2005 IEEE International
Symposium on Signal Processing
and Information Technology

Prediction of the eigenvectors for spatial multiplexing MIMO systems in time-varying channels

Hung Tuan Nguyen
Department of Communication Technology
Aalborg University
Niels Jernes Vej 12, DK-9220 Aalborg
Email: htn@kom.auc.dk

Geert Leus
Faculty of Electrical Engineering
Delft University of Technology
Mekelweg 4 2628 CD Delft
The Netherlands
Email:leus@cas.et.tudelft.nl

Nadia Khaled
Interuniversity Micro-Electronics Center
Kapeldreef 75, B-3001 Leuven
Belgium
Email:khaledn@imec.be

***Abstract*— In mobile communications, time varying channels make the available channel information out of date. Timely updating the channel state is an obvious solution to improve the system performance in a time varying channel. However, a better knowledge of the channel comes at the cost of a decrease in the system throughput. Thus, predicting the future channel conditions can improve not only the performance but also the throughput of many types of wireless systems. This is especially true for a wireless system where multiple antennas are applied at both link ends. In this paper we propose and evaluate the performance of a prediction scheme for multiple input multiple output (MIMO) systems that apply spatial multiplexing. We aim at predicting the future precoder/decoder directly without going through the prediction of the channel matrix. The results show that in a slowly time varying channel an increase in the system performance by a factor of two is possible.**

I. Introduction

Multiple input multiple output (MIMO) systems have a potential of offering higher capacity than the traditional single input single output (SISO) systems by utilizing space, polarization or pattern diversity [1], [2]. In a MIMO system, it is possible to transmit a few data streams in parallel, called spatial multiplexing. Decoupling the data streams can be done by using the channel knowledge at the receiver only. One can use zero forcing (ZF), minimum mean square error (MMSE), successive interference cancellation or ordered successive interference cancellation (VBLAST) to decouple the subchannels. However, since the transmitted signals are not matched to the channel, degradation in system performance is inevitable. Once the channel information is available at both ends of the transmission link the singular value decomposition (SVD) transmission structure appears to be an elegant technique to diagonalize the channel matrices [3].

In a time-varying channel, the schemes mentioned above are subject to a performance degradation. The variation of the channel with time causes the available channel state information (CSI) at both sides to be out of date. While prediction of the future CSI for a SISO channel is possible using available methods (i.e. [4], [5], [6], [7], [9] among others) predicting all components of the CSI matrix in a MIMO system appears to be cumbersome. Moreover the precoder/decoder obtained from the SVD of the predicted channel matrix is more prompt to estimation errors.

Having an orthonormal property and a square structure, the precoders and decoders belong to a unitary group, denoted as $\mathcal{U}(D)$ where D is the dimension. This $\mathcal{U}(D)$ group is a subgroup of the Stiefel manifold which contains all rectangular matrices with orthonormal columns. On $\mathcal{U}(D)$, one can use the so-called geodesic interpolation to find the smoothest trajectory or geodesic flow between two successive points [8], [10], [11]. In this paper, by extending the geodesic interpolation idea we investigate the possibility of predicting the precoder and decoder in a time-varying frequency flat MIMO channel. The paper is organized in the following way. First, the system model is presented in section II. Section III describes the prediction algorithm. Performance evaluation of the algorithm is investigated in section IV. Finally, some conclusions and remarks wrap up the paper in section V.

II. System model

Let us consider a spatial multiplexing narrowband MIMO system consisting of N_t transmitting antennas and N_r receiving antennas. Without using the precoder and decoder the received symbol vector has the form

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{x}_i + \mathbf{n}_i \tag{1}$$

where $\mathbf{x}_i$ is the transmitted symbol vector and $\mathbf{n}_i$ is the additive noise, subscript i is the transmitted symbol index. The precoder and decoder are obtained from the SVD of the channel matrix $\mathbf{H}_i$

$$\mathbf{H}_i = \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{V}_i^H \tag{2}$$

where $\mathbf{\Lambda}_i$ is a diagonal matrix containing the singular values and $(.)^H$ denotes the complex conjugate transpose operation. The two unitary matrices $\mathbf{U}$ and $\mathbf{V}$ are in $\mathcal{U}(N_r)$ and $\mathcal{U}(N_t)$, respectively.

0-7803-9314-7/05/$20.00©2005 IEEE

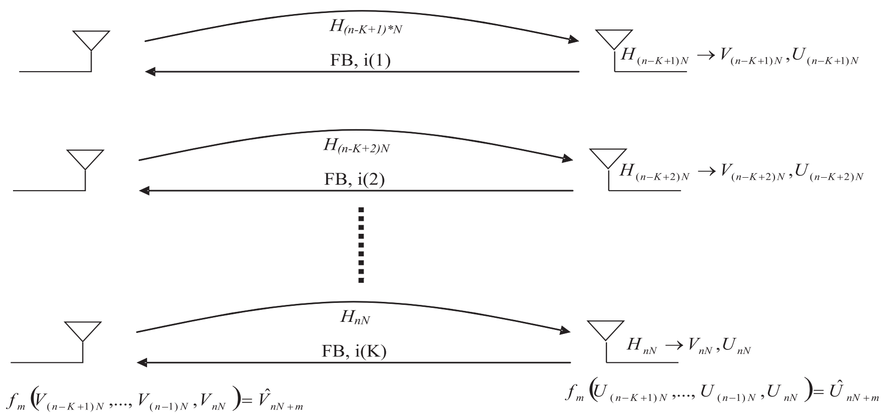


Fig. 1. Precoder/decoder prediction in a feedback delay scheme

Applying the precoder $\mathbf{V}_i$ and the decoder $\mathbf{U}_i^H$, at the transmitter and receiver, respectively, it is possible to decouple the MIMO channel into $S = min(N_t, N_r)$ subchannels, which we can use to transmit $Q \leq S$ data streams in parallel. Applying the precoder and decoder, we obtain

$$\begin{aligned} \mathbf{y}_i &= \mathbf{U}_i^H \mathbf{U}_i \mathbf{\Lambda}_i \mathbf{V}_i^H \mathbf{V}_i \mathbf{x}_k + \mathbf{U}_i^H \mathbf{n}_i \\ &= \mathbf{\Lambda}_i \mathbf{x}_i + \widetilde{\mathbf{n}}_i \end{aligned} \tag{3}$$

As long as the leakage among the subchannels is not severe, the individual substreams can be detected separately. Because no joint detection is required, the detection algorithm becomes rather simple.

We use the well known Jakes' model, [13] to model the time-varying channel. The maximum relative velocity is related to the maximum Doppler frequency f_d by $v = c\frac{f_d}{f_c}$; where f_c is the carrier frequency and c is the velocity of light. Each element of the channel matrix $\mathbf{H}_i$ is simulated as a superposition of a few tens of uncorrelated plane waves. For a narrowband MIMO system the discrete time channel state information (CSI) can be described in baseband by

$$[\mathbf{H}_i]_{m,n} = \frac{1}{\sqrt{L}} \sum_{l=1}^{L} a_l e^{-j2\pi f_d T_s cos\phi_l} \tag{4}$$

where ϕ_l is uniformly distributed over $(0, 2\pi]$, a_l is a random complex Gaussian number with zero mean and variance 1, f_d is the maximum Doppler frequency, T_s is the symbol period and L is the number of scatterers.

At the start of each transmitted frame, we assume the precoder and decoder are derived from the SVD of the estimated channel matrix $\mathbf{H}_i$ and the precoder is fed back to the transmitter. For simplicity we assume that the noise free precoder and decoder are instantaneously updated at the start of each frame. Because of the time-varying channel, the precoder and decoder gradually become out of date at the end of each transmitted frame. Leakage among the subchannels is more severe at the end of each frame and performance degradation is inevitable.

Figure 1 shows the proposed precoder/decoder prediction scheme in a time-varying channel. In the figure, N is the number of symbols within the frame and n is the frame index. The prediction scheme is based on the information that would be available for any MIMO system applying spatial multiplexing. Using the nearest K past precoders/decoders, the precoders/decoders for the transmitted/received symbols within the next frame are predicted. Therefore, in a predicted precoder/decoder system the overhead required for channel probing is the same as for the unpredicted ones but the performance improves.

III. Prediction of the Precoder and Decoder

Unlike other methods used to predict the future CSI, in the prediction of the precoder/decoder, the orthonormal constraint must be retained. One can use a projection based method where the precoder/decoder is first predicted in the Grassmann manifold and then projected onto the Stiefel manifold. However, for interpolation purposes, the performance of this scheme is shown to be lower than that of other methods [12].

The orthonormal property and square structure of the precoder/decoder matrix allow us to perform the exponential map, a key transformation step in geodesic interpolation. Based on the geodesic interpolation, interpolation of the precoder for spatial multiplexing MIMO-OFDM systems has been recently illustrated in [12]. Therefore, we decided to extend the geodesic interpolation method to extrapolate the precoder and decoder for a frequency flat time-varying MIMO channel.

Since the precoder/decoder as a solution of the SVD of the channel matrix $\mathbf{H}_i$ is not unique, the correlation of the consecutive precoder/decoder elements are always lower than that of the channel matrix $\mathbf{H}_i$. Therefore, to enhance the prediction performance, the precoder/decoder needs to be transformed in a way to reduce the ambiguity. For simplicity, in the following we formulate a prediction scheme for the precoder only. The future decoders are predicted in the same manner.

On the K past precoder matrices denoted as $\mathbf{V}_{nN}, \mathbf{V}_{(n-1)N}, ..., \mathbf{V}_{(n-K+1)N}$ we perform the following transformation

$$
\begin{aligned}
\mathbf{V}_{nN} &\rightarrow \mathbf{I} = \mathbf{V}_{n,o}^{Tr} = expm(\mathbf{S}_{n,0}) \\
\mathbf{V}_{(n-1)N} &\rightarrow \mathbf{V}_{nN}^{-1}\mathbf{V}_{(n-1)N}\mathbf{\Theta}_{n,-1} = \\
&\quad\; \mathbf{V}_{n,-1}^{Tr} = expm(\mathbf{S}_{n,-1}) \\
&\quad\vdots \\
\mathbf{V}_{(n-K+1)N} &\rightarrow \mathbf{V}_{nN}^{-1}\mathbf{V}_{(n-K+1)N}\mathbf{\Theta}_{n,-K+1} = \\
&\quad\; \mathbf{V}_{n,-K+1}^{Tr} = expm(\mathbf{S}_{n,-K+1}) \qquad (5)
\end{aligned}
$$

where $expm(.)$ is the matrix exponential operator and $(.)^{Tr}$ denotes the transformed matrix. Further information on the exponential map of matrices in $\mathcal{U}(D)$ can be found in [10] and [11].

In (5), $\mathbf{\Theta}_{n,k}$, with $k \in \{-K+1, \ldots, -1, 0\}$ is the orientation matrix that makes the two matrices $\mathbf{V}_{nN}$ and $\mathbf{V}_{(n+k)N}\mathbf{\Theta}_{n,k}$ as close as possible in Euclidean distance. We use the same solution as the one proposed in [12] to find the orientation matrix $\mathbf{\Theta}_{n,k}$

$$\mathbf{\Theta}_{n,k} = diag(\mathbf{V}_{(n-k+1)N}^{-1}\mathbf{V}_{nN}) \oslash |diag(\mathbf{V}_{(n-k+1)N}^{-1}\mathbf{V}_{nN})| \quad (6)$$

where $\oslash$ represents element-wise division. The $\mathbf{S}_{n,k}$ matrix is a skew-Hermitian matrix. It can be calculated by

$$\mathbf{S}_{n,k} = \mathbf{A}_{n,k} ln(\mathbf{\Xi}_{n,k})\mathbf{A}_{n,k}^{-1} \quad (7)$$

where $\mathbf{A}_{n,k}$ and $\mathbf{\Xi}_{n,k}$ are derived from the eigenvalue decomposition of the transformed matrix $\mathbf{V}_{n,k}^{Tr}$, $\mathbf{V}_{n,k}^{Tr} = \mathbf{A}_{n,k}\mathbf{\Xi}_{n,k}\mathbf{A}_{n,k}^{-1}$. Through these K skew-Hermitian matrices $(\mathbf{S}_{n,-K+1}, \mathbf{S}_{n,-K+2}, ..., \mathbf{S}_{n,0})$ we try to fit a P^{th} order polynomial. When $P+1$ is equal to K, the P^{th} order matrix polynomial goes exactly through the K skew-Hermitian matrices $\mathbf{S}_{n,k}$. The $P+1$ unknown matrix coefficients can be solved by a set of K linear matrix equations:

$$\mathbf{S}_{n,k} = \sum_{p=0}^{P} \mathbf{C}_{n,p}\left((n+k)N\right)^{p} \quad (8)$$

where $k \in \{-K+1, \ldots, -1, 0\}$.

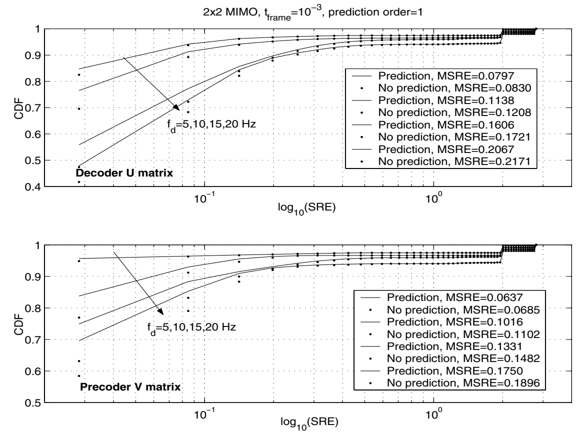
When $P+1$ is strictly smaller than K, the $P+1$ unknown coefficients can be obtained by using the least squares fitting. Note that the skew-Hermitian property of the K matrices $\mathbf{S}_{n,k}$ is translated to a skew-Hermitian property for the coefficient matrices $\mathbf{C}_{n,p}$. Hence, any prediction using the obtained P^{th} order matrix polynomial leads to a skew-Hermitian matrix and thus to a unitary precoder. The skew-Hermitian matrix $\hat{\mathbf{S}}_{nN+m}$ at time index $nN+m$ is estimated by

$$\hat{\mathbf{S}}_{nN+m} = \sum_{p=0}^{P} \mathbf{C}_{n,p}(nN+m)^{p} \quad (9)$$

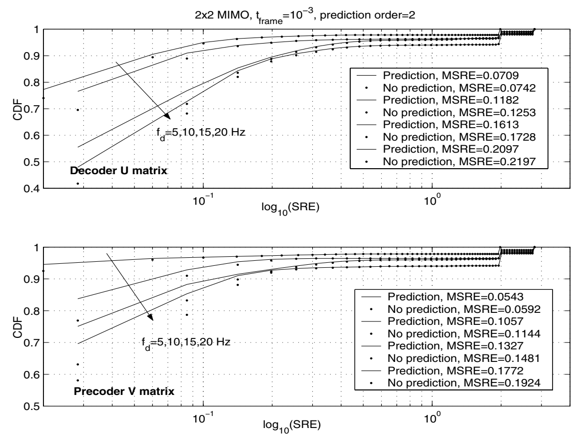
where $m \in \{1, 2, \ldots, N-1\}$. The corresponding future precoders for the $(K+1)^{th}$ frame can thus be constructed as

$$\hat{\mathbf{V}}_{nN+m} = \mathbf{V}_{nN} expm(\hat{\mathbf{S}}_{nN+m}) \quad (10)$$

Note also that in the proposed scheme there is no restriction on the prediction resolution or the number of symbol N within a frame.



(a) 2x2 Prediction order P=1



(b) 2x2 Prediction order P=2

Fig. 2. The distribution of the precoder/decoder prediction error for various settings and maximum Doppler spread values in a 2x2 MIMO setting

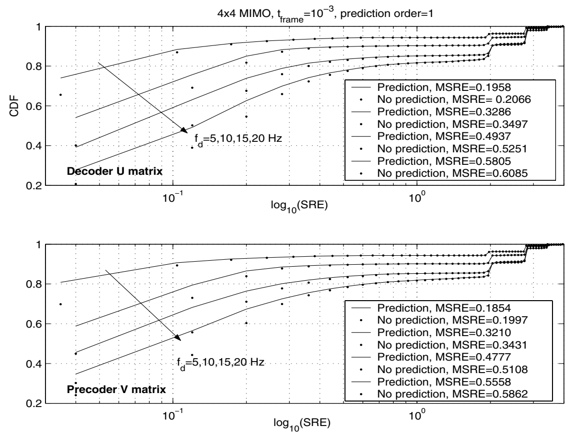
IV. Performance evaluation

A natural criterion for evaluating the performance of the prediction scheme is the following square root error (SRE) measure

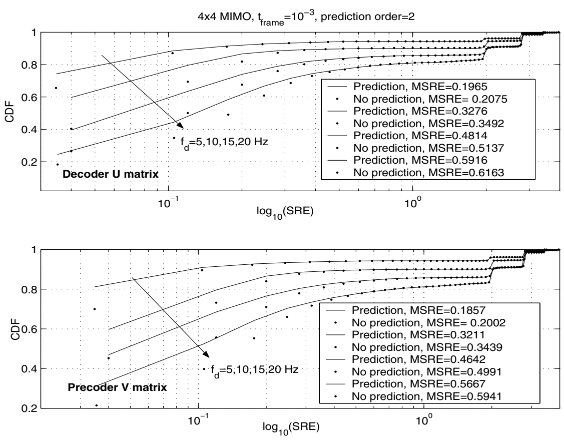
$$SRE_V = ||\hat{\mathbf{V}}_i - \mathbf{V}_i||_F \text{ or } SRE_U = ||\hat{\mathbf{U}}_i - \mathbf{U}_i||_F \quad (11)$$

where $||.||_F$ denotes the Frobenius norm. The SRE essentially is the Euclidean distance between the predicted precoder/decoder and the true ones.

Since the precoder and decoder obtained from the SVD of the channel matrix $\mathbf{H}_i$ are ambiguous up to an orientation matrix, comparing the predicted precoder/decoder with the true ones may not be a good way of evaluating the prediction performance. The predicted precoder/decoder when applied at the transmitter and receiver should create the least power leakage between the subchannels. In other words, the off-diagonal components of the matrix $\hat{\mathbf{U}}_i^H\mathbf{H}_i\hat{\mathbf{V}}_i$ should be as close



(a) 4x4 Prediction order P=1



(b) 4x4 Prediction order P=2

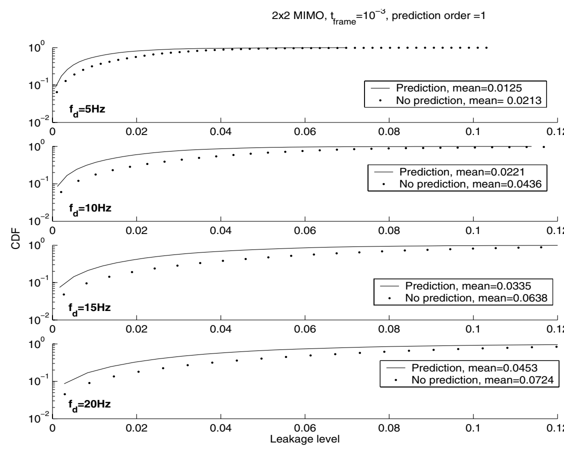
Fig. 3. The distribution of the precoder/decoder prediction error for various settings and maximum Doppler spread values in a 4x4 MIMO setting

as possible to zero. Therefore, we chose another metric which we call the leakage level to evaluate the performance of the prediction scheme, that is

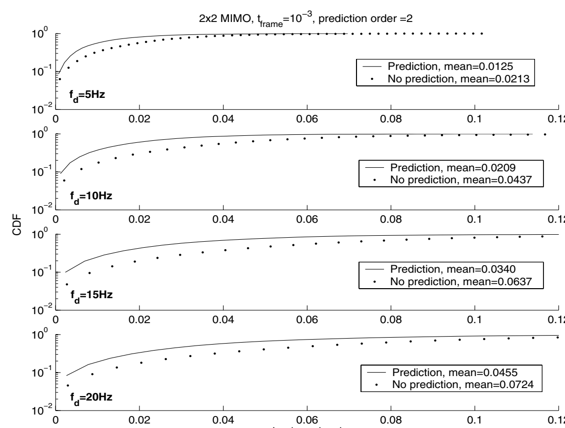
$$||\hat{\mathbf{U}}_i^H \mathbf{H}_i \hat{\mathbf{V}}_i - \mathbf{\Lambda}_i||_F \qquad (12)$$

In the prediction of the precoder/decoder we aim at a slowly time-varying channel with a maximum Doppler spread ranging from a couple of Hz to a few tens of Hz. This type of channel can occur in an indoor environment. The time required to transmit a data frame is $t_{frame} = NT_s = 10^{-3}$s. The channel matrices were generated using the model described in section II. We consider 1000 channel realizations. The total number of simulated frames was 1000.

Figures 2 and 3 show the cumulative distribution function (CDF) of the SRE for different values of the maximum Doppler frequency, prediction order and MIMO settings. For comparison, we also calculate the SRE for the time-varying MIMO channel where the same precoder/decoder is used for the whole data frame (without prediction). The mean values of the SRE are shown in the same figure. From the results it can be seen that when applying prediction the SRE is indeed lower than for the case where no prediction is made. However, for a low Doppler spread (5Hz) and a low number of transmitting and receiving antennas (2x2) the improvement in the SRE is moderate. Including more than two points ($K > 2$) in order to predict the future precoder/decoder may not enhance the prediction performance. The past frames which do not follow the variation of the newly updated frames spoil the prediction preciseness. This also reflects a general trend which can be observed in predicting the time-varying channel coefficients.



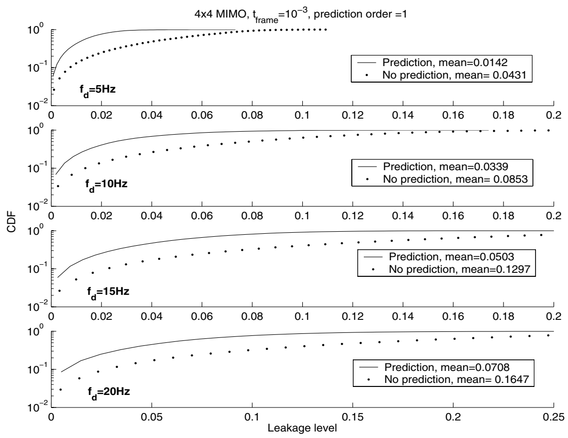
(a) 2x2 MIMO, Prediction order=1



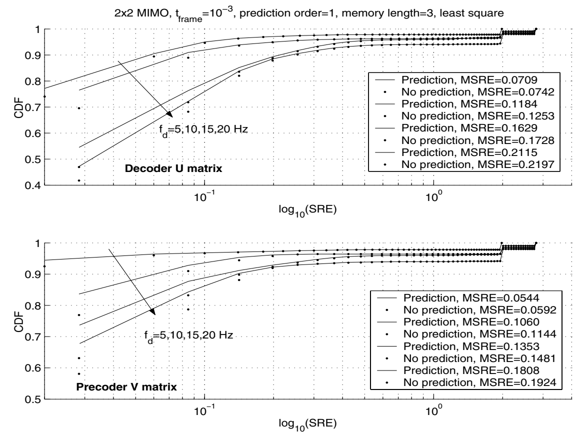
(b) 2x2 MIMO, Prediction order=2

Fig. 4. CDF of the leakage for the 2x2 setting

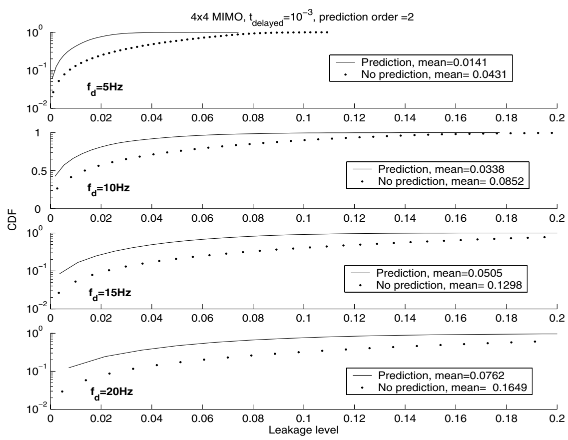
Figures 4 and 5 show the leakage level for a 2x2 and 4x4 MIMO system with and without precoder/decoder prediction. Using the leakage metric defined in (12) the performance improvement of the prediction scheme with the first and second



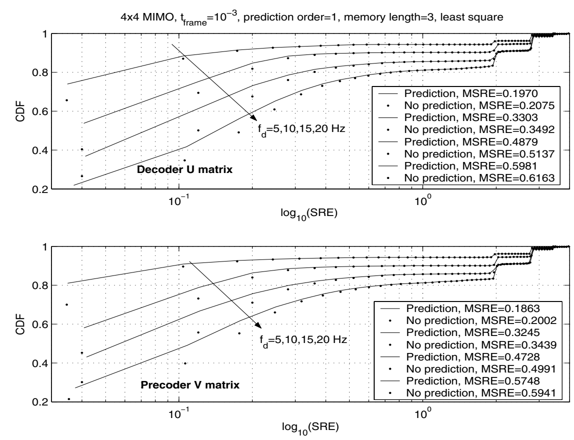
(a) 4x4 MIMO, Prediction order=1



(a) 2x2 setting



(b) 4x4 MIMO, Prediction order=2



(b) 4x4 Setting

Fig. 5. CDF of the leakage for the 4x4 setting

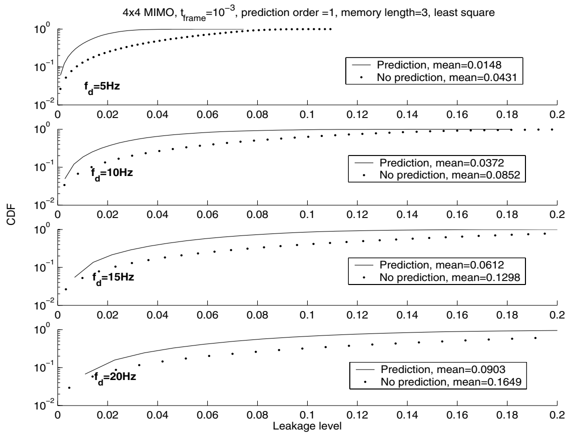
Fig. 6. The distribution of the precoder/decoder prediction error for various MIMO settings and maximum Doppler spread values

order polynomial prediction can be clearly seen. With the precoder/decoder prediction the mean leakage levels for most of the MIMO settings and time-varying channel conditions are reduced by a factor of two.

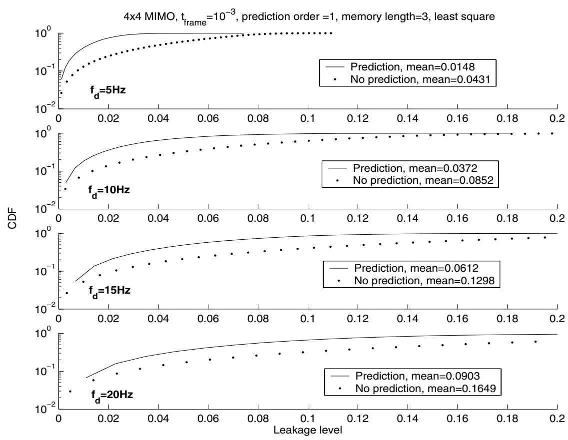
We also evaluated the performance of the prediction scheme when $P+1$ is smaller than the number of points used for prediction K. In that case, the polynomial does not exactly go through the K points. In the simulation, $P+1$ and K was chosen to be 2 and 3, respectively. Least squares fitting was used to find the coefficient matrices $\mathbf{C}_{n,p}$ presented in (8). The results in Figure 6 and 7 show that the prediction performance is in between linear prediction and second order prediction. Although no significant performance improvement can be obtained, it is expected that when the noise is present this scheme could lead to a smaller prediction error.

In general, based on the two metrics presented above the proposed precoder/decoder prediction scheme always outperforms the scheme with no prediction (i.e. only using a delayed version of the precoder/decoder). A reduction in the leakage level by a factor of two is observed for most MIMO settings and time-varying channel conditions. Despite using only two past samples for the prediction, linear prediction of the precoder/decoder shows a reasonable performance improvement.

For completeness we evaluate the performance of a spatial multiplexing MIMO system using the precoder/decoder prediction scheme and compare it with the case no prediction is used. Again we consider a 2x2 and 4x4 MIMO system in a varying channel with a maximum Doppler spread f_d=20Hz. The precoder/decoder was predicted using a first order polynomial. For each data stream independent QPSK symbols were transmitted. At the receiver coherent detection is assumed and each data stream was detected separately. The BER of each data stream is shown in Figure 8. The results show that the performance improvement in terms of the BER is moderate for the 2x2 MIMO setting. For the 4x4 MIMO



(a) 2x2 setting



(b) 4x4 setting

Fig. 7. The distribution of the leakage for various MIMO settings and maximum Doppler spread values

setting, an improvement by a few dBs in SNR can be observed on the subchannel with low channel gain. Nevertheless, the proposed prediction scheme still remains attractive considering its simplicity.

V. Conclusions and Remarks

In this paper, we have proposed and evaluated the performance of a precoder/decoder prediction scheme for a time-varying MIMO channel. The proposed prediction scheme is an expansion of the geodesic interpolation method in a unitary group where any unitary matrix can be expressed as the matrix exponential of a skew-Hermitian matrix. The prediction of the precoder/decoder is made based on the information that would be available for any MIMO system deploying spatial multiplexing. Therefore, the amount of overhead required for channel probing is the same as for the case with no percoder/decoder prediction. To evaluate the prediction performance, two metrics were defined namely the Euclidean

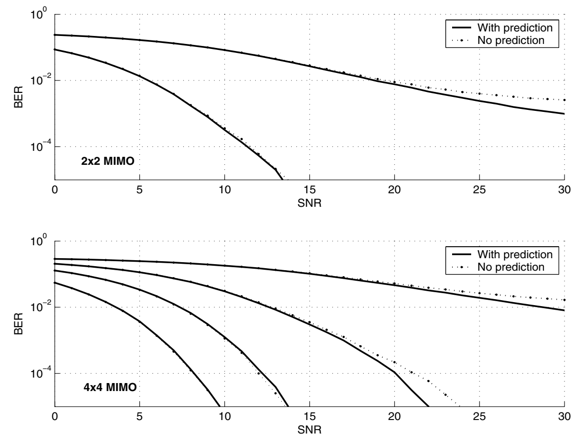


Fig. 8. BER of the time-varying MIMO channel for the 2x2 and 4x4 setting

distance between the predicted precoder/decoder and the true ones and the leakage level. Based on these two metrics, it has been shown that the proposed precoder/decoder prediction scheme can work well for a slowly time-varying MIMO channel. Evaluating the performance of the prediction scheme when the channel estimation error and quantization error are taken into account is one of the interesting problems for future work.

References

[1] G.J.Foschini, ”Layered space-time architecture for wireless communication in fading environment when using multi-element antennas” Bell Labs Tech. J., 4159, 1996.

[2] I.E.Telatar ”Capacity of Multi-Antenna Gaussian Channels. AT&T Bell Labs”. 522526, 1995.

[3] J.B.Andersen ”Array gain and capacity for known random channels with multiple element arrays at both ends” IEEE Journal on Selected Areas in Communications, Volume 18, Issue 11, Nov. 2000 Page(s):2172 - 2178.

[4] A.Duel-Hallen, S.Hu and H.Hallen, ”Long-range Prediction of Fading Signals”, IEEE Signal Processing Magazine, vol. 17, no. 3, pp. 62-75, May 2000.

[5] R.Vaughan, P.Teal and R.Raich, ”Short-term Mobile Channel Prediction Using Discrete Scatterer Propagation Model and Subspace Signal Processing Algorithms”, in Proc. IEEE International Vehicular Technology Conference, pp. 751-758, Sep. 2000.

[6] T.Eyceoz, A.Duel-Hallen and H. Hallen, ”Deterministic Channel Modeling and Long Range Prediction of Fast Fading Mobile Radio Channels”, IEEE Communications Letters, vol. 2, pp. 254-256, Sep. 1998.

[7] J.B.Andersen, J.Jensen, S.Jensen and F.Frederiksen, ”Prediction of Future Fading Based on Past Measurements”, in Proc. IEEE International Vehicular Technology Conference, pp. 15-155, Sep. 1999.

[8] A.Edelman, T.A.Arias adn S.T.Smith, ”The geometry of algorithms with orhogonal constraints”, SIAM J.MAtrix Anal. Appl. Vol. 20, No.2, pp.303-305, 1998.

[9] L.Dong, G.Xu, and H.Ling, ”Prediction of Fast Fading Mobile Radio Channels in Wideband Communication Systems”, in Proc. IEEE Global Communications Conference, pp. 3287-3291, Nov. 2001.

[10] D.Asimov and A.Buja” The grand tour via geodesic interpolation of 2 frames”, NAS-NASA RNR technical report RNR-94-004 Feb 1994.

[11] C.Belta and V.Kumar ”An SVD based projection method for interpolation on SE(3)”

[12] N.Khaled , B.Mondal, R.W.Heath J.r, G.Leus and F.Petre ”Interpolation-based multi-mode precoding for MIMO-OFDM systems with limited feedback” submitted to VTC’ 05.

[13] W.C.Jakes”Microwave Mobile Communications”, New York Wiley 1974.