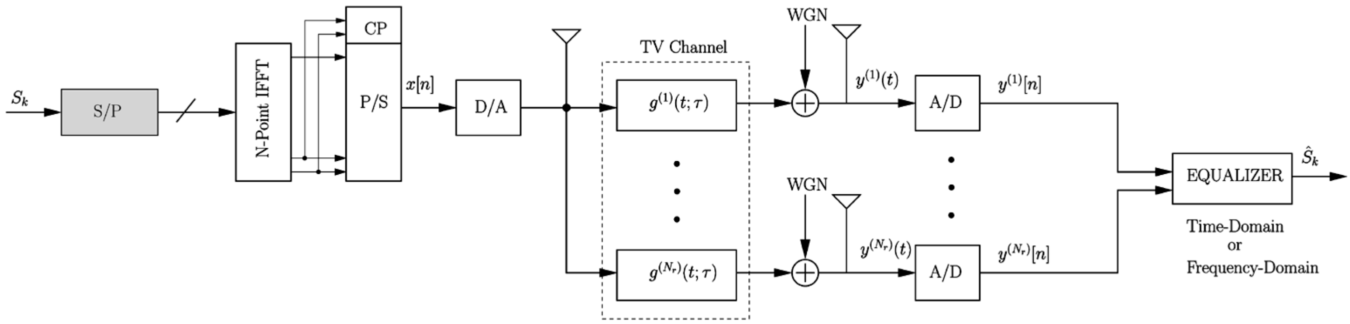

Fig. 1. System model.

The baseband-equivalent doubly selective channel $g^{(r)}(t;\tau)$ includes the physical channel $g_{\mathrm{ch}}^{(r)}(t;\tau)$ as well as the transmit filter $g_{\mathrm{tr}}(t)$ and receive filter $g_{\mathrm{rec}}(t)$

$$g^{(r)}(t;\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_{\mathrm{rec}}(s)g_{\mathrm{tr}}(\tau-s-\theta)g_{\mathrm{ch}}^{(r)}(t-s;\theta)\,\mathrm{d}s\,\mathrm{d}\theta.$$

Sampling each receive antenna at the symbol period T, the received sample sequence at the rth receive antenna $y^{(r)}[n]=y^{(r)}(nT)$ can be written as

$$y^{(r)}[n]=\sum_{\theta=-\infty}^{\infty} g^{(r)}[n;\theta]x[n-\theta]+\eta^{(r)}[n] \qquad (1)$$

where $\eta^{(r)}[n]=\eta^{(r)}(nT)$ and $g^{(r)}[n;\theta]=g^{(r)}(nT;\theta T)$.

Most wireless links experience multipath fading propagation due to scattering and reflection of the transmitted signal. Each resolvable path corresponds to a superposition of scattered rays that arrive at the receiver almost simultaneously with a common propagation delay, called a cluster. Each ray within the cluster is characterized by its own complex gain and Doppler shift. Hence, the physical channel $g_{\mathrm{ch}}^{(r)}(t;\tau)$ can be written as [17], [18]

$$g_{\mathrm{ch}}^{(r)}(t;\tau)=\sum_c \delta\left(\tau-\tau_c^{(r)}\right)\sum_\mu G_{c,\mu}^{(r)}e^{j2\pi f_{c,\mu}^{(r)}t} \qquad (2)$$

where $\tau_c^{(r)}$ is the propagation delay of the cth cluster of the rth receive antenna, and $G_{c,\mu}^{(r)}$ and $f_{c,\mu}^{(r)}$ are the complex gain and the frequency offset, respectively, of the μth ray of the cth cluster characterizing the link between the transmitter and the rth receive antenna.

Assuming the time variation of the physical channel $g_{\mathrm{ch}}^{(r)}(t;\tau)$ is negligible over the span of the receive filter $g_{\mathrm{rec}}(t)$, we obtain

$$\begin{aligned} g^{(r)}(t;\tau) &= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} g_{\mathrm{rec}}(s)g_{\mathrm{tr}}(\tau-s-\theta)\,\mathrm{d}s\right)g_{\mathrm{ch}}^{(r)}(t;\theta)\,\mathrm{d}\theta \\ &= \int_{-\infty}^{\infty}\psi(\tau-\theta)g_{\mathrm{ch}}^{(r)}(t;\theta)\mathrm{d}\theta \\ &= \sum_c \psi\left(\tau-\tau_c^{(r)}\right)\sum_\mu G_{c,\mu}^{(r)}e^{j2\pi f_{c,\mu}^{(r)}t} \qquad (3)\end{aligned}$$

where $\psi(t)=g_{\mathrm{tr}}(t)\star g_{\mathrm{rec}}(t)$. Hence, we can express $g^{(r)}[n;\theta]$ as

$$g^{(r)}[n;\theta]=\sum_c \psi\left(\theta T-\tau_c^{(r)}\right)\sum_\mu G_{c,\mu}^{(r)}e^{j2\pi f_{c,\mu}^{(r)}nT}.$$

The channel model described in (3) is a rather complex model, with a huge (possibly infinite) number of parameters to be identified/equalized. This motivates the use of an alternative channel model with fewer parameters. In this paper, we use the BEM to approximate the discrete-time baseband-equivalent doubly selective channel.

A. Basis Expansion Channel Model

In this section, we describe the BEM channel [19]–[22]. In this BEM, the doubly selective channel $g^{(r)}[n;\theta]$ is modeled as an FIR filter where the taps are expressed as a superposition of complex exponential basis functions with frequencies on a discrete grid. Assuming $g^{(r)}(t;\tau)=0$ for $\tau\notin[0,(L+1)T)$, each channel $g^{(r)}[n;\theta]$ can be approximated for $n\in\{i(N+\nu)+\nu+d-L',\ldots,(i+1)(N+\nu)+d-1\}$ (L' and d to be defined later) by a BEM

$$h^{(r)}[n;\theta]=\sum_{l=0}^{L}\delta[\theta-l]\sum_{q=-Q/2}^{Q/2} h_{q,l}^{(r)}[i]e^{j2\pi qn/K} \qquad (4)$$

where Q and K should be selected such that $Q/(KT)\geq 2f_{\max}$, with $f_{\max}$ the maximum Doppler spread of all channels

$$f_{\max}=\max_{r,c,\mu}\left\{\left|f_{c,\mu}^{(r)}\right|\right\}.$$

Under this BEM the TV channel can be modeled as shown in Fig. 2.

In this expansion model, L represents the delay spread (expressed in multiples of T, the delay resolution of the model), and $Q/2$ represents the Doppler spread (expressed in multiples of $1/(KT)$, the Doppler resolution of the model). Note that the coefficients $h_{q,l}^{(r)}[i]$ remain invariant over a period of length $(N+L')T$ but may change from block to block.

Substituting (4) in (1), the received sample sequence at the rth receive antenna can be written as

$$y^{(r)}[n]=\sum_{l=0}^{L}\sum_{q=-Q/2}^{Q/2} e^{j2\pi qn/K}h_{q,l}^{(r)}[i]x[n-l]+\eta^{(r)}[n]. \quad (5)$$

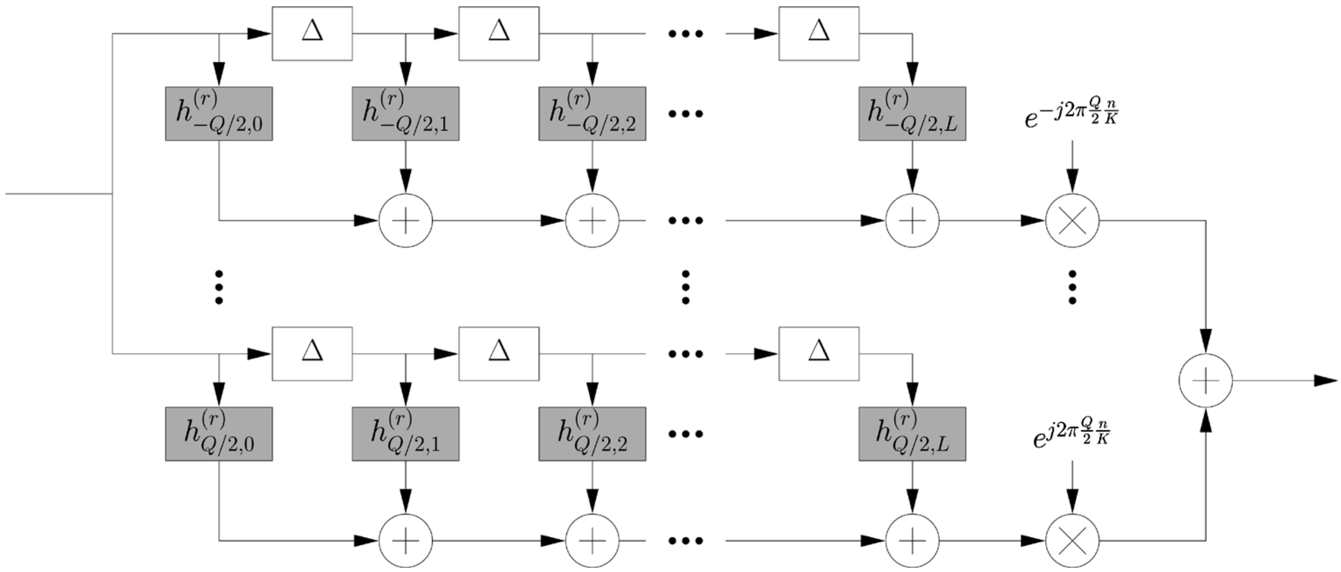

Fig. 2. Block diagram of the BEM realization of a doubly selective channel.

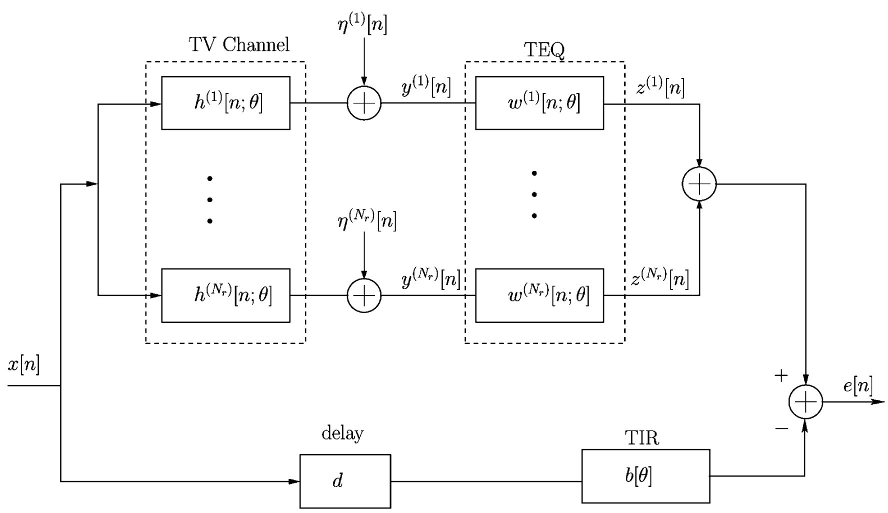

Fig. 3. Block diagram of the TEQ.

Note that in (4) and (5), due to the equalizer filter span L' (to be defined later), some overlap between consecutive blocks occurred. This overlap implies that some channel samples and received samples are defined twice, which may be inconsistent. Upon block processing of the received sequence, we take first the block index i, and accordingly we take $n \in \{i(N+\nu)+\nu+d-L', \ldots, (i+1)(N+\nu)+d-1\}$. Note that the definitions in (4) and (5) are merely approximations of the true channel and the received sequence, respectively. These equations are used to simplify the derivation of the proposed equalizers (time domain and frequency domain), which are then used to equalize the true channel.

III. Time-Domain Equalization

In this section, we introduce TEQ for OFDM systems over doubly selective channels. We assume the most general case, where the TV channel delay spread is larger than the CP. The TEQ is implemented by a TV-FIR filter, i.e., at the rth receive antenna, we apply a TV-FIR TEQ denoted by $w^{(r)}[n;\theta]$. The purpose of the TEQ is to convert the doubly selective channel into a frequency-selective channel with a delay spread that fits within the CP, i.e., to convert the doubly selective channel of order $L > \nu$ and $f_{\max} \neq 0$ into a target impulse response (TIR) $b[\theta]$ that is purely frequency selective with order $L'' \leq \nu$ and $f_{\max} = 0$. The purpose of the TV-FIR TEQ is thus to mitigate both IBI and ICI. As shown in Fig. 3, we require to design a TV-FIR TEQ and TIR such that the difference term $e[n]$ is minimized in the mean-square error (MSE) sense, subject to some decision delay d.

The output of the TV-FIR TEQ at the rth receive antenna subject to some decision delay d can be written as

$$z^{(r)}[n] = \sum_{\theta=-\infty}^{\infty} w^{(r)}[n;\theta]y^{(r)}[n+d-\theta]. \tag{6}$$

Since we approximate the doubly selective channel using the BEM, it is also convenient to model the TV-FIR TEQ using the BEM. In other words, we design the TV-FIR TEQ $w^{(r)}[n;\theta]$ to have $L'+1$ taps, where the time variation of each tap is modeled by $Q'+1$ time-varying complex exponential basis functions. Hence, we can write the TV-FIR TEQ $w^{(r)}[n;\theta]$ for $n \in \{i(N+\nu)+\nu,\ldots,(i+1)(N+\nu)-1\}$ as

$$w^{(r)}[n;\theta] = \sum_{l'=0}^{L'} \delta[\theta - l'] \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)}[i] e^{j2\pi q' n/K}. \quad (7)$$

Substituting (7) in (6), we obtain

$$\begin{aligned} z^{(r)}[n] &= \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)}[i] e^{j2\pi q' n/K} y^{(r)}[n+d-l'] \\ &= \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} w_{q',l'}^{(r)}[i] \\ &\quad \times h_{q,l}^{(r)}[i] e^{j2\pi q' n/K} e^{j2\pi q n/K} x[n+d-l-l'] \\ &\quad + \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)}[i] e^{j2\pi q' n/K} \eta^{(r)}[n+d-l']. \end{aligned} \quad (8)$$

It is more convenient at this point to switch to a block level formulation. Defining $\mathbf{z}^{(r)}[i] = [z^{(r)}[i(N+\nu)+\nu],\ldots,z^{(r)}[(i+1)(N+\nu)-1]]^T$, $\mathbf{x}[i] = [x[i(N+\nu)+\nu+d-L-L'],\ldots,x[(i+1)(N+\nu)+d-1]]^T$, and $\boldsymbol{\eta}^{(r)}[i] = [\eta^{(r)}[i(N+\nu)+d-L'],\ldots,\eta^{(r)}[(i+1)(N+\nu)+d-1]]^T$, then (8) on a block level can be formulated as

$$\begin{aligned} \mathbf{z}^{(r)}[i] &= \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} w_{q',l'}^{(r)}[i] \\ &\quad \times h_{q,l}^{(r)}[i] \mathbf{D}_{q'}[i] \mathbf{Z}_{l'} \tilde{\mathbf{D}}_q[i] \tilde{\mathbf{Z}}_l \mathbf{x}[i] \\ &\quad + \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)} \mathbf{D}_{q'}[i] \mathbf{Z}_{l'} \boldsymbol{\eta}[i] \end{aligned} \quad (9)$$

where

$$\begin{aligned} \mathbf{D}_{q'}[i] =& \mathrm{diag}\{[e^{j2\pi q'(i(N+\nu)+\nu)/K},\ldots, \\ & e^{j2\pi q'((i+1)(N+\nu)-1)/K}]^T\} \\ \mathbf{Z}_{l'} =& [\mathbf{0}_{N\times(L'-l')}, \mathbf{I}_N, \mathbf{0}_{N\times l'}] \\ \tilde{\mathbf{D}}_q[i] =& \mathrm{diag}\{[e^{j2\pi q(i(N+\nu)+\nu-L')/K},\ldots, \\ & e^{j2\pi q((i+1)(N+\nu)-1)/K}]^T\} \end{aligned}$$

and

$$\tilde{\mathbf{Z}}_l = [\mathbf{0}_{(N+L')\times(L-l)}, \mathbf{I}_{N+L'}, \mathbf{0}_{(N+L')\times l}].$$

Using the property $\mathbf{Z}_{l'}\tilde{\mathbf{D}}_q[i] = e^{j2\pi q(L'-l')/K}\mathbf{D}_q[i]\mathbf{Z}_{l'}$, and defining $p=q+q'$ and $k=l+l'$, we can write (9) as

$$\begin{aligned} \mathbf{z}[i] =& \sum_{p=-(Q+Q')/2}^{(Q+Q')/2} \sum_{k=0}^{L+L'} f_{p,k}[i] \mathbf{D}_p[i] \bar{\mathbf{Z}}_k \mathbf{x}[i] \\ &+ \sum_{r=1}^{N_r} \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} w_{q',l'}^{(r)}[i] \mathbf{D}_{q'}[i] \mathbf{Z}_{l'} \boldsymbol{\eta}[i] \end{aligned} \quad (10)$$

where $\mathbf{z}[i] = \sum_{r=1}^{N_r} \mathbf{z}^{(r)}[i]$, $\bar{\mathbf{Z}}_k = [\mathbf{0}_{N\times(L+L'-k)}, \mathbf{I}_N, \mathbf{0}_{N\times k}]$, and $f_{p,k}[i]$ can be written as

$$f_{p,k}[i] = \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \sum_{l'=0}^{L'} e^{j2\pi(p-q')l'/K} w_{q',l'}^{(r)}[i] h_{p-q',k-l'}^{(r)}[i]. \quad (11)$$

Defining $\mathbf{f}[i] = [f_{-(Q+Q')/2,0}[i],\ldots,f_{(Q+Q')/2,L+L'}[i]]^T$, we can further write (10) as

$$\begin{aligned} \mathbf{z}[i] &= (\mathbf{f}^T[i] \otimes \mathbf{I}_N) \mathbf{A}[i] \mathbf{x}[i] \\ &\quad + \sum_{r=1}^{N_r} \left(\mathbf{w}^{(r)T}[i] \otimes \mathbf{I}_N\right) \mathbf{B}[i] \boldsymbol{\eta}^{(r)}[i] \\ &= (\mathbf{f}^T[i] \otimes \mathbf{I}_N) \mathbf{A}[i] \mathbf{x}[i] \\ &\quad + (\mathbf{w}^T[i] \otimes \mathbf{I}_N)(\mathbf{I}_{N_r} \otimes \mathbf{B}[i]) \boldsymbol{\eta}[i] \end{aligned} \quad (12)$$

where $\mathbf{w}^{(r)}[i] = [w_{-Q'/2,0}^{(r)}[i],\ldots,w_{Q'/2,L'}^{(r)}[i]]^T$, $\mathbf{w}[i] = [\mathbf{w}^{(1)T}[i],\ldots,\mathbf{w}^{(N_r)T}[i]]^T$, $\boldsymbol{\eta}[i] = [\boldsymbol{\eta}^{(1)T}[i],\ldots,\boldsymbol{\eta}^{(N_r)T}[i]]^T$, and $\mathbf{A}[i]$ and $\mathbf{B}[i]$ are given by

$$\mathbf{A}[i] = \begin{bmatrix} \mathbf{D}_{-(Q+Q')/2}[i]\bar{\mathbf{Z}}_0 \\ \vdots \\ \mathbf{D}_{-(Q+Q')/2}[i]\bar{\mathbf{Z}}_{L+L'} \\ \vdots \\ \mathbf{D}_{(Q+Q')/2}[i]\bar{\mathbf{Z}}_{L+L'} \end{bmatrix}$$

$$\mathbf{B}[i] = \begin{bmatrix} \mathbf{D}_{-Q'/2}[i]\mathbf{Z}_0 \\ \vdots \\ \mathbf{D}_{-Q'/2}[i]\mathbf{Z}_{L'} \\ \vdots \\ \mathbf{D}_{Q'/2}[i]\mathbf{Z}_{L'} \end{bmatrix}.$$

Note that the term in $f_{p,k}[i]$ corresponding to the rth receive antenna is related to a two-dimensional (2-D) convolution of the BEM coefficients of the doubly selective channel for the rth receive antenna and the BEM coefficients of the TV-FIR TEQ for the rth receive antenna. This allows us to derive a linear relationship between $\mathbf{f}[i]$ and $\mathbf{w}[i]$. We first define the $(L'+1)\times(L'+L+1)$ Toeplitz matrix

$$\mathcal{T}_{l,L'+1}\left(h_{q,l}^{(r)}[i]\right) = \begin{bmatrix} h_{q,0}^{(r)}[i] & \cdots & h_{q,L}^{(r)}[i] & & 0 \\ & \ddots & & \ddots & \\ 0 & & h_{q,0}^{(r)}[i] & \cdots & h_{q,L}^{(r)}[i] \end{bmatrix}.$$

We then define $\boldsymbol{\mathcal{H}}_q^{(r)}[i] = \boldsymbol{\Omega}_q \mathcal{T}_{l,L'+1}(h_{q,l}^{(r)}[i])$, where $\boldsymbol{\Omega}_q = \mathrm{diag}\{[e^{j2\pi qL'/K},\ldots,1]^T\}$ and introduce the $(Q'+1)(L'+1)\times(Q+Q'+1)(L+L'+1)$ block Toeplitz matrix

$$\mathcal{T}_{q,Q'+1}\left(\boldsymbol{\mathcal{H}}_q^{(r)}[i]\right) = \begin{bmatrix} \boldsymbol{\mathcal{H}}_{-Q/2}^{(r)}[i] & \cdots & \boldsymbol{\mathcal{H}}_{Q/2}^{(r)}[i] & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \boldsymbol{\mathcal{H}}_{-Q/2}^{(r)}[i] & \cdots & \boldsymbol{\mathcal{H}}_{Q/2}^{(r)}[i] \end{bmatrix}.$$

Introducing the definitions $\boldsymbol{\mathcal{H}}^{(r)}[i] = \mathcal{T}_{q,Q'+1}(\boldsymbol{\mathcal{H}}_q^{(r)}[i])$ and $\boldsymbol{\mathcal{H}}[i] = [\boldsymbol{\mathcal{H}}^{(1)T}[i],\ldots,\boldsymbol{\mathcal{H}}^{(N_r)T}[i]]^T$, we can finally derive from (11) that

$$\mathbf{f}^T[i] = \mathbf{w}^T[i]\boldsymbol{\mathcal{H}}[i]. \quad (13)$$

As already mentioned, the purpose of the TEQ is to convert the doubly selective channel into a frequency-selective equivalent channel with order less than or equal the CP. To this aim, we define the so-called TIR denoted by $b[\theta]$ and of order $L'' \leq \nu$, which can be modeled for $n \in \{i(N+\nu)+\nu, \ldots, (i+1)(N+\nu)-1\}$ as

$$b[\theta] = \sum_{l''=0}^{L''} \delta[\theta - l'']b_{l''}[i].$$

As shown in Fig. 3, we will now design a TEQ $\mathbf{w}[i]$, a TIR $\mathbf{b}[i] = [b_0[i], \ldots, b_{L''}[i]]^T$ and a synchronization delay d such that the difference between the outputs of the upper branch and the lower branch is minimized. Defining $\mathbf{e}[i] = [e[i(N+\nu)], \ldots, e[i(N+\nu)+N-1]]^T$, we can express $\mathbf{e}[i]$ as

$$\begin{aligned}\mathbf{e}[i] &= (\mathbf{f}^T[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] + (\mathbf{w}^T[i] \otimes \mathbf{I}_N)(\mathbf{I}_{N_r} \otimes \mathbf{B}[i])\boldsymbol{\eta}[i] \\ &\quad - \sum_{l''=0}^{L''} b_{l''}[i]\bar{\mathbf{Z}}_{l''}\mathbf{x}[i] \\ &= (\mathbf{f}^T[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] + (\mathbf{w}^T[i] \otimes \mathbf{I}_N)(\mathbf{I}_{N_r} \otimes \mathbf{B}[i])\boldsymbol{\eta}[i] \\ &\quad - (\tilde{\mathbf{b}}[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{x}[i] \qquad (14)\end{aligned}$$

where the augmented vector $\tilde{\mathbf{b}}[i]$ can be written as $\tilde{\mathbf{b}}[i] = \mathbf{C}\mathbf{b}[i]$ with the selection matrix $\mathbf{C}$ given by

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{((Q+Q')(L+L'+1)/2+d)\times(L''+1)} \\ \hline \mathbf{I}_{L''+1} \\ \hline \mathbf{0}_{((Q+Q')(L+L'+1)/2-L''-d-1)\times(L''+1)} \end{bmatrix}.$$

Hence, we can write the following cost function:

$$\begin{aligned}\mathcal{J}[i] &= \mathcal{E}\{\mathbf{e}^H[i]\mathbf{e}[i]\} \\ &= \mathrm{tr}\left\{(\mathbf{f}^T[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^H[i](\mathbf{f}^*[i] \otimes \mathbf{I}_N)\right\} \\ &\quad + \mathrm{tr}\{(\mathbf{w}^T[i] \otimes \mathbf{I}_N)\tilde{\mathbf{B}}[i]\mathbf{R}_\eta\tilde{\mathbf{B}}^H[i](\mathbf{w}^*[i] \otimes \mathbf{I}_N)\} \\ &\quad + \mathrm{tr}\{(\tilde{\mathbf{b}}^T[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^H[i](\tilde{\mathbf{b}}^*[i] \otimes \mathbf{I}_N)\} \\ &\quad - 2\mathrm{tr}\{\Re\{(\mathbf{f}^T[i] \otimes \mathbf{I}_N)\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^H[i](\tilde{\mathbf{b}}^*[i] \otimes \mathbf{I}_N)\}\} \qquad (15)\end{aligned}$$

where $\tilde{\mathbf{B}}[i] = \mathbf{I}_{N_r} \otimes \mathbf{B}[i]$. Let us now introduce the following properties:

$$\mathrm{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_N)\mathbf{X}(\mathbf{x}^* \otimes \mathbf{I}_N)\} = \mathbf{x}^T\,\mathrm{subrt}\,\{\mathbf{X}\}\mathbf{x}^*$$
$$\mathrm{tr}\{(\mathbf{x}^T \otimes \mathbf{I}_N)\mathbf{V}(\mathbf{y}^* \otimes \mathbf{I}_N)\} = \mathbf{x}^T\,\mathrm{subrt}\,\{\mathbf{V}\}\mathbf{y}^*$$

where $\mathrm{subtr}\{\cdot\}$ splits the matrix up into $N \times N$ submatrices and replaces each submatrix by its trace.[1] Note that $\mathbf{X}$ is a square matrix while $\mathbf{V}$ is not necessarily square. Hence, $\mathrm{subtr}\{\cdot\}$ reduces the row and column dimension by a factor N. Therefore, the cost function in (15) reduces to

$$\begin{aligned}\mathcal{J}[i] &= \mathbf{w}^T[i](\boldsymbol{\mathcal{H}}[i]\mathbf{R}_A[i]\boldsymbol{\mathcal{H}}^H[i] + \mathbf{R}_{\tilde{B}}[i])\mathbf{w}^*[i] \\ &\quad + \tilde{\mathbf{b}}^T[i]\mathbf{R}_A[i]\tilde{\mathbf{b}}^*[i] - 2\Re\{\mathbf{w}^T[i]\boldsymbol{\mathcal{H}}[i]\mathbf{R}_A[i]\tilde{\mathbf{b}}^*[i]\} \qquad (16)\end{aligned}$$

where $\mathbf{R}_A[i] = \mathrm{subtr}\{\mathbf{A}[i]\mathbf{R}_x\mathbf{A}^H[i]\}$, and $\mathbf{R}_{\tilde{B}}[i] = \mathrm{subtr}\{\tilde{\mathbf{B}}[i]\mathbf{R}_\eta\tilde{\mathbf{B}}^H[i]\}$. This cost function is now to be optimized with regard to $\mathbf{w}[i]$ and $\mathbf{b}[i]$. In order to avoid the trivial solution (zero vector $\mathbf{w}[i]$ and zero vector $\mathbf{b}[i]$), nontriviality constraints need to be added, e.g., a unit tap constraint $b_0[i] = 1$, a unit-norm constraint $\|\mathbf{b}[i]\|^2 = 1$ or $\|\mathbf{w}[i]\|^2 = 1$, or a unit-energy constraint $\mathbf{b}^H[i]\mathbf{R}_A[i]\mathbf{b}[i] = 1$ or $\mathbf{w}^H[i]\mathbf{R}_{\tilde{B}}[i]\mathbf{w}[i] = 1$. More details about these constraints for TI channels can be found in [23] and [13] for the unit-tap and unit-norm constraints, and in [24] for the unit-energy constraint. A TEQ for the MIMO case is proposed in [25].

In this paper, we only consider the unit-norm constraint and the unit energy constraint for their superior performance to the other constraint (this is proven to be the case in DMT systems, and here is no exception).

1) Unit-Norm Constraint: In this case, we have the following optimization problem:

$$\min_{\mathbf{w}[i],\mathbf{b}[i]} \mathcal{J}[i] \quad \text{such that } \|\mathbf{b}[i]\|^2 = 1.$$

The solution to this problem is given by

$$\begin{aligned}\mathbf{w}^T[i] &= \tilde{\mathbf{b}}^T[i]\left(\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\boldsymbol{\mathcal{H}}[i] + \mathbf{R}_A^{-1}[i]\right)^{-1}\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i], \\ \mathbf{b}[i] &= \mathrm{eig}_{\min}(\mathbf{R}^\perp[i])\end{aligned}$$

where $\mathbf{R}^\perp[i]$ is given by

$$\mathbf{R}^\perp[i] = \mathbf{C}^T\left(\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\boldsymbol{\mathcal{H}}[i] + \mathbf{R}_A^{-1}[i]\right)^{-1}\mathbf{C}.$$

2) Unit-Energy Constraint: In this case, we have the following optimization problem:

$$\min_{\mathbf{w}[i],\mathbf{b}[i]} \mathcal{J}[i] \quad \text{such that } \tilde{\mathbf{b}}^H[i]\mathbf{R}_A[i]\tilde{\mathbf{b}}[i] = 1.$$

The solution to this optimization problem is given by

$$\begin{aligned}\mathbf{w}^T[i] &= \tilde{\mathbf{b}}^T[i]\left(\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\boldsymbol{\mathcal{H}}[i] + \mathbf{R}_A^{-1}[i]\right)^{-1}\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i] \\ \mathbf{b}[i] &= \mathrm{eig}_{\max}(\tilde{\mathbf{R}}^\perp[i])\end{aligned}$$

where $\tilde{\mathbf{R}}^\perp[i]$ is given by

$$\begin{aligned}\tilde{\mathbf{R}}^\perp[i] &= \mathbf{C}^T\left(\boldsymbol{\mathcal{H}}^H[i]\mathbf{R}_{\tilde{B}}^{-1}[i]\boldsymbol{\mathcal{H}}[i] + \mathbf{R}_A^{-1}[i]\right)^{-1} \\ &\qquad \times \boldsymbol{\mathcal{H}}^H\mathbf{R}_{\tilde{B}}^{-1}[i]\boldsymbol{\mathcal{H}}\mathbf{R}_A[i]\mathbf{C}.\end{aligned}$$

Note that $\mathrm{eig}_{\min}(\mathbf{A})$ ($\mathrm{eig}_{\max}(\mathbf{A})$) is the eigenvector corresponding to the minimum (maximum) eigenvalue of the matrix $\mathbf{A}$.

[1]Let $\mathbf{A}$ be the $pN \times qN$ matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \cdots & \mathbf{A}_{1q} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{p1} & \cdots & \mathbf{A}_{pq} \end{bmatrix}$$

where $\mathbf{A}_{ij}$ is the (i,j)th $N \times N$ submatrix of $\mathbf{A}$. The $p \times q$ matrix $\mathrm{subtr}\{\mathbf{A}\}$ is then given by

$$\mathrm{subtr}\{\mathbf{A}\} = \begin{bmatrix} \mathrm{tr}\{\mathbf{A}_{11}\} & \cdots & \mathrm{tr}\{\mathbf{A}_{1q}\} \\ \vdots & \ddots & \vdots \\ \mathrm{tr}\{\mathbf{A}_{p1}\} & \cdots & \mathrm{tr}\{\mathbf{A}_{pq}\} \end{bmatrix}.$$

In conjunction with the devised TEQ, a one-tap FEQ applied to the filtered received sequence in the frequency domain is still necessary to fully recover the transmitted QAM symbols. Define $\hat{S}_k[i]$ as the estimate of the transmitted QAM symbol on the kth subcarrier of the ith OFDM symbol. This estimate is then obtained by applying a 1-tap FEQ to the TEQ output after the discrete Fourier transform (DFT) demodulation

$$\hat{S}_k[i] = \frac{1}{d_k[i]} \mathcal{F}^{(k)} \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \mathbf{D}_{q'}[i] \mathbf{W}_{q'}^{(r)}[i] \mathbf{y}^{(r)}[i] \quad (17)$$

where $\mathcal{F}^{(k)}$ is the $(k+1)$st row of the unitary fast Fourier transform (FFT) matrix $\boldsymbol{\mathcal{F}}, \mathbf{W}_{q'}^{(r)}[i]$ is an $N \times (N+L')$ Toeplitz matrix, with first column $[w_{q',L'}^{(r)}[i], \mathbf{0}_{1\times(N-1)}]^T$ and first row $[w_{q',L'}^{(r)}[i], \ldots, w_{q',0}^{(r)}[i], \mathbf{0}_{1\times(N-1)}], d_k[i]$ is the frequency response of the TIR on the kth subcarrier of the ith OFDM block ($1/d_k[i]$ represents the 1-tap FEQ), and $\mathbf{y}^{(r)}[i] = [y^{(r)}[i(N+\nu)+\nu+d-L'], \ldots, y^{(r)}[(i+1)(N+\nu)+d-1]]^T$.

The existence of a perfect shortening TEQ (a TEQ that completely eliminates IBI/ICI in the noiseless case) requires that $\boldsymbol{\mathcal{H}}$ is of full column rank. A necessary condition for $\boldsymbol{\mathcal{H}}$ to have full column rank is that $Nr(Q'+1)(L'+1) \geq (Q+Q'+1)(L+L'+1)$. For sufficiently large Q' and L', it is clear that we need at least two receive antennas, i.e., $N_r \geq 2$. This justifies our assumption of SIMO systems. Conditions like column reduced property and irreducibility can be deduced from the MIMO time-invariant FIR case. These are discussed in more detail in [16].

IV. Frequency-Domain Per-Tone Equalization

In Section III, a time-domain equalizer is proposed to combat the effect of the propagation channel. The purpose of the proposed TEQ is to convert the doubly selective channel into a purely frequency-selective channel. The proposed TEQ optimizes the performance on all subcarriers in a joint fashion. An optimal frequency-domain PTEQ can be obtained by transferring the TEQ operations to the frequency domain. Hence, the estimate of the transmitted QAM symbol on the kth subcarrier in the ith OFDM block is then obtained as

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \mathcal{F}^{(k)} \mathbf{D}_{q'}[i] \mathbf{Y}^{(r)}[i] \mathbf{w}_{q'}^{(r)}[i] / d_k[i] \quad (18a)$$

$$= \sum_{r=1}^{N_r} \sum_{q'=-Q'/2}^{Q'/2} \mathcal{F}^{(k)} \underbrace{\mathbf{D}_{q'}[i] \mathbf{Y}^{(r)}[i] \hat{\mathbf{D}}_{q'}^*}_{\tilde{\mathbf{Y}}_{q'}^{(r)}[i]} \underbrace{\hat{\mathbf{D}}_{q'} \mathbf{w}_{q'}^{(r)}[i] / d_k[i]}_{\tilde{\mathbf{w}}_{q'}^{(r,k)}[i]} \quad (18b)$$

where $\mathbf{Y}^{(r)}[i]$ is an $N \times (L'+1)$ Toeplitz matrix, with first column $[y^{(r)}[i(N+\nu)+\nu+d], \ldots, y^{(r)}[(i+1)(N+\nu)+d-1]]^T$ and first row $[y^{(r)}[i(N+\nu)+\nu+d], \ldots, y^{(r)}[i(N+\nu)+\nu+d-L']], \mathbf{w}_{q'}^{(r)}[i] = [w_{q',0}^{(r)}[i], \ldots, w_{q',L'}^{(r)}[i]]^T$, and $\hat{\mathbf{D}}_{q'} = \text{diag}\{[1, \ldots, e^{j2\pi q' L'/K}]^T\}$. Note that the right multiplication of $\mathbf{Y}^{(r)}[i]$ with the diagonal matrix $\hat{\mathbf{D}}_{q'}$ in (18b) is done here to restore the Toeplitz structure in $\mathbf{Y}_{q'}^{(r)}[i] = \mathbf{D}_{q'} \mathbf{Y}^{(r)}[i]$, which will simplify the analysis and implementation as will be clear later. From (18b), we can see that each subcarrier has its own $(L'+1)$-tap FEQ. This allows us to optimize the equalizer coefficients $\tilde{\mathbf{w}}_{q'}^{(r,k)}[i]$ for each subcarrier k separately, without taking into account the specific relation that existed originally between $\tilde{\mathbf{w}}_{q'}^{(r,k)}[i], \mathbf{w}_{q'}^{(r,k)}[i]$, and $d_k[i]$.

Defining $\tilde{\mathbf{Y}}^{(r)}[i] = [\tilde{\mathbf{Y}}_{-Q'/2}^{(r)}[i], \ldots, \tilde{\mathbf{Y}}_{Q'/2}^{(r)}[i]]$ and $\tilde{\mathbf{w}}^{(r,k)}[i] = [\tilde{\mathbf{w}}_{-Q'/2}^{(r,k)T}[i], \ldots, \tilde{\mathbf{w}}_{Q'/2}^{(r,k)T}[i]]^T$, (18b) reduces to

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \mathcal{F}^{(k)} \tilde{\mathbf{Y}}^{(r)}[i] \tilde{\mathbf{w}}^{(r,k)}[i]. \quad (19)$$

Transferring the TEQ operation to the frequency domain by interchanging the TEQ with the DFT in (19), we obtain

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \tilde{\mathbf{w}}^{(r,k)T}[i] \mathbf{F}^{(k)}[i] \mathbf{y}^{(r)}[i] \quad (20)$$

where $\mathbf{F}^{(k)}[i] = (\mathbf{I}_{Q'+1} \otimes \tilde{\boldsymbol{\mathcal{F}}}^{(k)})[\tilde{\mathbf{D}}_{-Q'/2}^T[i], \ldots, \tilde{\mathbf{D}}_{Q'/2}^T[i]]^T$, and $\tilde{\boldsymbol{\mathcal{F}}}^{(k)}$ is given by

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)} = \begin{bmatrix} 0 & \cdots & 0 & \boxed{\mathcal{F}^{(k)}} \\ \vdots & 0 & \boxed{\mathcal{F}^{(k)}} & 0 \\ 0 & \iddots & \iddots \; 0 & \vdots \\ \boxed{\mathcal{F}^{(k)}} & 0 & \cdots & 0 \end{bmatrix},$$

which corresponds to a sliding DFT operation, which will be implemented as a sliding FFT.

To compute (20), we require $(Q'+1)$ sliding FFTs per receive antenna. Each sliding FFT is applied to a modulated version of the sequence received on a particular antenna. The q'th sliding FFT on the rth receive antenna is shown in Fig. 4. To estimate the transmitted QAM symbol on the kth subcarrier, we then have to combine the outputs of the $N_r(Q'+1)$ PTEQs corresponding to the kth subcarrier. This results in a complexity of $(Q'+1)(L'+1)$ multiply–add (MA) operations per receive antenna per subcarrier, i.e., $N_r N(Q'+1)(L'+1)$ MA operations for a block of N symbols. In Section V, we show how we can further reduce the complexity of the proposed PTEQ by replacing the $Q'+1$ sliding FFTs by only a few sliding FFTs, the number of which is entirely independent of Q' but rather depends on the BEM frequency resolution K. The removed sliding FFTs are compensated for by combining the PTEQ outputs of neighboring subcarriers on the remaining sliding FFTs.

In the following, we will show how the PTEQ coefficients can be computed in order to minimize the MSE. Defining $\tilde{\mathbf{w}}^{(k)}[i] = [\tilde{\mathbf{w}}^{(1,k)T}[i], \ldots, \tilde{\mathbf{w}}^{(N_r,k)T}[i]]^T$ and $\mathbf{y}[i] = [\mathbf{y}^{(1)T}[i], \ldots, \mathbf{y}^{(N_r)T}[i]]^T$, (20) can be written as

$$\hat{S}_k[i] = \tilde{\mathbf{w}}^{(k)T}[i] \left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i] \right) \mathbf{y}[i]. \quad (21)$$

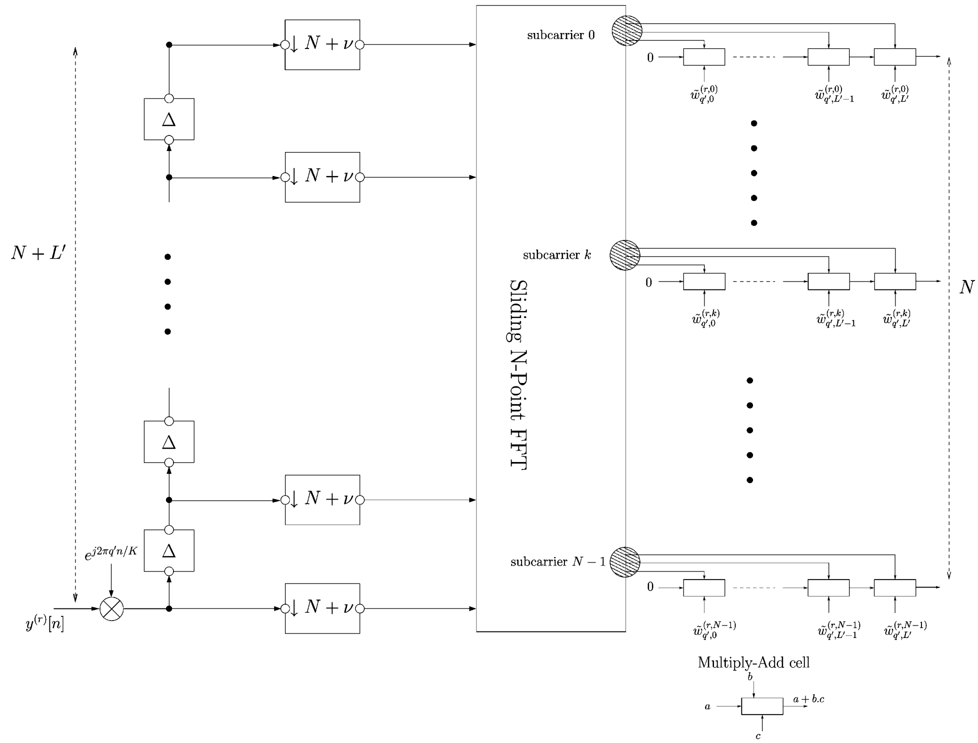

Fig. 4. Sliding FFT of the q'th modulated version of the received sequence $y^{(r)}[n]$.

At this point, we may introduce a model for the received sequence on the rth receive antenna $\mathbf{y}^{(r)}[i]$ as

$$\mathbf{y}^{(r)}[i] = \underbrace{\sum_{q=-Q/2}^{Q/2} \tilde{\mathbf{D}}_q[i]\left[\boldsymbol{O}_1, \mathbf{H}_q^{(r)}[i], \boldsymbol{O}_2\right](\mathbf{I}_3 \otimes \mathbf{P})(\mathbf{I}_3 \otimes \boldsymbol{\mathcal{F}}^H)}_{\mathbf{G}^{(r)}[i]} \times \underbrace{\begin{bmatrix} \mathbf{s}[i-1] \\ \mathbf{s}[i] \\ \mathbf{s}[i+1] \end{bmatrix}}_{\tilde{\mathbf{s}}} + \boldsymbol{\eta}^{(r)}[i] \quad (22)$$

where $\boldsymbol{O}_1 = \mathbf{0}_{(N+L')\times(N+2\nu+d-L-L')}$, $\boldsymbol{O}_2 = \mathbf{0}_{(N+L')\times(N+\nu-d)}$, $\mathbf{H}_q^{(r)}[i]$ is an $(N+L')\times(N+L'+L)$ Toeplitz matrix with first column $[h_{q,L}^{(r)}[i], \mathbf{0}_{1\times(N+L'-1)}]^T$ and first row $[h_{q,L}^{(r)}[i], \ldots, h_{q,0}^{(r)}[i], \mathbf{0}_{1\times(N+L'-L-1)}]$, and $\mathbf{P}$ is the CP insertion matrix given by

$$\mathbf{P} = \begin{bmatrix} \mathbf{0}_{\nu\times(N-\nu)} & \mathbf{I}_\nu \\ \mathbf{I}_N & \end{bmatrix}.$$

To obtain the PTEQ coefficients for the kth subcarrier, we define the following MSE cost function

$$\mathcal{J}[i] = \mathcal{E}\left\{\left\| S_k[i] - \tilde{\mathbf{w}}^{(k)T}[i]\left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right)\mathbf{y}[i]\right\|^2\right\}.$$

Hence, the minimum MSE (MMSE) PTEQ coefficients for the kth subcarrier are given by

$$\tilde{\mathbf{w}}_{\mathrm{MMSE}}^{(k)}[i] = \arg\min_{\tilde{\mathbf{w}}^{(k)}[i]} \mathcal{J}[i]. \quad (23)$$

The solution of (23) is obtained by solving $\partial\mathcal{J}[i]/\partial\tilde{\mathbf{w}}^{(k)}[i] = \mathbf{0}$, which reduces to

$$\begin{aligned}\tilde{\mathbf{w}}_{\mathrm{MMSE}}^{(k)T}[i] = &\left(\left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right)\left(\mathbf{G}[i]\mathbf{R}_{\tilde{s}}\mathbf{G}^H[i] + \mathbf{R}_\eta\right)\right. \\ &\left.\times\left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)H}[i]\right)\right)^{-1} \\ &\times\left(\mathbf{I}_{N_r} \otimes \mathbf{F}^{(k)}[i]\right)\mathbf{G}[i]\mathbf{R}_{\tilde{s}}\mathbf{e}^{(k)} \end{aligned} \quad (24)$$

where $\mathbf{G}[i] = [\mathbf{G}^{(1)T}[i], \ldots, \mathbf{G}^{(N_r)T}[i]]^T$ and $\mathbf{e}^{(k)}$ is the unit vector with a 1 in the position $N+k+1$.

Note that, in contrast to the time-domain approach, where the BEM resolution of the channel model and the BEM resolution of the TV-FIR TEQ are assumed to be equal, the BEM resolution of the channel model and the BEM resolution of the PTEQ can be different.

V. Efficient Implementation of the PTEQ

In Section IV, we have shown that to implement the proposed PTEQ, we basically require $(Q'+1)$ sliding FFTs. In this section, we show how we can lower the complexity of the proposed PTEQ by further exploiting the special structure of $\tilde{\mathbf{Y}}_{q'}^{(r)}[i]$ (see (18b)). Our complexity reduction will proceed in two steps.

Step 1: In general, the BEM frequency resolution K is greater than or equal to the DFT size N. We will assume that K is an integer multiple of the FFT size, i.e., $K = PN$, where P is an integer greater than or equal to 1 ($P \in \mathbb{Z}^+$). We start by defining $\mathbb{Q} = \{-Q'/2, \ldots, Q'/2\}$, and

$\mathbb{Q}_p = \{q \in \mathbb{Q} | q \bmod P = p\}$. Based on these definitions, (18b) and (19) can be written as

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \mathcal{F}^{(k-l_p)} \underbrace{\mathbf{D}_p[i]\mathbf{Y}^{(r)}[i]\hat{\mathbf{D}}_p^*}_{\tilde{\mathbf{Y}}_p^{(r)}[i]} \underbrace{\hat{\mathbf{D}}_p \mathbf{w}_{p,l_p}^{(r,k)}[i]/d_k[i]}_{\bar{\mathbf{w}}_{p,l_p}^{(r,k)}[i]} \tag{25a}$$

$$= \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \mathcal{F}^{(k-l_p)} \tilde{\mathbf{Y}}_p^{(r)}[i] \bar{\mathbf{w}}_{p,l_p}^{(r,k)}[i] \tag{25b}$$

where $l_p = ((q_p - p)/P)$, and $\mathbf{w}_{p,l_p}^{(r,k)}[i] = \mathbf{w}_{q_p}^{(r,k)}[i]$. Note that (25b) splits the $Q'+1$ different terms of (19) into P different groups, with the pth group containing $|\mathbb{Q}_p|$ terms, where $|\mathbb{Q}_p|$ denotes the cardinality of the set $\mathbb{Q}_p$ for $p = 0, \ldots, P-1$. Transferring the TEQ operation to the frequency domain, we obtain

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \sum_{q_p \in \mathbb{Q}_p} \bar{\mathbf{w}}_{p,l_p}^{(r,k)T}[i] \tilde{\boldsymbol{\mathcal{F}}}^{(k-l_p)} \underbrace{\tilde{\mathbf{D}}_p[i]\mathbf{y}^{(r)}[i]}_{\tilde{\mathbf{y}}_p^{(r)}[i]}. \tag{26}$$

Note that $\tilde{\mathbf{y}}_p^{(r)}[i] = [\tilde{y}_p^{(r)}[i(N+\nu)+\nu+d-L'], \ldots, \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-1]]^T$ with $\tilde{y}_p^{(r)}[n] = e^{j2\pi pn/K} y^{(r)}[n]$, which is the pth modulated version of the received sequence.

Defining $\bar{\mathbf{w}}_p^{(r,k)}[i] = [\ldots, \bar{\mathbf{w}}_{p,-1}^{(r,k)T}[i], \bar{\mathbf{w}}_{p,0}^{(r,k)T}[i], \bar{\mathbf{w}}_{p,1}^{(r,k)T}[i], \ldots]^T$, (26) can now be written as

$$\hat{S}_k[i] = \sum_{r=1}^{N_r} \sum_{p=0}^{P-1} \bar{\mathbf{w}}_p^{(r,k)T}[i] \tilde{\mathbf{F}}_p^{(k)} \tilde{\mathbf{y}}_p^{(r)}[i] \tag{27}$$

where $\tilde{\mathbf{F}}_p^{(k)} = [\ldots, \tilde{\boldsymbol{\mathcal{F}}}^{(k-1)T}, \tilde{\boldsymbol{\mathcal{F}}}^{(k)T}, \tilde{\boldsymbol{\mathcal{F}}}^{(k+1)T}, \ldots]^T$. Let us now define $\tilde{\mathbf{F}}^{(k)} = \mathrm{diag}\{\tilde{\mathbf{F}}_0^{(k)}, \ldots, \tilde{\mathbf{F}}_{P-1}^{(k)}\}$, $\tilde{\mathbf{y}}^{(r)}[i] = [\tilde{\mathbf{y}}_0^{(r)T}[i], \ldots, \tilde{\mathbf{y}}_{P-1}^{(r)T}[i]]^T$, and $\tilde{\mathbf{y}}[i] = [\tilde{\mathbf{y}}^{(1)T}[i], \ldots, \tilde{\mathbf{y}}^{(N_r)T}[i]]^T$. Further defining $\bar{\mathbf{w}}^{(r,k)}[i] = [\bar{\mathbf{w}}_0^{(r,k)T}[i], \ldots, \bar{\mathbf{w}}_{P-1}^{(r,k)T}[i]]^T$ and $\bar{\mathbf{w}}^{(k)}[i] = [\bar{\mathbf{w}}^{(1,k)T}[i], \ldots, \bar{\mathbf{w}}^{(N_r,k)T}[i]]^T$, (27) can finally be written as

$$\hat{S}_k[i] = \bar{\mathbf{w}}^{(k)T}[i] \left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}\right) \tilde{\mathbf{y}}[i]. \tag{28}$$

To implement (27), we require P sliding FFTs per receive antenna rather than $Q'+1$ sliding FFTs per receive antenna as in Section IV (in practice and in our simulations $P \ll Q'+1$). Each sliding FFT is applied to a modulated version of the received sequence. This reduction in the number of sliding FFTs per receive antenna is compensated for by combining $|\mathbb{Q}_p|$ neighboring subcarriers on the pth sliding FFT. Notice here that apart from the reduction in the number of sliding FFTs, the implementation complexity remains the same as in Section IV, i.e., $N_r N(Q'+1)(L'+1)$ MA operations for a block of N symbols.

Similar to (23), the MMSE solution can be obtained by minimizing the following cost function:

$$\mathcal{J}[i] = \mathcal{E}\left\{ \left\| S_k[i] - \bar{\mathbf{w}}^{(k)T}[i] \left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}\right) \tilde{\mathbf{y}}[i] \right\|^2 \right\}. \tag{29}$$

The solution of (29) is

$$\begin{aligned}\bar{\mathbf{w}}_{\mathrm{MMSE}}^{(k)T}[i] = &\left(\left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}[i]\right)(\mathbf{G}[i]\mathbf{R}_{\tilde{s}}\mathbf{G}^H[i] + \mathbf{R}_\eta)\right.\\ &\left.\times \left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)H}[i]\right)\right)^{-1}\\ &\times \left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}[i]\right) \mathbf{G}[i]\mathbf{R}_{\tilde{s}}\mathbf{e}^{(k)}\end{aligned} \tag{30}$$

which is equivalent to the one obtained in (24).

Step 2: We can further simplify the computational complexity associated with the proposed PTEQ by replacing each sliding FFT by only one full FFT and L' difference terms that are common to all subcarriers similar to the procedure in [26]. To explain this, we will consider only one sliding FFT. Let us consider the kth subcarrier of the pth sliding FFT, i.e., $\tilde{\boldsymbol{\mathcal{F}}}^{(k)} \tilde{\mathbf{y}}_p^{(r)}[i]$. Define $\tilde{Y}_p^{(r,k)} = \mathcal{F}^{(k)}[\tilde{y}_p^{(r)}[i(N+\nu)+\nu+d], \ldots, \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-1]]^T$ as the frequency response on the kth subcarrier of the pth modulated version of the received sequence on the rth receive antenna. It can then be shown that

$$\tilde{\boldsymbol{\mathcal{F}}}^{(k)} \tilde{\mathbf{y}}_p^{(r)}[i] = \mathbf{T}^{(k)} \begin{bmatrix} \tilde{Y}_p^{(r,k)} \\ \Delta\tilde{\mathbf{y}}_p^{(r)}[i] \end{bmatrix} \begin{matrix} \updownarrow 1 \times 1 \\ \updownarrow L' \times 1 \end{matrix} \tag{31}$$

where $\mathbf{T}^{(k)}$ is an $(L'+1) \times (L'+1)$ lower triangular Toeplitz matrix given by

$$\mathbf{T}^{(k)} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \beta^k & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \beta^{kL'} & \cdots & \beta^k & 1 \end{bmatrix} \tag{32}$$

with $\beta = e^{-j2\pi/N}$. The difference terms $\Delta\tilde{\mathbf{y}}_p^{(r)}[i]$ are given by the equation shown at the bottom of the page. In a similar fashion, we can obtain an expression for the neighboring subcarriers on the same sliding FFT by replacing the subcarrier index. The symbol estimate (28) can then be written as follows. We first define $\mathbf{u}_{p,l_p}^{(r,k)T}[i] = \bar{\mathbf{w}}_{p,l_p}^{(r,k)T}[i]\mathbf{T}^{(k+l_p)}$ and also define the following $|\mathbb{Q}_p|(L'+1) \times (|\mathbb{Q}_p| + L')$ selection matrix

$$\mathbf{S}_p = \tilde{\mathbf{I}} \otimes \begin{bmatrix} 1 \\ \mathbf{0}_{L' \times 1} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{1 \times (|\mathbb{Q}_p| + L')} \\ \tilde{\tilde{\mathbf{I}}} \end{bmatrix} \otimes \mathbf{1}_{|\mathbb{Q}_p|}$$

where $\tilde{\mathbf{I}}$ is the first $|\mathbb{Q}_p|$ rows of the matrix $\mathbf{I}_{|\mathbb{Q}_p|+L'}$, and $\tilde{\tilde{\mathbf{I}}}$ is the last L' rows of the matrix $\mathbf{I}_{|\mathbb{Q}_p|+L'}$. Introducing

$$\Delta\tilde{\mathbf{y}}_p^{(r)}[i] = \begin{bmatrix} \tilde{y}_p^{(r)}[i(N+\nu)+\nu+d-1] - \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-1] \\ \vdots \\ \tilde{y}_p^{(r)}[i(N+\nu)+\nu+d-L'] - \tilde{y}_p^{(r)}[(i+1)(N+\nu)+d-L'-1] \end{bmatrix}.$$

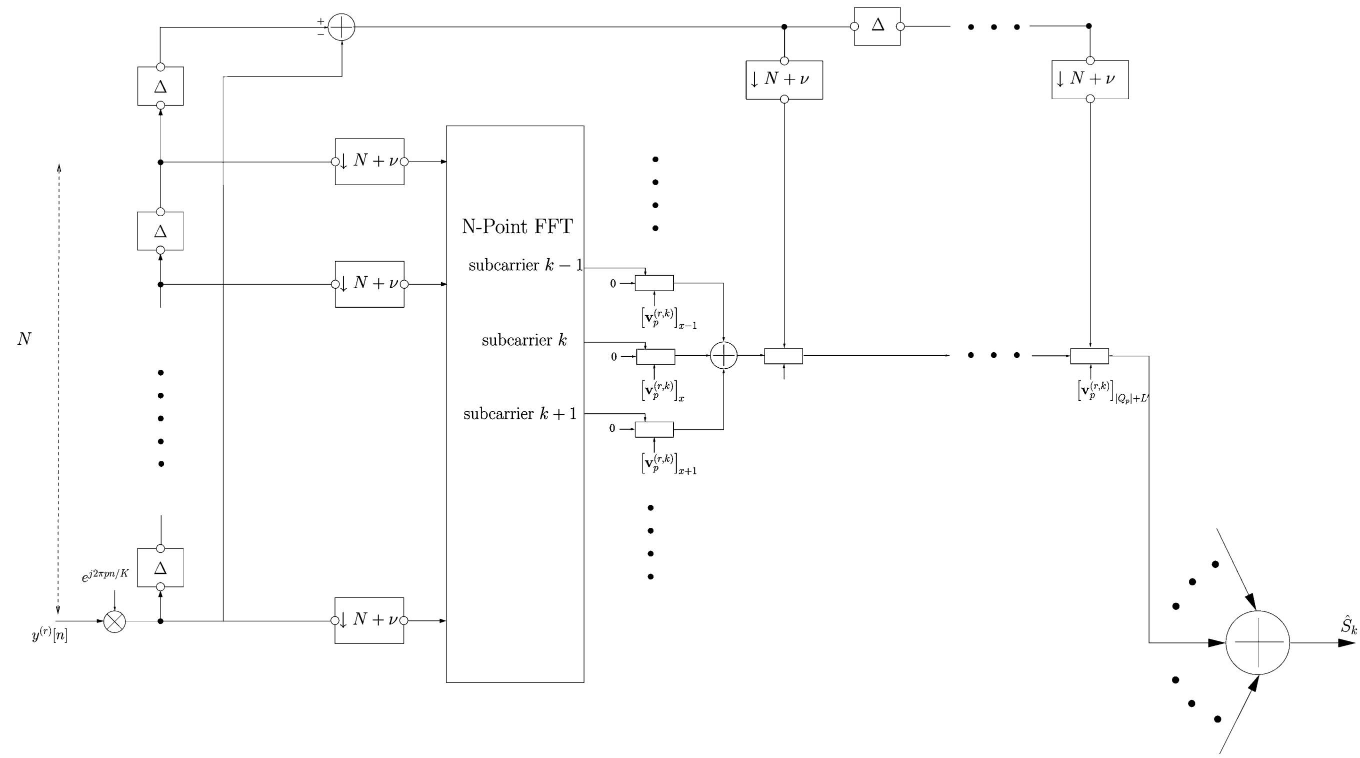

Fig. 5. Low-complexity PTEQ on the pth branch of the rth receive antenna.

$\mathbf{u}_p^{(r,k)T}[i] = [\ldots, \mathbf{u}_{p,-1}^{(r,k)T}[i], \mathbf{u}_{p,0}^{(r,k)T}[i], \mathbf{u}_{p,1}^{(r,k)T}[i], \ldots]^T$ and $\mathbf{v}_p^{(r,k)T}[i] = \mathbf{u}_p^{(r,k)T}[i]\mathbf{S}_p$, (28) can then be written as

$$\hat{S}_k[i] = \sum_{r=1}^{N_r}\sum_{p=0}^{P-1} \mathbf{v}_p^{(r,k)T}[i] \left[\begin{array}{c} \vdots \\ \tilde{Y}_p^{(r,k-1)}[i] \\ \tilde{Y}_p^{(r,k)}[i] \\ \tilde{Y}_p^{(r,k+1)}[i] \\ \vdots \\ \hline \Delta\tilde{\mathbf{y}}_p^{(r)}[i] \end{array}\right] \begin{array}{l} \updownarrow |\mathbb{Q}_p| \times 1 \\ \\ \updownarrow L' \times 1 \end{array}$$

$$= \sum_{r=1}^{N_r}\sum_{p=0}^{P-1} \mathbf{v}_p^{(r,k)T}[i] \times \underbrace{\left[\begin{array}{ccc} \vdots & \vdots & \\ \mathbf{0}_{1\times L'} & \mathcal{F}^{(k-1)} & \\ \mathbf{0}_{1\times L'} & \mathcal{F}^{(k)} & \\ \mathbf{0}_{1\times L'} & \mathcal{F}^{(k+1)} & \\ \vdots & \vdots & \\ \hline \bar{\mathbf{I}}_{L'} & \mathbf{0}_{L'\times(N-L')} & -\bar{\mathbf{I}}_{L'} \end{array}\right]}_{\tilde{\mathbf{F}}_p^{(k)}} \tilde{\mathbf{y}}_p^{(r)}[i] \quad (33)$$

where $\bar{\mathbf{I}}_{L'}$ is the anti-diagonal identity matrix of size $L' \times L'$. Defining $\mathbf{v}^{(r,k)}[i] = [\mathbf{v}_0^{(r,k)T}[i], \ldots, \mathbf{v}_{P-1}^{(r,k)T}[i]]^T, \mathbf{v}^{(k)}[i] = [\mathbf{v}^{(1,k)T}[i], \ldots, \mathbf{v}^{(N_r,k)T}[i]]^T$ and $\tilde{\mathbf{F}}^{(k)} = \mathrm{diag}\{\tilde{\mathbf{F}}_0^{(k)}, \ldots, \tilde{\mathbf{F}}_{P-1}^{(k)}\}$, (33) can finally be written as

$$\hat{S}_k = \mathbf{v}^{(k)T}[i]\left(\mathbf{I}_{N_r} \otimes \tilde{\mathbf{F}}^{(k)}\right)\tilde{\mathbf{y}}[i]. \quad (34)$$

TABLE I
IMPLEMENTATION COMPLEXITY COMPARISON

	MA/tone/rx	FFT/rx
TEQ	$(Q'+1)(L'+1)+1$	1
PTEQ (Figure 4)	$(Q'+1)(L'+1)$	$(Q'+1)$ sliding FFTs
PTEQ (Figure 5)	$P(L'+1)+Q'+1$	P FFTs

Note that, due to the fact that the difference terms are common to all subcarriers in a particular sliding FFT, the implementation complexity is $P(L'+1)+Q'+1$ MA operations per receive antenna per subcarrier, compared with $(Q'+1)(L'+1)$ per receive antenna per subcarrier in Section IV. In Fig. 5, we show how (34) can be realized for the pth sliding FFT on the rth receive antenna. Note that replacing the sliding FFT with one full DFT and L' difference terms in Section IV will not reduce the implementation complexity. This is due to the fact that we only consider a single-subcarrier output for each sliding FFT to estimate a particular symbol. The implementation complexity of the TEQ and the different configurations of the PTEQ is summarized in Table I.[2] On the other hand, the design complexity of the PTEQ is higher than the design complexity of the TEQ. We can easily show that the design complexity of the TEQ requires $\mathcal{O}((Q+Q'+1)^3(L+L'+1)^3)$ MA operations, while it requires $\mathcal{O}((Q'+1)(L'+1)N)$ MA operations per subcarrier to design the PTEQs. The design complexity of the TEQ is mainly due to a matrix inversion of size $(Q+Q'+1)(L+L'+1)\times(Q+Q'+1)(L+L'+1)$. The complexity associated with computing the max (min) eigenvector of an $(L''+1)\times(L''+1)$ matrix,

[2]These figures do not take into account the BEM channel coefficients estimation/computation.

which requires $\mathcal{O}((L''+1)^2)$ MA operations [27], is negligible compared with the above matrix inversion.

We finally note that our approach unifies and extends many existing frequency-domain approaches, for the case of TI as well as TV channels as follows.

1) TI channels ($Q=0$, and hence $Q'=0$):
   a) $\nu \geq L$, and $L'=0$: the proposed PTEQ comes down to the 1-tap MMSE FEQ as in [28];
   b) $\nu < L$, and $L' \neq 0$: the proposed PTEQ comes down to the per-tone equalizer proposed in [14] for DMT-based transmission (e.g., for DSL modems).
2) TV channels ($Q' \neq 0$):
   a) $\nu \geq L, L'=0$, and $P=1$: the proposed PTEQ comes down to the FEQ proposed in [4];
   b) $\nu \geq L, L'=0$, and $P \geq 1$: the proposed PTEQ comes down to the FEQ proposed in [29].

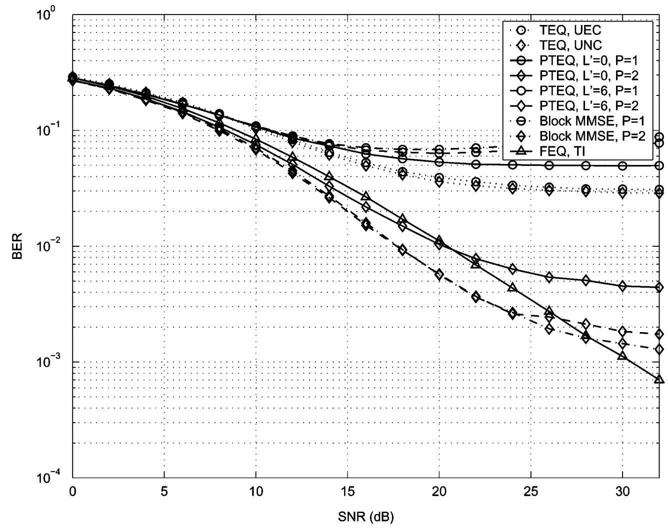

Fig. 6. BER versus SNR for TEQ and PTEQ, $N_r=1$ receive antenna.

VI. Simulations

In this section, we show some simulation results for the proposed IBI/ICI mitigation techniques. We consider a SISO system as well as a SIMO system with $N_r=2$ receive antennas. The channel is assumed to be doubly selective of order $L=6$ with a maximum Doppler frequency of $f_{\max}=100$ Hz (corresponds to a speed of 120 km/h on the GSM band of 900 MHz). The channel taps are simulated as independent identically distributed (i.i.d.) random variables, correlated in time with a correlation function according to Jakes' model $\mathcal{E}\{h^{(r)}[n_1;l_1]h^{(r')*}[n_2;l_2]\} = \sigma_h^2 J_0(2\pi f_{\max}T(n_1-n_2))\delta[l_1-l_2]\delta[r-r']$, where J_0 is the zeroth-order Bessel function of the first kind and σ_h^2 denotes the variance of the channel. We consider an OFDM transmission with $N=128$ subcarriers. Quadrature-phase-shift-keying (QPSK) signaling is assumed. The sampling time is $T=50\,\mu$s, which corresponds to a data rate of 40 kb/s, which is suitable for applications like mobile multimedia (M^3). The normalized Doppler frequency is then obtained as $f_{\max}T=0.005$. We define the SNR as $\mathrm{SNR}=\sigma_h^2(L+1)E_s/\sigma_\eta^2$, where E_s is the QPSK symbol power. The decision delay d is always chosen as $d=\lfloor (L+L')/2 \rfloor + 1$.

We use a BEM to approximate the channel. We assume that the BEM coefficients are known at the receiver (obtained through an LS fit of the true channel in the noiseless case). The BEM coefficients of the approximated channel are used to design the time-domain and the frequency-domain per-tone equalizers. These equalizers, however, are used to equalize the true channel (Jakes' model). The BEM resolution is determined by $K=PN$ with P is chosen as $P=1,2$. The number of TV basis functions of the channel is chosen such that $Q/(2KT) \geq f_{\max}$ is satisfied, which results in $Q=2$ for $P=1$, and $Q=4$ for $P=2$.

- First, we consider a SISO system, where the channel impulse response fits within the CP, i.e., $\nu = L$. Hence, the total OFDM symbol duration is 6.7 ms. We measure the performance in terms of the BER versus SNR. We consider a TEQ with $Q'=14, L'=14$, and $P=2$. We consider the unit-energy constraint (UEC), and the unit-norm constraint (UNC). For the PTEQ, we use different scenarios. More specifically, we consider a PTEQ resulting from the following:
  — a purely time-selective TEQ with $Q'=10$ and $L'=0$, and $P=1$ as in [15];
  — a purely time-selective TEQ with $Q'=10$ and $L'=0$, and $P=2$ as in [15];
  — a doubly selective TEQ with $Q'=10$ and $L'=6$, and $P=1$;
  — a doubly selective TEQ with $Q'=10$ and $L'=6$, and $P=2$.

  As a benchmark, we consider the case of OFDM transmission over purely frequency-selective (TI) channels where the equalizer is the conventional 1-tap MMSE FEQ, as well as OFDM transmission over doubly selective (TV) channels with a block MMSE equalizer. The block MMSE equalizer for OFDM used here is similar to the block MMSE proposed in [16] designed for SC transmission with CP. As shown in Fig. 6, the performance of the TEQ suffers from an early error floor for both UEC and UNC. The PTEQ exhibits a similar performance when $P=1$ for both $L'=0$ and $L'=6$. However, the performance of the PTEQ is significantly improved when $P=2$. For $P=2$ with $L'=0$, we see that the PTEQ slightly outperforms the 1-tap MMSE equalizer for OFDM over TI channels for low SNR ($\mathrm{SNR} \leq 20$ dB), and it experiences a 3-dB loss in SNR compared with the block MMSE for OFDM over TV channels. On the other hand, when $P=2$ with $L'=6$, the PTEQ outperforms the conventional 1-tap MMSE FEQ of OFDM over TI channels, with an SNR gain of 2 dB at $\mathrm{BER}=10^{-2}$ and coincides with the performance of the block MMSE equalizer for OFDM over TV channels.
- Second, we consider a SIMO system with $N_r=2$ receive antennas. We consider the case where the cyclic prefix ν is shorter than the channel order, ν is chosen to be $\nu=3$. The OFDM symbol duration is then 6.55 ms. We consider a TEQ with $Q'=8$ and $L'=8$. A PTEQ is then obtained by transferring this TEQ to the frequency domain.

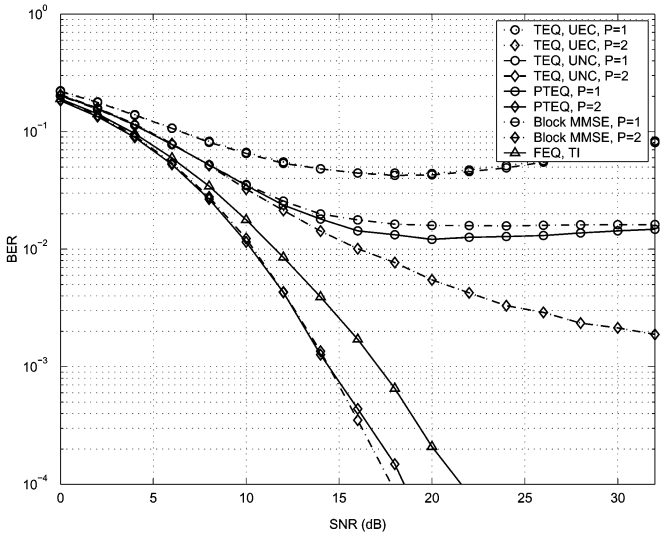

Fig. 7. BER versus SNR for TEQ and PTEQ, $N_r = 2$ receive antennas.

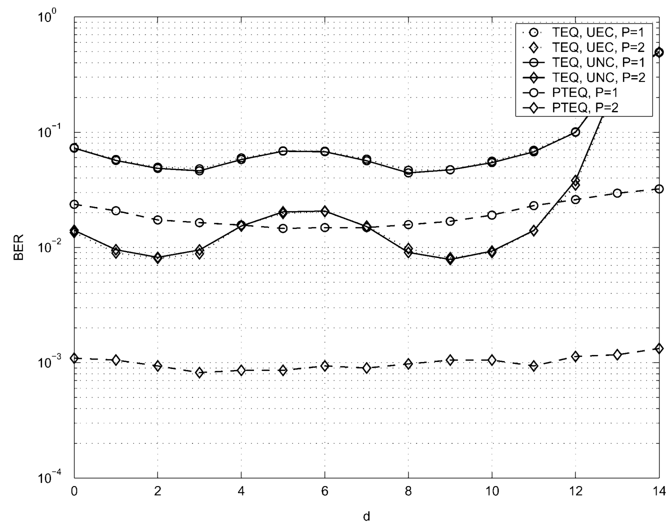

Fig. 8. BER versus decision delay d for TEQ and PTEQ, $N_r = 2$ receive antennas at SNR = 15 dB.

We consider the case when $P = 1$ and $P = 2$. As shown in Fig. 7, both the TEQ and the PTEQ suffers from an early error floor when $P = 1$, where the first exhibits an error floor at BER $= 4 \times 10^{-2}$ and SNR $= 20$ dB and the latter at BER $= 10^{-2}$ and SNR $= 20$ dB. The performance is significantly improved when $P = 2$ for both the TEQ and the PTEQ. The PTEQ significantly outperforms the TEQ, where we can see a 6-dB gain in SNR for the PTEQ over the TEQ at BER $= 10^{-2}$. The TEQ experiences a 4-dB loss in SNR compared with the 1-tap MMSE FEQ for OFDM over TI channels, and 6-dB loss in SNR compared with the PTEQ which coincides with the block MMSE for OFDM over TV channels.

- Third, we examine the effect of the decision delay d on the BER performance for the TEQ considering the UEC and UNC, and the PTEQ. We consider again the cases $P = 1$ and $P = 2$ for a SIMO system with $N_r = 2$ receive antennas and the same equalizer parameters as before. We examine the performance at SNR = 15 dB. As shown in Fig. 8, the performance of the PTEQ approach is a much smoother function of the synchronization delay than the performance of the TEQ approach. Hence, for the PTEQ approach the synchronization delay setting is less critical than for the TEQ approach.
- In our setup so far, we use the BEM coefficients (i.e., the approximated channel) to design the PTEQ. Here, we consider the performance of the PTEQ when the true channel is used for equalizer designs as well as evaluations. The channel is assumed to be a doubly selective channel with maximum Doppler frequency $f_{\max} = 100$ Hz, and sampling time $T = 50$ μs. The channel order is assumed to be $L = 6$. We consider a SISO system as well as a SIMO system with $N_r = 2$ receive antennas. For both cases, we consider an OFDM transmission with $N = 128$ subcarriers and a cyclic prefix of length $\nu = 3$. We examine the performance of the PTEQ for $P = 1$ and $P = 2$. For the SISO system, the equalizer parameters are chosen as $Q' = 4$ and $L' = 10$ for $P = 1$ and $Q' = 8$ and $L' = 10$ for $P = 2$. For the SIMO system, the equalizer parameters are chosen as $Q' = 2$ and $L' = 8$ for $P = 1$ and $Q' = 4$ and $L' = 8$ for $P = 2$. Note that, in order to keep the same subcarrier span, the Q' for $P = 2$ is chosen to be twice as large as the Q' for $P = 1$. As shown in Fig. 9, the PTEQ with $P = 2$ outperforms the PTEQ with $P = 1$ for the same subcarrier span. For the SISO case, an SNR gain of 4 dB is obtained for the PTEQ with $P = 2$ over the PTEQ with $P = 1$ at BER $= 10^{-2}$. Similarly, an SNR gain of 2 dB is obtained for the PTEQ with $P = 2$ over the PTEQ with $P = 1$ at BER $= 10^{-2}$ for the SIMO case.

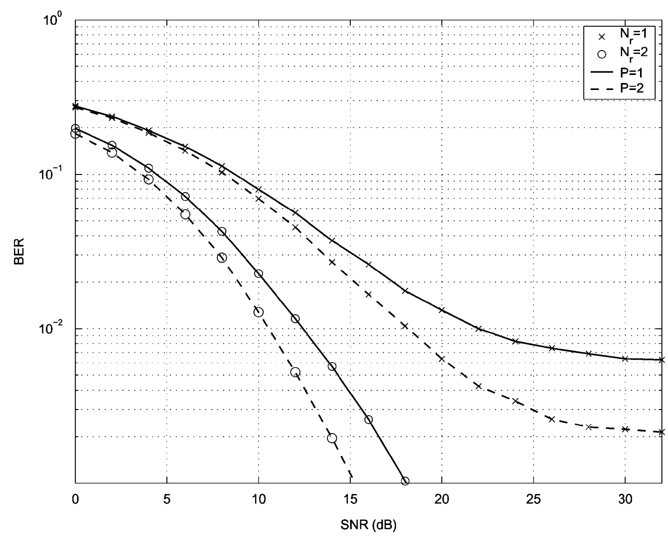

Fig. 9. BER versus SNR for the PTEQ, when the true channel is used to design the equalizer.

VII. Conclusion

In this paper, we propose a time-domain (TEQ) and a frequency-domain per-tone equalizer (PTEQ) for orthogonal frequency-division multiplexing (OFDM) over doubly selective channels. We consider the most general case where the channel delay spread is larger than the CP. The TV channel is approximated using the basis expansion model (BEM). The TEQ is implemented as a time-varying finite-impulse-response (TV-FIR) filter. We use a BEM to model the TV-FIR TEQ. The PTEQ is then obtained by transferring the TEQ operation to the frequency domain. Comparing the TEQ to the PTEQ, we arrive at the following conclusions.[3]

- While the TEQ optimizes the performance on all subcarriers in a joint fashion, the PTEQ optimizes the performance on each subcarrier separately, leading to improved performance.
- The design complexity of the PTEQ is higher than the design complexity of the TEQ.
- The implementation complexity of the PTEQ is comparable to the implementation complexity of the TEQ [apart from the fact that the PTEQ may require additional fast Fourier transforms (FFTs)].

From the simulations, we arrive at the following conclusions.

- The PTEQ always outperforms the TEQ.
- The PTEQ is less sensitive to the choice of the decision delay.
- A key role in the performance of the TEQ and PTEQ is the BEM frequency resolution. We show that a BEM resolution equal to twice the discrete Fourier transform (DFT) resolution (the DFT size) is enough to get an acceptable performance.
- The PTEQ outperforms the conventional 1-tap FEQ for OFDM over TI channels.
- The PTEQ approaches the performance of the block minimum mean-square error (MMSE) equalizer for OFDM over doubly selective channels.
- Oversampling the received sequence while keeping the same intercarrier interference (ICI) span pays off.

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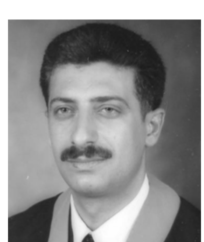

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[3]Note that some of these conclusions are analogous to the results obtained for the time-invariant case.

Geert Leus (M'01) was born in Leuven, Belgium, in 1973. He received the electrical engineering degree and the Ph.D. degree in applied sciences from the Katholieke Universiteit Leuven (K.U. Leuven), Leuven, Belgium, in 1996 and 2000, respectively.

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