PRECODER AND DECODER PREDICTION IN TIME-VARYING MIMO CHANNELS

Hung Tuan Nguyen [1], *Geert Leus* [2], *Nadia Khaled* [3]

[1] Department of Communication Technology, Aalborg University, Email: htn@kom.auc.dk
[2] Faculty of Electrical Engineering, Mathematics, and Computer Science, Delft University of Technology, Email:leus@cas.et.tudelft.nl
[3] Wireless Research, Interuniversity Micro-Electronics Center (IMEC), Email:khaledn@imec.be

ABSTRACT

In wireless communications, mobility can make the available channel information out of date. A timely update of the channel state information is an obvious solution to improve the system performance in a time-varying channel. However, this comes at the cost of a decrease in the system throughput since many pilots have to be inserted. Thus, predicting the future channel conditions can improve not only the performance but also the throughput of many types of wireless systems. This is especially true when multiple antennas are applied at both link ends. In this paper, we propose and evaluate the performance of a prediction scheme for multiple input multiple output (MIMO) systems that apply spatial multiplexing. We aim at predicting the future precoder/decoder directly without going through the prediction of the channel matrix. The results show that in a slowly time-varying channel an increase in the system performance by a factor of two is possible.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have the potential of offering higher capacity than traditional single-input single-output (SISO) systems by utilizing space, polarization or pattern diversity [1], [2]. In a MIMO system, it is possible to transmit a few data streams in parallel, called spatial multiplexing. Decoupling the data streams can be done by using the channel knowledge at the receiver only. One can use zero forcing (ZF), minimum mean square error (MMSE), or (random or ordered) successive interference cancellation (VBLAST) to decouple the subchannels. However, since the transmitted signals are not matched to the channel, degradation in system performance is inevitable. Once the channel information is available at both ends of the transmission link the singular value decomposition (SVD) transmission structure appears to be an elegant technique to diagonalize the channel matrices [3].

In a time-varying channel, the schemes mentioned above are subject to a performance degradation. The variation of the channel with time causes the available channel state information (CSI) at both sides to be out of date. While prediction of the future CSI for a SISO channel is possible using available methods (i.e. [4], [5], [6], [7], [9] among others) predicting all components of the CSI matrix in a MIMO system appears to be cumbersome. Moreover the precoder/decoder obtained from the SVD of the predicted channel matrix is more prone to estimation errors.

Having a unitary matrix structure, the precoders and decoders belong to the unitary group, denoted as $\mathcal{U}(N)$, where N is the dimension. On $\mathcal{U}(N)$, one can use the so-called geodesic interpolation to find the smoothest trajectory or geodesic flow between two successive points [8], [10], [11]. In this paper, by extending the geodesic interpolation idea we investigate the possibility of predicting the precoder and decoder in a time-varying frequency-flat MIMO channel.

2. SYSTEM MODEL

Let us consider a spatial multiplexing narrowband MIMO system consisting of N_t transmit antennas and N_r receive antennas. Without using a precoder or decoder, the received sample vector at time index i has the form

$$\mathbf{y}_i = \mathbf{H}_i\mathbf{x}_i + \mathbf{n}_i, \tag{1}$$

where $\mathbf{n}_i$, $\mathbf{x}_i$, and $\mathbf{H}_i$, are respectively, the additive noise vector, the transmitted symbol vector, and the channel matrix at time index i. A reasonable precoder and decoder can then be obtained from the SVD of the channel matrix $\mathbf{H}_i$:

$$\mathbf{H}_i = \mathbf{U}_i\mathbf{\Lambda}_i\mathbf{V}_i^H, \tag{2}$$

where $\mathbf{U}_i$ and $\mathbf{V}_i$ are in $\mathcal{U}(N_r)$ and $\mathcal{U}(N_t)$, respectively, $\mathbf{\Lambda}_i$ is an $N_r \times N_t$ matrix containing the singular values on its main diagonal, and $(.)^H$ denotes the complex conjugate transpose operation.

Precoding $\mathbf{x}_i$ as $\mathbf{V}_i\mathbf{x}_i$ and decoding $\mathbf{y}_i$ as $\mathbf{U}_i^H\mathbf{y}_i$, we obtain

$$\begin{aligned}\mathbf{U}_i^H\mathbf{y}_i &= \mathbf{U}_i^H\mathbf{H}_i\mathbf{V}_i\mathbf{x}_i + \mathbf{U}_i^H\mathbf{n}_i \\ &= \mathbf{\Lambda}_i\mathbf{x}_i + \mathbf{U}_i^H\mathbf{n}_i.\end{aligned} \tag{3}$$

As a result, we have decoupled the MIMO channel into $S = \min(N_t, N_r)$ subchannels (only $S = \min(N_t, N_r)$ entries of $\mathbf{\Lambda}_i$ are non-zero). As a result, we only use the first $Q \leq S$ entries in $\mathbf{x}_i$ to transmit information. The other entries are set to zero. As long as the leakage among the subchannels is not severe, the individual substreams can be detected separately. Because no joint detection is required, the detection algorithm becomes rather simple.

We use the well-known Jakes' model [13] to model the time-varying channel. The maximum relative velocity v is related to the maximum Doppler frequency f_d by $v = cf_d/f_c$, where f_c is the carrier frequency and c the velocity of light. Each element of the channel matrix $\mathbf{H}_i$ is simulated as a superposition of a few tens of uncorrelated plane waves. For a narrowband MIMO system the discrete-time channel state information (CSI) can be described in baseband by

$$[\mathbf{H}_i]_{m,n} = \frac{1}{\sqrt{L}}\sum_{l=1}^{L} a_l e^{-j2\pi f_d i T_s cos\phi_l}, \tag{4}$$

where ϕ_l is uniformly distributed over $(0, 2\pi]$, a_l is a random complex Gaussian number with zero mean and variance one, f_d is the maximum Doppler frequency, T_s is the symbol period, and L is the number of scatterers. Since each channel coefficient is generated independently, the channel matrix can be considered as a matrix with no spatial correlation between its components.

We assume a system where at the start of each frame, the precoder and decoder are derived from the SVD of the estimated channel matrix $\mathbf{H}_i$ and the precoder is fed back to the transmitter. For simplicity, we assume that the noise free precoder and decoder are instantaneously updated at the start of each frame, and that the feedback link is ideal (no delay and no noise). Because of the time-varying behavior of the channel, the precoder and decoder gradually become out of date at the end of each transmitted frame. Leakage among the subchannels is more severe at the end of each frame and performance degradation is inevitable. We try to solve this problem by prediction.

3. PREDICTION OF THE PRECODER AND DECODER

Unlike methods used to predict the future CSI, in the prediction of the precoder/decoder, the orthogonality constraint must be retained. One can use a projection based method where the precoder/decoder is first predicted in the space of complex square matrices and then projected onto the unitary group. However, for interpolation purposes, the performance of this scheme has been shown to be lower than that of other methods [12].

The unitary structure of the precoder/decoder matrix allows us to perform an exponential map, a key step in geodesic interpolation. Based on geodesic interpolation, interpolation of the precoder for spatial multiplexing MIMO-OFDM systems has been recently illustrated in [12]. Therefore, we decided to extend the geodesic interpolation method to predict the precoder and decoder for a frequency-flat time-varying MIMO channel.

Since the precoder/decoder as a solution of the SVD of the channel matrix $\mathbf{H}_i$ is not unique, the correlation of the consecutive precoder/decoder elements is always lower than that of the channel matrix $\mathbf{H}_i$. Therefore, to enhance the prediction performance, the precoder/decoder needs to be transformed in a way to reduce this ambiguity. For simplicity, in the following we formulate a prediction scheme for the precoder only. The future decoders are predicted in the same manner.

Suppose that at frame n, we have K past precoder matrices at our disposal, denoted as $\mathbf{V}_{nN}, \mathbf{V}_{(n-1)N}, ..., \mathbf{V}_{(n-K+1)N}$, where N is the number of symbols within a frame. The first thing we do is transforming these precoder matrices as:

$$\begin{aligned}
\mathbf{V}_{nN} &\rightarrow \mathbf{I} = \mathbf{V}_{n,o}^{Tr} = \text{expm}(\mathbf{S}_{n,0}), \\
\mathbf{V}_{(n-1)N} &\rightarrow \mathbf{V}_{nN}^{-1}\mathbf{V}_{(n-1)N}\mathbf{\Theta}_{n,-1} \\
&\quad = \mathbf{V}_{n,-1}^{Tr} = \text{expm}(\mathbf{S}_{n,-1}), \\
&\quad \vdots \\
\mathbf{V}_{(n-K+1)N} &\rightarrow \mathbf{V}_{nN}^{-1}\mathbf{V}_{(n-K+1)N}\mathbf{\Theta}_{n,-K+1} \\
&\quad = \mathbf{V}_{n,-K+1}^{Tr} = \text{expm}(\mathbf{S}_{n,-K+1}),
\end{aligned} \tag{5}$$

where expm$(.)$ is the matrix exponential operator and $(.)^{Tr}$ denotes the transformed matrix. Further information on the exponential map of matrices in $\mathcal{U}(D)$ can be found in [10] and [11].

In (5), $\mathbf{\Theta}_{n,k}$, with $k \in \{-K+1, .., -1, 0\}$ is the orientation matrix that makes the two matrices $\mathbf{V}_{nN}$ and $\mathbf{V}_{(n+k)N}\mathbf{\Theta}_{n,k}$ as close as possible in Euclidean distance. We use the same solution as the one proposed in [12] to find the orientation matrix $\mathbf{\Theta}_{n,k}$:

$$\mathbf{\Theta}_{n,k} = \text{diag}(\mathbf{V}_{(n-k+1)N}^{-1}\mathbf{V}_{nN}) \oslash |\text{diag}(\mathbf{V}_{(n-k+1)N}^{-1}\mathbf{V}_{nN})|,$$

where $\oslash$ represents element-wise division. The matrix $\mathbf{S}_{n,k}$ is a skew-Hermitian matrix. It can be calculated by

$$\mathbf{S}_{n,k} = \mathbf{A}_{n,k}\ln(\mathbf{\Xi}_{n,k})\mathbf{A}_{n,k}^{-1}, \tag{6}$$

where $\mathbf{A}_{n,k}$ and $\mathbf{\Xi}_{n,k}$ are derived from the eigenvalue decomposition of the transformed matrix $\mathbf{V}_{n,k}^{Tr}$: $\mathbf{V}_{n,k}^{Tr} = \mathbf{A}_{n,k}\mathbf{\Xi}_{n,k}\mathbf{A}_{n,k}^{-1}$. To these K skew-Hermitian matrices $\mathbf{S}_{n,k}$ we try to fit a Pth order matrix polynomial as

$$\min_{\mathbf{C}_{n,p}} \sum_{k=-K+1}^{0} \left\| \mathbf{S}_{n,k} - \sum_{p=0}^{P} \mathbf{C}_{n,p}\left((n+k)N\right)^p \right\|_F^2, \tag{7}$$

where $\|.\|_F$ represents the Frobenius norm. We only consider situations where $P+1 \leq K$. When $P+1 = K$, the Pth order matrix polynomial goes exactly through the K skew-Hermitian matrices $\mathbf{S}_{n,k}$. When $P+1 < K$, this is not the case, but when noise is present, taking $P+1 < K$ could lead to a smaller prediction error. Note that the skew-Hermitian property of the K matrices $\mathbf{S}_{n,k}$ is translated into a skew-Hermitian property for the matrix coefficients $\mathbf{C}_{n,p}$. Hence, any prediction using the obtained Pth order matrix polynomial leads to a skew-Hermitian matrix and thus to a unitary precoder. The skew-Hermitian matrix at time index $nN+m$ is estimated as

$$\hat{\mathbf{S}}_{nN+m} = \sum_{p=0}^{P} \mathbf{C}_{n,p}(nN+m)^p, \tag{8}$$

where $m \in \{1, \ldots, N-1\}$ is the symbol index within the $(n+1)$st frame. The corresponding precoder matrix at time index $nN+m$ can then be constructed as

$$\hat{\mathbf{V}}_{nN+m} = \mathbf{V}_{nN}\text{expm}(\hat{\mathbf{S}}_{nN+m}). \tag{9}$$

4. PERFORMANCE EVALUATION

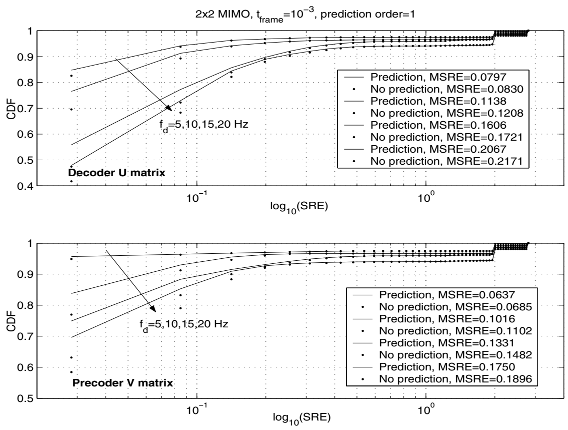
A natural criterion for evaluating the performance of the prediction scheme is the following square root error (SRE) measure

$$SRE_V = \|\hat{\mathbf{V}}_i - \mathbf{V}_i\|_F \quad \text{or} \quad SRE_U = \|\hat{\mathbf{U}}_i - \mathbf{U}_i\|_F. \tag{10}$$

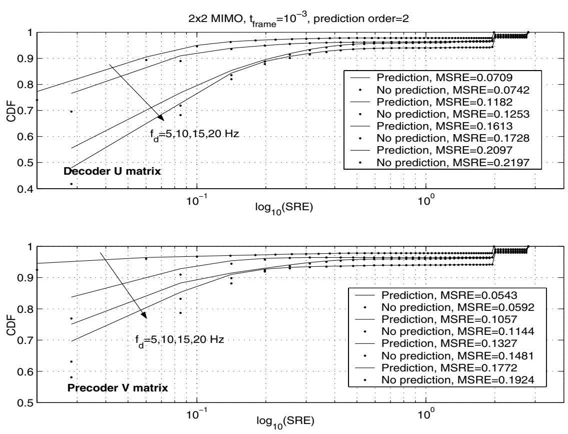
The SRE essentially is the Euclidean distance between the predicted precoder/decoder and the true one.

Since the precoder and decoder obtained from the SVD of the channel matrix $\mathbf{H}_i$ are ambiguous up to an orientation matrix, comparing the predicted precoder/decoder with the true one may not be a good way of evaluating the prediction performance. The predicted precoder/decoder when applied at the transmitter and receiver should create the least power leakage between the subchannels. In other words, the off-diagonal components of the matrix $\hat{\mathbf{U}}_i^H\mathbf{H}_i\hat{\mathbf{V}}_i$ should be as small as possible. Therefore, we chose another metric which we call the leakage level to evaluate the performance of the prediction scheme, that is

$$\|\hat{\mathbf{U}}_i^H\mathbf{H}_i\hat{\mathbf{V}}_i - \mathbf{\Lambda}_i\|_F. \tag{11}$$

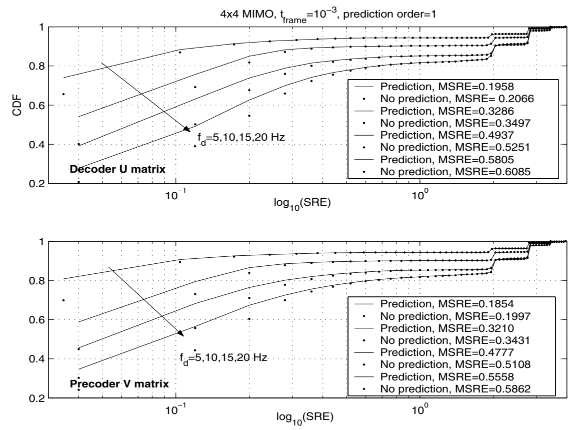


(a) 2×2, prediction order $P = 1$, memory length $K = 2$

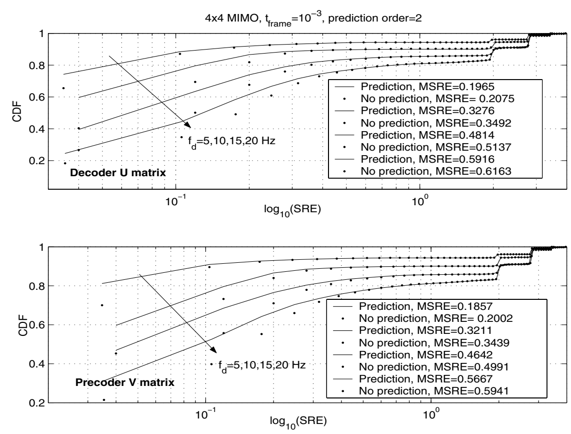


(b) 2×2, prediction order $P = 2$, memory length $K = 3$

Fig. 1. The distribution of the precoder/decoder prediction error for various settings and maximum Doppler spread values in a 2×2 MIMO setting.



(a) 4×4, prediction order $P = 1$, memory length $K = 2$



(b) 4×4, prediction order $P = 2$, memory length $K = 3$

Fig. 2. The distribution of the precoder/decoder prediction error for various settings and maximum Doppler spread values in a 4×4 MIMO setting.

In the prediction of the precoder/decoder we aim at a slowly time-varying channel with a maximum Doppler spread ranging from a couple of Hz to a few tens of Hz. This type of channel can occur in an indoor environment. The time required to transmit a data frame is assumed to be $NT_s = 10^{-3}$s. The channel matrices were generated using the model described in section 2. We consider 1000 channel realizations. The total number of simulated frames was 1000.

Figures 1 and 2 show the cumulative distribution function (CDF) of the SRE for different values of the maximum Doppler frequency, prediction order, memory length, and MIMO setting. For comparison, we also calculate the SRE for the time-varying MIMO channel where the same precoder/decoder is used for the whole data frame (without prediction). The mean values of the SRE are shown in the legend. From the results it can be seen that when applying prediction, the SRE is indeed lower than when no prediction is made. However, for a low Doppler spread (5 Hz) and a low number of transmit and receive antennas (2×2) the improvement in the SRE is moderate. Increasing the prediction order and the memory length ($P > 2$, $K > 3$) in order to predict the future precoder/decoder may not enhance the prediction performance. The past frames which do not follow the variation of the newly updated frames spoil the prediction preciseness. This also reflects a general trend which can be observed in predicting the time-varying channel coefficients.

Table 1 shows the mean leakage level for a 2×2 and 4×4 MIMO system with and without precoder/decoder prediction. Using the leakage metric defined in (11) the performance improvement of the prediction scheme with the first and second order polynomial prediction can be clearly seen. With the precoder/decoder prediction the mean leakage levels for most of the MIMO settings and time-varying channel conditions are reduced by a factor of

Doppler frequency	2 × 2 MIMO			4 × 4 MIMO		
	No cod.	With cod.		No cod.	With cod.	
		$P = 1$	$P = 2$		$P = 1$	$P = 2$
5 Hz	0.0213	0.0125	0.0125	0.0431	0.0142	0.0141
10 Hz	0.0426	0.0221	0.0209	0.0853	0.0339	0.0338
15 Hz	0.0638	0.0335	0.0304	0.1297	0.0503	0.0505
20 Hz	0.0724	0.0453	0.0455	0.1647	0.0708	0.0762

Table 1. Mean leakage level level of a 2×2 and 4×4 MIMO system with and without precoder/decoder prediction.

two.

For completeness we evaluate the performance of a spatial multiplexing MIMO system using the precoder/decoder prediction scheme and compare it with the case no prediction is used. Again we consider a 2×2 and 4×4 MIMO system in a time-varying channel with a maximum Doppler spread $f_d = 20$ Hz. The precoder/decoder was predicted using prediction order $P = 1$ and memory length $K = 2$. For each data stream, independent QPSK symbols were transmitted. At the receiver coherent detection is assumed and each data stream was detected separately. The BER of each data stream is shown in Figure 3. The results show that the performance improvement in terms of the BER is moderate for the 2×2 MIMO setting. For the 4×4 MIMO system, an improvement by a few dBs in SNR can be observed on the subchannel with a low channel gain. Nevertheless, when higher Doppler speeds are considered (i.e., when outdoor applications are envisioned), the difference will be much larger and the proposed prediction scheme will become very attractive.

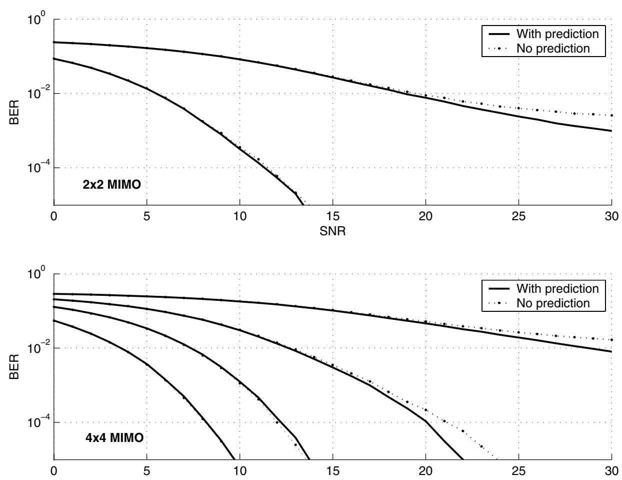


Fig. 3. BER of time-varying 2×2 and 4×4 MIMO channel.

5. CONCLUSIONS AND REMARKS

In this paper, we have proposed and evaluated the performance of a precoder/decoder prediction scheme for a time-varying MIMO channel. The proposed prediction scheme is an extension of the geodesic interpolation method in a unitary group where any unitary matrix can be expressed as the matrix exponential of a skew-Hermitian matrix. The prediction of the precoder/decoder is made based on the information that would be available for any MIMO system deploying spatial multiplexing. Therefore, the amount of overhead required for channel probing is the same as for the case when no precoder/decoder prediction is used. To evaluate the prediction performance, two metrics were defined namely the Euclidean distance between the predicted precoder/decoder and the true one and the leakage level. Based on these two metrics, it has been shown that the proposed precoder/decoder prediction scheme can work well for a time-varying MIMO channel. Evaluating the performance of the prediction scheme when the channel estimation error and quantization error are taken into account is one of the interesting problems for future work.

6. REFERENCES

[1] G.J.Foschini, "Layered space-time architecture for wireless communication in fading environment when using multi-element antennas," Bell Labs Tech. J., 4159, 1996.

[2] I.E.Telatar, "Capacity of Multi-Antenna Gaussian Channels," AT&T Bell Labs, 522526, 1995.

[3] J.B.Andersen, "Array gain and capacity for known random channels with multiple element arrays at both ends," IEEE JSAC, Volume 18, Issue 11, Nov. 2000, pp. 2172–2178.

[4] A.Duel-Hallen, S.Hu and H.Hallen, "Long-Range Prediction of Fading Signals," IEEE Signal Processing Magazine, vol. 17, no. 3, pp. 62–75, May 2000.

[5] R.Vaughan, P.Teal and R.Raich, "Short-Term Mobile Channel Prediction Using Discrete Scatterer Propagation Model and Subspace Signal Processing Algorithms," in Proc. IEEE VTC, pp. 751–758, Sep. 2000.

[6] T.Eyceoz, A.Duel-Hallen and H. Hallen, "Deterministic Channel Modeling and Long Range Prediction of Fast Fading Mobile Radio Channels," IEEE Com. Letters, vol. 2, pp. 254–256, Sep. 1998.

[7] J.B.Andersen, J.Jensen, S.Jensen and F.Frederiksen, "Prediction of Future Fading Based on Past Measurements," in Proc. IEEE VTC, pp. 151–155, Sep. 1999.

[8] A.Edelman, T.A.Arias adn S.T.Smith, "The geometry of algorithms with orhogonal constraints," SIAM J.MAtrix Anal. Appl. Vol. 20, No.2, pp. 303–305, 1998.

[9] L.Dong, G.Xu, and H.Ling, "Prediction of Fast Fading Mobile Radio Channels in Wideband Communication Systems," in Proc. IEEE Globecom, pp. 3287–3291, Nov. 2001.

[10] D.Asimov and A.Buja, "The grand tour via geodesic interpolation of 2 frames," NAS-NASA RNR technical report RNR-94-004, Feb. 1994.

[11] C.Belta and V.Kumar, "An SVD based projection method for interpolation on SE(3)," IEEE Transactions on Robotics and Automation, Volume 18, Issue 3, June 2002, pp. 334–345.

[12] N.Khaled , B.Mondal, R.W.Heath J.r, G.Leus and F.Petre, "Interpolation-based multi-mode precoding for MIMO-OFDM systems with limited feedback," in Proc. IEEE VTC, Sep. 2005.

[13] W.C.Jakes, "Microwave Mobile Communications," New York Wiley 1974.