

# An Interference-Suppressing RAKE Receiver for the CDMA Downlink

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**Abstract**—In this letter, we propose an interference-suppressing RAKE receiver for the code division multiple-access (CDMA) downlink. In the downlink, the received signal has a special structure that makes it possible for a RAKE receiver (which is a simple low-complexity linear receiver) with appropriately chosen weights to suppress interference efficiently. While there have been a few other interference-suppressing RAKE receivers proposed recently, our design is based on a different motivation, and we show that our approach significantly outperforms them especially when the number of active users in the cell is not large.

**Index Terms**—Direct-sequence code division multiple-access (DS-CDMA), linear equalization, linear receivers, RAKE receivers.

## I. INTRODUCTION

THE PROBLEM of multiuser detection and interference suppression [1] in direct-sequence code division multiple-access (DS-CDMA) systems has been of significant research interest for a number of years. Most of the work has focused on the uplink in general, due to complexity reasons. More recently, low-complexity RAKE receivers have been proposed for the downlink based on its special signal structure [2]–[4].

A simple examination of a CDMA downlink system reveals the following three important properties.

- All the users (desired and interfering) have symbol-synchronous transmissions at the base station (BS).
- The spreading codes used by the BS for all the users (desired and interfering) are orthogonal.
- As seen by any mobile station (MS), all the transmitted signals pass through the same propagation channel.

As we show shortly, these properties make it possible for RAKE receivers to suppress interference effectively. Hence, there has been a significant amount of research interest in this direction. On the other hand, these properties are not available in the uplink. Hence, RAKE receivers for the uplink are not efficient in suppressing interference.

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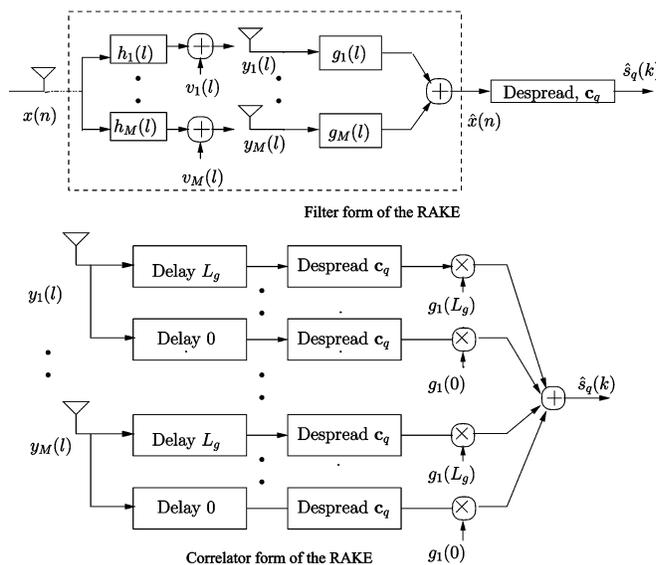


Fig. 1. System model.

Let us now discuss two equivalent forms of the RAKE receiver. The well-known RAKE receiver is the conventional RAKE that involves a bank of correlators operating at various delays on the received signal. These correlator outputs are then summed up with certain weights that are matched to the channel taps at those delays. One can change the weights on these correlator outputs to get a different RAKE receiver. We refer to this form of the RAKE receiver as the correlator or combiner form (see Fig. 1). In another equivalent form, a simple filter is applied to the received signal at the chip level, and the resulting output is despread at symbol level to obtain the symbol estimates. It can be shown that both forms are equivalent if the filter taps are chosen to be the same as the weights in the combiner form as shown in the figure.

The interference-suppressing RAKE receivers proposed to date [2]–[4] are broadly based on the following idea. In the absence of multipath, it is clear that the receiver can simply despread the received signal with the desired user's spreading code (since it is orthogonal to the rest of the codes) in order to eliminate multiple-access interference (MAI). In practice, there is multipath, and so these receivers try to equalize the channel at chip level to remove the effect of multipath and then simply despread the equalizer output with the spreading code of the desired user. If the channel is equalized perfectly, MAI is completely eliminated, since the codes are orthogonal. Note that a channel equalizer followed by a despreader is simply a RAKE receiver expressed in the filter form. It is not easy to motivate these receivers based on the correlator form.

Receivers based on the above approach treat multipath to be *detrimental*, since it destroys the orthogonality of the spreading codes and causes MAI. Therefore, they aim to *cancel* it. However, it is well known that multipath provides diversity in a fading environment that can be tapped fully by coherently *combining* them (as done by the conventional RAKE receiver).

In this letter, we derive a new RAKE receiver based on a simple MMSE criterion that neatly balances the positive effect (diversity) of multipath described above with its negative effect (MAI). We show that the performance of this RAKE receiver is much better than both equalization-based RAKE and conventional RAKE receivers. It can be implemented in a simple manner from training symbols.

We describe the system model in the next section and then derive the proposed RAKE receiver in Section III. We then compare the performance of our receiver with other existing RAKE receivers. While receiver performance [e.g., bit-error rate (BER)] has typically been obtained as the raw SNR varies, it turns out that for the present problem it is more insightful to obtain performance as the number of users in the cell varies (at a suitable, fixed SNR). In Section V, we discuss how the choice of the filter taps or combiner weights effects the resulting interference and noise at the output, and explain in detail why the proposed receiver outperforms the other receivers. We then summarize the contributions of the letter and conclude in Section VI.

## II. SYSTEM MODEL

Let the spreading factor be  $P$ , and let the number of active users in the cell be  $Q$ . Let  $k$  denote the symbol index and  $n$  the chip index, both beginning from zero. We assume that each user transmits a white symbol stream that is independent of the other users' symbol streams. Let matrix  $C$  denote the set of orthogonal spreading codes, the  $q$ th column being the spreading code,  $\mathbf{c}_q$ , of user  $q$ . In practical systems, which employ long codes, this varies with the symbol index  $k$ , but here we assume a short code system ( $C$  remains fixed in time), as is the typical assumption in linear receiver theory.

The system model is depicted in Fig. 1. The transmitted signal  $x(n)$ , the receiver noise,  $\{v_m(n)\}_{m=1}^M$  (assumed additive white Gaussian and of variance,  $\sigma_v^2$ ), and the received signal  $\{y_m(n)\}_{m=1}^M$  are at the chip rate, and  $M$  is the number of receive antennas (or is the product of number of receive antennas and oversampling factor in an oversampled system). The channel impulse responses in this multichannel/single-input multiple-output (SIMO) system are denoted by  $h_m(l)$ , where  $m$  is the branch index. User amplitudes are denoted by  $A_q$ . Out-of-cell interference, soft handoff, and multicode transmission have been ignored in this letter and are considered in a future publication.

## III. RECEIVER ALGORITHM AND DERIVATION

Recall that we said that the idea of using a channel equalizer followed by a despreader is more apparent from the filter form although both forms are equivalent. A channel equalizer gives an output  $\bar{x}(n) \simeq x(n)$  (see Fig. 1). Then, the output of the despreader is  $\hat{s}_1(k) \simeq s_1(k)$  (assuming that the desired user is

$q = 1$ ), since the spreading codes are orthogonal. However, this does not capture the multipath diversity well.

We now derive the RAKE weights for the proposed receiver, and explain how it outperforms existing RAKE receivers for the downlink. It is easier to motivate and derive the proposed receiver based on the combiner form of the RAKE receiver.

Let  $L_h$  denote the maximum order of the channel impulse responses,  $h_m(l)$ . Let  $L_g$  be the order of the combiner or equivalently, let  $L_g + 1$  be the number of taps on each branch (indexed by  $m$ ) of the RAKE filter  $g_m(l)$ . Like any (finite-impulse response) linear receiver, the RAKE receiver operates on a window of the received signal in order to obtain  $\hat{s}_1(k)$ . The duration of this window in chips is  $L_e = L_g + P$ , and its position (in time) determines the delay,  $D$  in estimating the symbols. The quantities  $L_g$  and  $D$  are design parameters.

At each symbol index  $k$ , we stack  $L_e$  received samples,  $y_m(n)$  into the vector,  $\mathbf{y}_m(k)$ . Assuming a delay of  $D$ , we take

$$\mathbf{y}_m(k) = [y_m(kP + P + D - L_e), \dots, y_m(kP + P + D - 1)]^T. \quad (1)$$

Let  $\mathbf{x}(k)$  be the transmit signal window that generates this received signal. Then  $\mathbf{y}_m(k)$  is simply given by

$$\mathbf{y}_m(k) = \mathcal{H}_m \mathbf{x}(k) + \mathbf{v}_m(k) \quad (2)$$

where  $\mathbf{v}_m(k)$  is similarly defined as  $\mathbf{y}_m(k)$ , and  $\mathcal{H}_m$  is the toeplitz channel matrix obtained from  $h_m(l)$ . We assume for simplicity that the memory in the channel is such that  $\mathbf{x}(k)$  does not span more than three symbol periods. Then,  $\mathbf{x}(k)$ , can be written as a function of the symbols as follows:

$$\mathbf{x}(k) = \underbrace{[A_1 \mathbf{C}_1, \dots, A_Q \mathbf{C}_Q]}_C \underbrace{[s_1(k), \dots, s_Q(k)]^T}_{\mathbf{s}(k)}$$

where  $\mathbf{s}_q(k) = [s_q(k-1), s_q(k), s_q(k+1)]^T$ , and

$$\mathbf{C}_q = \begin{bmatrix} \mathbf{c}_q^{(-1)} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_q & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{c}_q^{(+1)} \end{bmatrix}$$

with  $\mathbf{c}_q^{(+1)}$  the first  $D$  elements and  $\mathbf{c}_q^{(-1)}$  the last  $L_g + L_h - D$  elements of  $\mathbf{c}_q$ . Hence, we have

$$\mathbf{y}_m(k) = \mathcal{H}_m \mathbf{C} \mathbf{s}(k) + \mathbf{v}_m(k).$$

Stacking the outputs of the  $M$  channels:  $\mathbf{y}(k) = [\mathbf{y}_1^T(k), \dots, \mathbf{y}_M^T(k)]^T$ , we obtain

$$\mathbf{y}(k) = \mathcal{H} \mathbf{C} \mathbf{s}(k) + \mathbf{v}(k)$$

where  $\mathbf{v}(k)$  is similarly defined as  $\mathbf{y}(k)$ , and  $\mathcal{H} = [\mathcal{H}_1^T, \dots, \mathcal{H}_M^T]^T$ . Define the matrix  $\mathbf{C}_1$  of size,  $(L_g + 1) \times (L_g + P)$  as

$$\mathbf{C}_1 = \begin{bmatrix} \boxed{\mathbf{c}_1^H} & & 0 \\ & \ddots & \\ 0 & & \boxed{\mathbf{c}_1^H} \end{bmatrix}. \quad (3)$$

The output of the RAKE receiver then becomes

$$\begin{aligned}\hat{s}_1(k) &= \sum_{m=1}^M \mathbf{g}_m^H \mathbf{C}_1 \mathbf{y}_m(k) \\ &= \sum_{m=1}^M \mathbf{g}_m^H \mathbf{C}_1 \mathbf{H}_m \mathbf{C}_s(k) + \mathbf{g}_m^H \mathbf{C}_1 \mathbf{v}_m(k)\end{aligned}$$

where  $\mathbf{g}_m$  is the vector of combiner weights. While the above equation gives the symbol estimate in terms of channel parameters and spreading codes, it does not show how it depends on the correlation properties of the codes. We now express it in an alternate form as shown below.

Using the commutativity of the convolution operation, it is easy to show that  $\mathbf{C}_1 \mathbf{H}_m$  can also be written as  $\tilde{\mathbf{H}}_m \tilde{\mathbf{C}}_1$ , where  $\tilde{\mathbf{H}}_m$  is similarly defined as  $\mathbf{H}_m$  but with  $L_g + 1$  rows instead of  $(L_g + P)$  rows, and  $\tilde{\mathbf{C}}_1$  is similarly defined as  $\mathbf{C}_1$  but with  $L_g + L_h + 1$  rows instead of  $L_g + 1$ . The matrix  $\mathbf{R}_1 = \tilde{\mathbf{C}}_1 \mathbf{C}$  captures the correlation properties of the desired user's code with all other codes and is channel-independent. Thus, the symbol estimate can alternately, be written as

$$\hat{s}_1(k) = \sum_{m=1}^M \mathbf{g}_m^H \tilde{\mathbf{H}}_m \tilde{\mathbf{C}}_1 \mathbf{C}_s(k) + \mathbf{g}_m^H \mathbf{C}_1 \mathbf{v}_m(k) \quad (4)$$

$$= \mathbf{g}^H \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{s}(k) + \mathbf{g}^H (\mathbf{I}_M \otimes \mathbf{C}_1) \mathbf{v}(k) \quad (5)$$

where  $\mathbf{g} = [\mathbf{g}_1^T, \dots, \mathbf{g}_M^T]^T$ ,  $\tilde{\mathbf{H}} = [\tilde{\mathbf{H}}_1^T, \dots, \tilde{\mathbf{H}}_M^T]^T$ , and  $\otimes$  denotes the Kronecker product.

We choose  $\mathbf{g}$  so as to minimize the following error function between the symbols and their estimates:

$$f(\mathbf{g}) = \mathcal{E}\{|\hat{s}_1(k) - s_1(k)|^2\}. \quad (6)$$

Since the symbols are independent in time, across users and with respect to the additive noise, with some algebra, it can be shown that

$$\begin{aligned}f(\mathbf{g}) &= \mathbf{g}^H \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{R}_1^H \tilde{\mathbf{H}}^H \mathbf{g} + \sigma_v^2 \mathbf{g}^H (\mathbf{I}_M \otimes \mathbf{C}_1) \\ &\quad \cdot (\mathbf{I}_M \otimes \mathbf{C}_1)^H \mathbf{g} - 2\Re\{\mathbf{g}^H \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{e}_2\} + 1\end{aligned} \quad (7)$$

where  $\mathbf{e}_2$  is the unit vector with a 1 as its second element, and the rest are all zeros. Hence, we obtain

$$\mathbf{g}_{\text{opt}} = \left( \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{R}_1^H \tilde{\mathbf{H}}^H + \sigma_v^2 (\mathbf{I}_M \otimes \mathbf{C}_1) (\mathbf{I}_M \otimes \mathbf{C}_1)^H \right)^{-1} \cdot \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{e}_2 \quad (8)$$

$$\begin{aligned}f(\mathbf{g}_{\text{opt}}) &= 1 - \mathbf{e}_2^H \mathbf{R}_1^H \tilde{\mathbf{H}}^H \left( \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{R}_1^H \tilde{\mathbf{H}}^H \right. \\ &\quad \left. + \sigma_v^2 (\mathbf{I}_M \otimes \mathbf{C}_1) (\mathbf{I}_M \otimes \mathbf{C}_1)^H \right)^{-1} \tilde{\mathbf{H}} \mathbf{R}_1 \mathbf{e}_2\end{aligned}$$

$$\text{SINR}_{\text{opt}} = \frac{1}{f(\mathbf{g}_{\text{opt}})} - 1. \quad (9)$$

The optimum signal-to-interference and noise ratio (SINR) above is a tight (i.e., achievable) *upper bound* on the SINR performance of any downlink RAKE receiver. In the next section we show that this is significantly higher than that achievable with existing RAKE receivers.

From (8), it may appear that the proposed receiver requires explicit knowledge of the channel, the number of users  $Q$  and, the interferer's codes and powers. However, one may determine  $\mathbf{g}_{\text{opt}}$  from the form in (6) using training symbols. Note that some form of training is also required for the equalizer receivers. Also, the weight vector in the proposed receiver is specific to the user

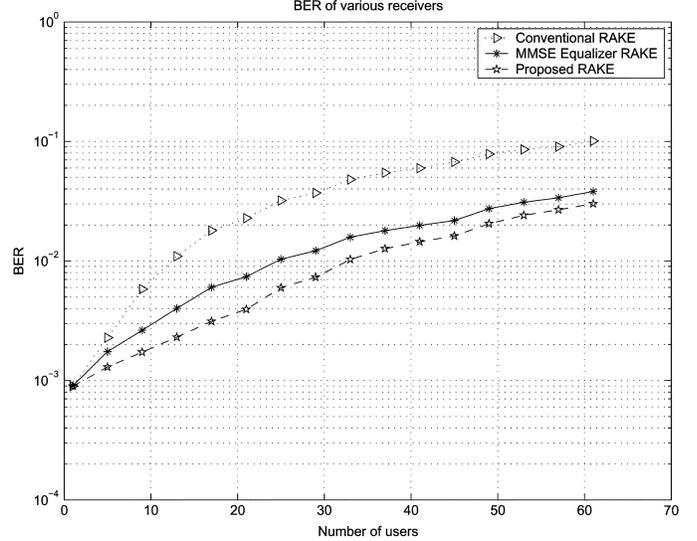


Fig. 2. BER of various receivers.

to be decoded. However, this is not a disadvantage, since in the downlink each user is interested in decoding only his symbols.

*Note:* In [5], a similar criterion has been suggested, but in an uplink setting. The motivation therein was simply based on a natural mathematical criterion arising out of the combiner form of the RAKE. Here, we independently considered this criterion for the downlink, and the idea was motivated by the following. An equalization approach and matched filtering approach have certain drawbacks and advantages over each other that are complementary. Hence, there may exist a solution differing from, or in between, these two extremes that achieves better performance. In the uplink, on the other hand, equalizing the desired user's multipath channel does not offer any significant benefit in terms of interference suppression, and therefore the two cases are fundamentally very different.

#### IV. SIMULATION RESULTS

Fig. 2 shows the average BER of the conventional, proposed, and minimum mean-square error (MMSE) equalizer-based RAKE receivers [2], [3] for a three-path frequency selective fading channel, as the number of users is varied (spreading factor,  $P = 64$ ). The signal power of all the users was assumed to be 8 dB below the noise power (or equivalently SNR per symbol =  $-8 + 10 \log_{10}(64) = 10$  dB). The paths were chosen to be at fixed delays (of zero, three, and eight chips), and the gain of each of these paths was assumed to be independent Rayleigh fading with unit variance. The value of  $L_g$  used was 9. A raised cosine pulse shaping filter of roll-off 0.22 was used at the transmitter. A single receive antenna (realistic for mobiles) with an oversampling factor of two was assumed. The proposed receiver can be seen to significantly outperform the conventional and MMSE equalizer-based RAKE receivers (the zero-forcing equalizer receiver has a very poor performance, i.e., independent of the number of users and, hence is not shown). It can be seen that the relative performance difference between the receivers exhibits an interesting trend as the number of users changes (this is not observed when the desired user power is changed at a fixed loading factor). The plots also enable one to determine the user capacity of the system with

various receivers. It can be seen that for a target BER of 0.01, while the conventional RAKE can support a user capacity of 13 users, and the MMSE equalizer RAKE a user capacity of 24 users, the proposed receiver can support as many as 34 users (this scenario may not be representative of the real cellular network, but it serves to compare the relative performance of the receivers well). The BER performance of the receivers as the raw SNR is varied, on the other hand, does not exhibit any interesting trends or offer any insights.

## V. DISCUSSION

Consider a fixed user. The performance of the receiver depends on the desired signal power and the amount of interchip interference (ICI), intersymbol interference (ISI), MAI, and noise power at its output. On careful examination (see also Fig. 1), the factors on which ICI, ISI, and MAI depend on can be seen to be as shown in the following (ACP refers to autocorrelation properties, CCP refers to cross-correlation properties and  $\bar{h}(l)$ , the composite response of the channel and RAKE filter):

	Q	Sidelobes in $\bar{h}(l)$	ACP	CCP
ICI, ISI	NO	Yes	Yes	No
MAI	Yes	Yes	No	Yes

Recall that if the channel is perfectly equalized, i.e., if  $\bar{h}(l)$  had no sidelobes and the filter response had only one tap, then MAI, ICI, and ISI are all absent but the noise power may be high. The equalization approach simply aims to suppress the sidelobes in  $\bar{h}(l)$  to reduce interference but fails to account for the other factors that interference depends on. For instance, if the number of users is small or if the interferers have low power and the codes have very good correlation properties, then it becomes less important to suppress the sidelobes in  $\bar{h}(l)$  than to tap the diversity in the channel (i.e., improve the SNR). It is well known that RAKE taps matched to the channel (conventional receiver) taps all the diversity in the channel, but this produces significant sidelobes in  $\bar{h}(l)$ . It is more suitable to use such a receiver when the number of interferers is small.

On the other extreme, when the number of interferers is high, it becomes desirable to cancel the multipath.

Depending on the amount of interference, the proposed receiver adjusts the taps so as to balance the amount of diversity and interference in a manner so as to maximize the SINR.

We now prove that the proposed receiver yields as desired, the conventional RAKE receiver when the code correlation properties are very good or ideal, and the zero-forcing equalizer RAKE when the noise is absent. Note that our MMSE criterion is for symbol detection and not channel equalization, and hence, it is not obvious that in the absence of noise, the proposed receiver leads to a zero-forcing equalizer solution (while, on the other hand, if the MMSE criterion was used for channel equalization, then this is obvious).

If the correlation properties of the codes are ideal, i.e., the cross correlation is zero and autocorrelation is a discrete-delta function., then the matrix  $\mathbf{R}_1$  has all zeros except the element in the  $(D+1)$ th row and second column, which is  $A_1P$ . And  $\mathbf{C}_1\mathbf{C}_1^H$  is simply scaled identity. Thus, from (8) we get

$$\mathbf{g}_{\text{opt}} = A_1 (A_1^2 P \mathbf{h} \mathbf{h}^H + \sigma_v^2 \mathbf{I})^{-1} \mathbf{h} \quad (10)$$

where  $\mathbf{h}$  is the  $(D+1)$ th column of  $\tilde{\mathbf{H}}$ . Using matrix inversion lemma, we get

$$\mathbf{g}_{\text{opt}} = \frac{A_1/\sigma_v^2}{1 + (A_1^2 P/\sigma_v^2 \mathbf{h}^H \mathbf{h})} \mathbf{h} \quad (11)$$

which is the conventional RAKE receiver within a scale factor. Now consider the noiseless case. Let  $\mathbf{g}$  be the zero-forcing equalizer within a scale factor, so that  $\mathbf{g}^H \mathbf{H} = \mathbf{e}_{D+1}/P$ . Then, we have from (5)

$$\hat{s}_1(k) = \frac{1}{P} \mathbf{e}_{D+1} \tilde{\mathbf{C}}_1 \mathbf{C} s(k). \quad (12)$$

From the definitions of  $\tilde{\mathbf{C}}_1$  and  $\mathbf{C}$ , this is simply  $s_1(k)$ , and hence the MSE is minimized to zero. This shows that one of the solutions for the proposed receiver is a zero-forcing equalizer in the absence of noise (interestingly, solutions that differ from the zero-forcing equalizer exist when the number of users is small, and we explore this in a future publication).

## VI. CONCLUSION

We proposed a simple interference-suppressing RAKE receiver for the CDMA downlink. The receiver balances the positive and negative effects of multipath well, unlike other RAKE receivers. By analysis, it is clear that the proposed RAKE receiver is the optimum in terms of SINR. We also presented numerical results to show the performance improvement over other downlink RAKE receivers.

We discussed some interesting insights into the working and performance of downlink RAKE receivers and explained how our design takes into account all the factors that MAI (especially loading factor), ICI, and ISI depend on, and balances these effects with the positive effect of multipath, namely diversity. We also derived the optimum SINR that gives the theoretical tight upper bound against which one can compare the performance of any fixed or adaptive RAKE receiver for the downlink (with short codes or long codes).

An interesting and direct application of this receiver is to the direct-sequence spread-spectrum mode of 802.11b standard [6] for wireless local area networks. While there is no MAI here, there is significant ISI and ICI present. Hence, it again becomes important to balance the amount of multipath diversity and, ISI and ICI.

## REFERENCES

- [1] S. Verdu, *Multuser Detection*. Cambridge, U.K.: Cambridge Univ. Press, 1998.
- [2] I. Ghauri and D. T. M. Stock, "Linear receivers for the DS-CDMA downlink exploiting orthogonality of spreading sequences," in *Conf. Rec. 32nd Asilomar Conf. Signals, System and Computers*, vol. 1, Monterey, CA, Nov. 1998, pp. 650–654.
- [3] T. Krauss, M. Zoltowski, and G. Leus, "Simple MMSE equalizers for CDMA downlink to restore chip sequence: Comparison to zero-forcing and RAKE," in *Proc. ICASSP*, vol. 5, Istanbul, Turkey, June 2000, pp. 2865–2868.
- [4] S. Mudulodu and A. Paulraj, "A blind multiuser receiver for the CDMA downlink," in *Proc. ICASSP*, vol. 5, Istanbul, Turkey, June 2000, pp. 2933–2936.
- [5] X. Wang and H. V. Poor, "Space-time multiuser detection in multipath CDMA channels," *IEEE Trans. Signal Processing*, vol. 47, pp. 2356–2374, Sept. 1999.
- [6] IEEE, "Information technology telecommunications and information exchange between system Part 11: Wireless LAN medium access control (MAC) and physical layer (PHY) specifications," IEEE Standard 802.11b, Mar. 1999.