Iterative Power Pricing for Distributed Spectrum Coordination in DSL

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***Abstract*—In this letter we propose a novel distributed technique for dynamic spectrum management of DSL lines. The proposed method generalizes several known techniques, by imposing pricing for use of spectrum. We propose a simple mechanism that allows each line to choose an appropriate pricing function independently of the other lines. Finally, by incorporating a total power constraint, the algorithm is capable of self-correcting an overly ambitious pricing function. We also provide simulated examples based on measured DSL lines.**

***Index Terms*—Interference channel, pricing, dynamic spectrum management, iterative water filling, multicarrier systems, DSL, game theory.**

I. Introduction

Recent years have shown great advances in digital subscriber line (DSL) spectrum management. The public telephone copper line network is limited by crosstalk between lines. As such dynamic management of the lines based on the actual crosstalk channels as well as the active lines becomes an important ingredient in enhancing the overall network performance at the physical layer. Joint transmission over all lines in a binder is still quite complicated. First, equipment already deployed uses the single input single output approach, where each line is operated independently. Second, the unbundling of the copper infrastructure and the deployment of remote terminals makes joint transmission impossible in certain cases. However, dynamic spectrum management (DSM) levels 1-2 [1] where the power spectral density is optimized to enhance overall network performance is still an important tool. The major difference between DSM level 1 and level 2 is the existence of a spectrum management center (SMC) performing the optimization jointly at level 2, while DSM level 1 requires distributed coordination of the lines, where each modem performs its optimization independently of the other lines. Level 1 coordination can be achieved using firmware upgrades for existing DSL modems (which already have a built in power spectral density (PSD) shaping capability). The common approach to distributed coordination is using the iterative water-filling (IWF) algorithm [2], [3]. In this case each modem iteratively optimizes its own transmit PSD against the actual noise caused by the other modems in the binder. A fixed point of the process is a Nash equilibrium [4] of the interference game. However, it is well known that Nash equilibrium points can be highly suboptimal due to the well known Prisoner's dilemma [5], [6]. This suggests that treating the interference scenario as a cooperative game where players can commit to follow certain strategies will improve not only the overall network capacity, but also each individual users' capacity (The payoff in the interference game is the achievable rate or capacity). Other cooperative techniques are the dynamic power back-off (DPBO) [7], the autonomous spectrum balancing (ASB) [8] and the band preference spectrum management (BPSM) [9]. We will discuss these in the next section.

In this paper we study a novel distributed power coordination scheme called the Iterative Power Pricing (IPP) algorithm. We show how other distributed strategies such as fixed margin iterative water-filling (FM-IWF) [2] and DPBO are special cases of the proposed method. In the proposed method each user has a predetermined power price function that depends on the actual line parameters, and therefore can be chosen independently by each user. Given the power price function, each user's objective is to minimize the weighted power under fixed rate and total power constraints. The solution is a generalization of the FM-IWF algorithm, where in each tone the power level depends not only on the interference and the insertion loss, but also on the power price function. In this aspect, this solution is similar to the one offered in the BPSM algorithm. However, unlike BPSM which is a rate adaptive algorithm (and hence uses all available power), the proposed solution guarantees a fixed rate while selectively optimizing the PSD of each user. This fact makes the proposed technique very appealing to operators with fixed service agreements.

II. The Spectrum Management Problem

Assume a binder with L many users. Each user transmits over frequency bands $I_k = [f_k + \delta/2, f_{k+1} - \delta/2] : k = 1, ..., K$, where δ is the tone spacing. δ should be chosen relatively small, so that the line transfer function, $h_{ii}(k)$, can be assumed approximately constant over each band. Let p_k^i be the power allocated by user i to band k. We assume that each user has a desired operating rate R_i and a total power limit of P_i given by:

$$\sum_{k=1}^{K} p_k^i = P_i \ . \quad (1)$$

The modem can measure the line transfer function $\{h_{ii}(k)\}_{k=1}^K$ and total noise plus interference distribution on its own line. Based on these measurements, it optimizes its own power such that it achieves its rate meeting some

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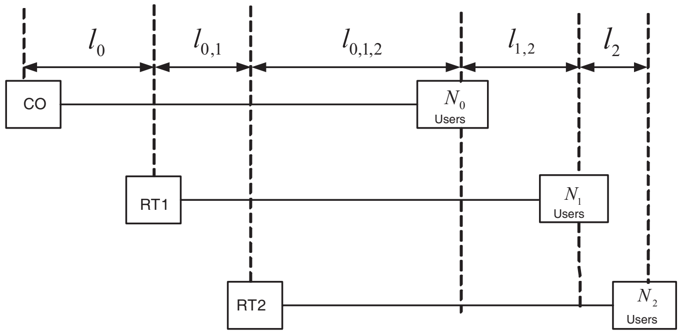

Fig. 1. Typical DSL deployment.

target function. For example the FM-IWF algorithm solves the following problem:

$$\begin{array}{ll} \min_{p_1^i,\ldots,p_K^i} & \sum_{k=1}^K p_k^i \\ \text{Subject to} & R_i = \sum_{k=1}^K \log_2\left(1+\frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i}|h_{ij}(k)|^2 p_k^j + N_i(k)}\right) \end{array} \quad (2)$$

where $N_i(k)$ is the AWGN experienced by user i at tone k. The process continues iteratively until all modems converge. A different approach is used in the rate adaptive iterative water-filling (RA-IWF) algorithm. In the RA-IWF algorithm each modem maximizes its rate under a total power constraint. This results in the following optimization problem:

$$\begin{array}{ll} \max_{p_1^i,\ldots,p_K^i} & \sum_{k=1}^K \log_2\left(1+\frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i}|h_{ij}(k)|^2 p_k^j + N_i(k)}\right) \\ \text{Subject to} & P_i = \sum_{k=1}^K p_k^i \end{array} \quad (3)$$

where P_i is the total power constraint of modem (user) i.

The RA-IWF is a selfish algorithm that lacks cooperation between the users. The RA-IWF algorithm convergence has been studied in [2], [10],[11],[3] and [12]. It converges to competitive Nash equilibria which can be highly suboptimal [5]. The convergence of the FM-IWF algorithm has been theoretically studied only recently in [13] although it has been extensively studied in simulations, showing large improvement over RA-IWF. The performance of the FM-IWF and RA-IWF algorithms can severely degrade on certain lines, especially in asymmetric scenarios as for upstream transmission where user modems are not co-located and different users have different loop lengths [5], [14]. A typical near-far DSL topology including a central office (CO) and remote terminals (RT) deployment is depicted in Fig. 1. Consider for example the case where $l_{1,2} = l_2 = 0$. In the downstream, this is the near-far scenario where the RT users are referred to as the near users and the CO users are the far users. Usually, the CO users experience higher direct channel attenuation than the RT served users due to longer loop lengths. Moreover, the RT users do not typically suffer severe interference from CO based lines, but do cause them substantial interference that reduces their channel capacity. This interference typically increases with frequency. For this reason, only the lower part of the band is available to the CO users, while RT users can use higher frequencies. In order to enhance the overall system performance, the ASB, DPBO and the BPSM algorithms were proposed. In the ASB algorithm every user maximizes the rate of a virtual reference line while maintaining a predetermined rate. This technique has shown an improvement in the rate region in several scenarios where reference line had been chosen to match to the weakest user [8]. However, choosing the reference line in the general case is an open issue. Furthermore, the ASB algorithm has higher computational complexity than the IWF and also require higher level of network information (choosing reference line, its crosstalks and its spectrum). Another approach is the DPBO algorithm which is designed for the near-far scenario. In this algorithm, the strong users restrict their PSD to the upper part of the spectrum by self-imposing a cutoff frequency. The cutoff frequency is maximized, so that lower frequencies are available to CO users.

A generalization of the RA-IWF algorithm is the BPSM algorithm in which each user solves the following problem:

$$\begin{array}{ll} \max_{p_1^i,\ldots,p_K^i} & \sum_{k=1}^K c_i(k)\log_2\left(1+\frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i}|h_{ij}(k)|^2 p_k^j + N_i(k)}\right) \\ \text{Subject to} & P_i = \sum_{k=1}^K p_k^i\ . \end{array} \quad (4)$$

The target function in this case is a weighted rate. The idea is that each user would prefer rates at frequencies with higher price. Note that the optimization is performed autonomously by each user once the weights are allocated. Therefore, the computational complexity is identical to that of the RA-IWF algorithm. The BPSM algorithm has a better rate region. However, it cannot operate in a fixed rate mode and always uses full power. This can be a major limitation for operators with a fixed rate agreement.

III. Iterative Power Pricing

In this section we present the IPP algorithm. We show how other algorithms form special cases of the new scheme and discuss some options for power pricing. The idea underlying the IPP algorithm is to impose a pricing policy on PSD. Each user minimizes a weighted power sum while meeting its target rate and total power constraints. The IPP algorithm is a generalization of the FM-IWF algorithm. In the proposed algorithm the cost of using each frequency band I_k by user i is $c_i(k)$ which satisfies $c_i(k) > 0$ for every k. Now the modem minimizes the total price of the spectrum while meeting its target rate. The problem can be posed as

$$\begin{array}{ll} \min_{p_1^i,\ldots,p_K^i} & \sum_{k=1}^K c_i(k) p_k^i \\ \text{Subject to} & R_i = \sum_{k=1}^K \log_2\left(1+\frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i}|h_{ij}(k)|^2 p_k^j + N_i(k)}\right) \\ & \sum_{k=1}^K p_k^i \le P^i\ , \end{array} \quad (5)$$

where P^i is the total power constraint and R_i is the target rate constraint. This problem is not convex since the equality constraint is not affine. However, simple relaxation of the rate constraint yields

$$\begin{array}{ll} \min_{p_1^i,\ldots,p_K^i} & \sum_{k=1}^K c_i(k) p_k^i \\ \text{Subject to} & R_i \le \sum_{k=1}^K \log_2\left(1+\frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i}|h_{ij}(k)|^2 p_k^j + N_i(k)}\right) \\ & \sum_{k=1}^K p_k^i \le P^i\ , \end{array} \quad (6)$$

which is convex since the inequality constraint is convex. Note however that for the optimal solution the rate is achieved on the boundary of the rate constraint, since if the rate is higher than R_i the power can be further reduced. It should be emphasized that the convexity of (6) holds only for each user individually assuming he optimizes (6) over $p_1^i, \ldots, p_K^i$ while all the other p_k^j are fixed for every $j \neq i$ and every $k = 1, ..., K$. This assumption is satisfied in practice since user i measures the noise plus interference and shapes his spectrum accordingly. However, this solution is no longer optimal once other users shape their spectra as well. Therefore, each user must iteratively optimize his spectrum until the entire system of users converges. The appropriate framework to analyze this process is game theory where each user is a player who competes to optimize his utility function.

To solve (6) we use duality theory. The Lagrangian is given by

$$\begin{aligned} &G\left(\mathbf{p}_i | \langle \mathbf{p}_j : j \neq i \rangle\right) = \sum_{k=1}^{K} c_i(k) p_k^i \\ &+\tilde{\lambda}_1 \left(R_i - \sum_{k=1}^{K} \log_2\left(1 + \frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i} |h_{ij}(k)|^2 p_k^j + N_i(k)}\right)\right) \\ &+\lambda_2 \left(\sum_{k=1}^{K} p_k^i - P^i\right) , \end{aligned} \tag{7}$$

where $\tilde{\lambda}_1$, λ_2 are the Lagrange coefficients related to the rate and power constraints respectively. This can be rewritten as

$$G\left(\mathbf{p}_i | \langle \mathbf{p}_j : j \neq i \rangle\right) = \sum_{k=1}^{K} G_k\left(\mathbf{p}_i | \langle \mathbf{p}_j : j \neq i \rangle\right) + \tilde{\lambda}_1 R_i - \lambda_2 P^i \tag{8}$$

where

$$\begin{aligned} &G_k\left(\mathbf{p}_i | \langle \mathbf{p}_j : j \neq i \rangle\right) = c_i(k) p_k^i \\ &-\tilde{\lambda}_1 \log_2\left(1 + \frac{|h_{ii}(k)|^2 p_k^i}{\sum_{j\neq i} |h_{ij}(k)|^2 p_k^j + N_i(k)}\right) + \lambda_2 p_k^i . \end{aligned} \tag{9}$$

Taking the derivative with respect to p_k^i we obtain (10) at the top of the following page where $\lambda_1 = \tilde{\lambda}_1 \log_2 e$. Further simplification yields

$$(c_i(k) + \lambda_2)\left(\sum_{j=1}^{L} |h_{ij}(k)|^2 p_k^j + N_i(k)\right) - \lambda_1 |h_{ii}(k)|^2 = 0 . \tag{11}$$

Separating p_k^i from the other variables and dividing by $|h_{ii}(k)|^2$ we obtain

$$p_k^i + g^i(k) = \frac{\lambda_1}{c_i(k) + \lambda_2} \tag{12}$$

where

$$g^i(k) = \frac{\sum_{j\neq i} |h_{ij}(k)|^2 p_k^j + N_i(k)}{|h_{ii}(k)|^2} \tag{13}$$

represents the normalized noise plus interference PSD. This is equivalent to water-filling except that the water level at each tone is normalized by $c_i(k) + \lambda_2$. Hence, a higher price at a particular frequency would reduce the power allocated to that frequency. The Lagrange multipliers are obtained via the constraints in (5). By complementarity (see e.g. [15]) the first multiplier, λ_1, is larger than zero since the target rate constraint is satisfied with equality at the optimal point. The second Lagrange multiplier λ_2 will be zero if the total power constraint is strictly met at the optimum

$$\sum_{k=1}^{K} {p_k^i}^* < P^i , \tag{14}$$

where $\left\{{p_k^i}^*\right\}_{k=1}^{K}$ is the optimal solution of (5). If the constraint in (14) is satisfied with equality, then $\lambda_2 > 0$. In this case λ_2 modifies the power price function such that the price is more uniform. The IPP algorithm approaches FM-IWF for $\lambda_2 >> \max_k c_i(k)$ since the power function becomes flat. Each user solves his optimization problem (5) independently as follows; First, λ_2 is set to zero, then p_k^i is obtained via (12) where λ_1 is determined via the rate constraint given in (5). If the resulting solution does not satisfy the total power constraint, then λ_2 is set to one, λ_1 and the power distributions are recomputed and the process is repeated iteratively (each time doubling λ_2) until the power constraint is satisfied. Once λ_2 is sufficiently large so that the total power constraint is met, a bi-section is used to optimize λ_2 between its current value and its previous value. If λ_2 diverges to infinity (our stopping rule requires that $\lambda_2 >> \max_k c_i(k)$), then the target rate of user i is infeasible under (14) even using FM-IWF. In this case, user i employs the RA-IWF algorithm to reach its maximum rate. This optimization is performed iteratively and autonomously by all users until they converge. By our stopping rule, convergence of each user is ensured even when the target rate is infeasible (in which case it settles for its best achievable rate). However, in practice whenever the target rate was feasible, the algorithm achieved the target rate. A single-user iteration of the IPP algorithm is summarized [1] in Table I. The FM-IWF algorithm is a special case of IPP where $c_i(k) = 1$ for every k and i. In addition, the DPBO algorithm is a special case in which $c_i(k) = C_0 > 1$ for every k such that $f_k < f_c$ and $c_i(k) = 1$ otherwise.

A. Power price function

Choosing the power price function is an important issue in the IPP algorithm. As discussed in Section II, lower-loss transfer function of a particular user results in stronger interference to other users, especially to those with longer loop-lengths. Therefore, the power price function should encourage users with a low-loss transfer function to exploit higher frequencies as much as possible. We propose the channel transfer function as a power price function, i.e.

$$c_i(k) = a_i e^{-\beta_i \sqrt{k}} \tag{15}$$

where a_i and β_i are chosen to fit the actual transfer function, $|h_{ii}(k)|$. This is a positive, monotonically decreasing function of k which gives preference to higher frequencies. Each user attempts to reach his target rate using the high frequency part of the spectrum. However, if user i cannot reach his target rate, the preference to higher frequencies is moderated by the Lagrange multiplier λ_2. Thus, in every asymmetric channel scenario, strong users allocate more power to higher

[1] In Table I, the algorithm is performed autonomously by every user. However it can be performed simultaneously by all of the users with some timing mechanism.

$$\frac{\partial G}{\partial p_k^i} = c_i(k) + \lambda_2 - \lambda_1 \frac{\sum_{j\neq i} |h_{ij}(k)|^2 p_k^j + N_i(k)}{\sum_{j=1}^{L} |h_{ij}(k)|^2 p_k^j + N_i(k)} \frac{|h_{ii}(k)|^2}{\sum_{j\neq i} |h_{ij}(k)|^2 p_k^j + N_i(k)} = 0 \tag{10}$$

TABLE I
THE IPP ALGORITHM

Assume a power constraint P^i of user i.
Let $R^{i,\text{target}}$, be the rate constraint. Then
Main function
Set $c_i(k) = a_i e^{-\beta_i \sqrt{k}}$ such that β_i and a_i fit the transfer function
  Initialize $\lambda_2 = 0$, and set a small $\epsilon > 0$
  Estimate noise plus interference $g^i(k)\ \forall k$
    (where $g^i(k)$ is defined in(13))
  $p_1^i, ..., p_K^i = \textbf{set\_power}(g^i(1), ..., g^i(K), \lambda_2)$
  If $\sum_{k=1}^{K} p_k^i > P^i$
   Initialize $\lambda_2^{\min} = 0,\ \lambda_2^{\max} = 1,\ \lambda_2^{\text{flag}} = 0$
   While $|\sum_{k=1}^{K} p_k^i - P^i| > \epsilon$
   $\lambda_2 = \lambda_2^{\text{flag}}(\lambda_2^{\max} + \lambda_2^{\min})/2 + (1 - \lambda_2^{\text{flag}})\lambda_2^{\max}$
   $p_1^i, ..., p_K^i = \textbf{set\_power}(g^i(1), ..., g^i(K), \lambda_2)$
   If $\sum_{k=1}^{K} p_k^i > P^i$
    $\lambda_2^{\min} = \lambda_2,\ \lambda_2^{\max} = \left(2 - \lambda_2^{\text{flag}}\right)\lambda_2^{\max}$
   else $\lambda_2^{\max} = \lambda_2,\ \lambda_2^{\text{flag}} = 1$
   end if
   if $\lambda_2 > 100 \max_k(c_i(k))$, perform RA-IWF and brake
   end if
  end while
  end if
Function $p_1^i, ..., p_K^i = \text{set\_power}(g^i(1), ..., g^i(K), \lambda_2)$
  Initialize $\lambda_1^{\min} = \lambda_1^{\text{flag}} = p_k^i = 0\ \forall k,\ \lambda_1^{\max} = 1$
  While $\left|\sum_{k=1}^{K} \log_2\left(1 + \frac{p_k^i}{g^i(k)}\right) - R^{i,\text{target}}\right| > \epsilon$
  $\lambda_1 = \lambda_1^{\text{flag}}(\lambda_1^{\min} + \lambda_1^{\mathbf{max}})/2 + \left(1 - \lambda_1^{\text{flag}}\right)\lambda_1^{\max}$
  . $p_k^i = \max\left\{\frac{\lambda_1}{c_i(k)+\lambda_2} - g^i(k), 0\right\}$,$\forall k$
   If $\sum_{k=1}^{K} \log_2(1 + \frac{p_k^i}{g^i(k)}) < R^{i,\text{target}}$
   $\lambda_1^{\min} = \lambda_1,\ \lambda_1^{\max} = \left(2 - \lambda_1^{\text{flag}}\right)\lambda_1^{\max}$
   else $\lambda_1^{\max} = \lambda_1,\ \lambda_1^{\text{flag}} = 1$
  end if
  end while

frequencies and reduce the interference seen by weak users at low frequencies. It is the case in every non-symmetrical scenario. This price function achieved good performance as shown Section IV. Moreover, this algorithm is simple and does not require centralized management, since each user can determine its power price function based on the measured line parameters. Note that the fixed rate constraint implies that users are not punished if they achieve their target rate while utilizing only the higher part of the spectrum.

B. convergence of the IPP algorithm

The convergence of the FM-IWF algorithm has been studied only recently. In [13], sufficient conditions for existence and uniqueness of a solution and for convergence of the FM-IWF algorithm without total power constraint were provided. This may represent a case in which users are limited by crosstalk much more than by thermal noise. Thus if the power vectors of all of the users are increased or decreased by the same factor, the rates remain approximately the same. To generalized these results to the IPP algorithm, we perform the following transformation:

$$\tilde{p}_k^i = \frac{p_k^i}{c_i(k)},\quad \tilde{h}_{ij}(k) = \frac{h_{ij}(k)}{\sqrt{c_j(k)}} \tag{16}$$

where we require that $c_i(k) > 0$. In this case (6) becomes (discarding the total power constraint)

$$\begin{array}{ll} \min_{\tilde{p}_1^i,\ldots,\tilde{p}_K^i} & \sum_{k=1}^{K} \tilde{p}_k^i \\ \text{Subject to} & R_i \leq \sum_{k=1}^{K} \log_2\left(1 + \frac{|\tilde{h}_{ii}(k)|^2 \tilde{p}_k^i}{\sum_{j\neq i} |\tilde{h}_{ij}(k)|^2 \tilde{p}_k^j + N_i(k)}\right) \end{array} \tag{17}$$

which is precisely the FM-IWF algorithm with scaled channel $\tilde{h}_{ij}(k)$. Thus all of the available results for the FM-IWF algorithm can be incorporated to the IPP algorithm using (16). A solution of the IPP algorithm is a power allocation $\left\{\left\{p_k^i\right\}_{k=1}^{K}\right\}_{i=1}^{L}$ such that (17) is satisfied for every $1 \leq i \leq L$.

Using (17) we obtain the following theorem (based on [13], Corollary 7).

Theorem 1: A sufficient condition for existence of a solution to the IPP algorithm is that

$$\begin{array}{c} \sum_{j\neq i} \frac{|h_{ij}(k)|^2 c_i(k)}{|h_{ii}(k)|^2 c_j(k)} < \frac{1}{e^{R_i}-1}, \\ \forall k \in \{1, \ldots, K\},\ i \in \{1, \ldots, L\} \end{array} \tag{18}$$

Note that in the case of the IPP algorithm, (18) is function of $c_i(k)$ while in the FM-IWF, it is only function of $h_{ij}(k)$ (since $c_i(k) = 1,\ \forall i, k$). This degree of freedom enables to choose a power price functions such that (18) is satisfied even when it is not satisfied for the FM-IWF algorithm.

Similarly to Theorem 1, we obtain the following theorem:

Theorem 2: A sufficient condition for uniqueness of a solution and convergence of the IPP algorithm without total power constraint, is that all principal minors of the matrix defined in (19) at the top of the following page, are positive. Where $\bar{p}_k^j$ is the j component of the vector

$$\begin{pmatrix} \bar{p}^1(k) \\ \vdots \\ \bar{p}^L(k) \end{pmatrix} = (\mathbf{Z}_k)^{-1} \begin{pmatrix} N_1(k)\left(e^{R_1} - 1\right) \\ \vdots \\ N_L(k)\left(e^{R_L} - 1\right) \end{pmatrix} \tag{20}$$

and $\mathbf{Z}_k$ is a matrix whose i^{th} diagonal entry is $|\tilde{h}_{jj}(k)|^2$ and its ij off diagonal entry is $-\left(e^{R_i} - 1\right)|\tilde{h}_{ij}(k)|^2$.
Note that the set of rates for which the IPP algorithm converges is greater than the equivalent set for the FM-IWF algorithm because of the additional degree of freedom in choosing power-price function.

IV. SIMULATION RESULTS

In this section we compare the IPP algorithm to the FM-IWF algorithm. The channel transfer matrix was a measured binder provided by France Telecom research labs[2][16]. Two

[2]We thank R. Tarafi, M. Ouzzief, F. Gauthier and H. Marriotte for providing the data which was used to generate the transfer functions. The data used was measured as part of the EU-FP6 U-BROAD project, contract no. 506790

$$\left[\bar{\mathbf{B}}(R_1,..,R_L)\right]_{ij} \triangleq \begin{cases} e^{-R_i}, & \text{if } i=j \\ -e^{-R_i}\max_k\left(\frac{|h_{ij}(k)|^2}{|h_{jj}(k)|^2}\frac{N_j(k)+\sum_{j\neq j'}|\tilde{h}_{jj'}(k)|^2\bar{p}_{j'}(k)}{N_i(k)}\right), & \text{otherwise} \end{cases} \tag{19}$$

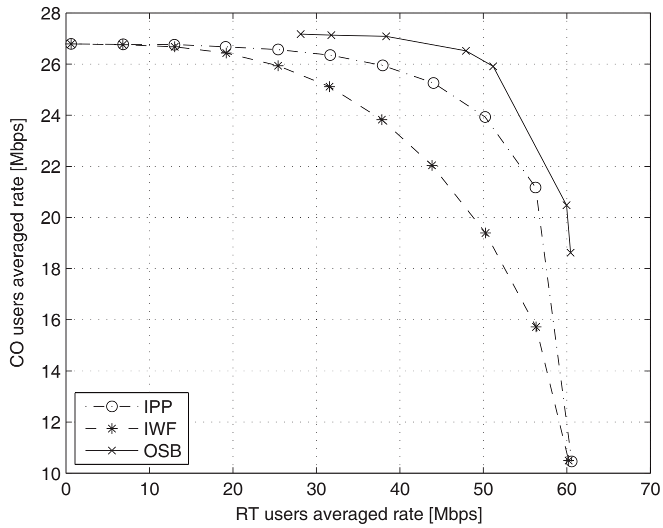

Fig. 2. Rate regions of the IPP algorithm versus the IWF algorithm for near far scenario. The OSB is presented as a bound.

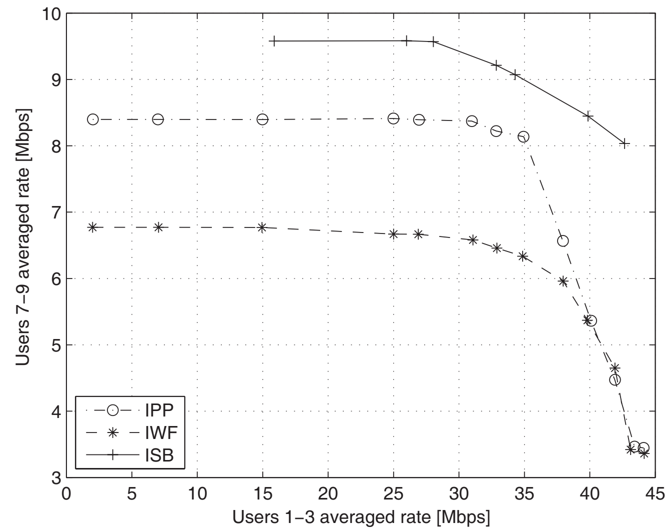

Fig. 3. Rate regions of the IPP (circles) versus the IWF (stars) for three distances scenario depicted in Fig. 1. Users 4-6 transmit at a constant rate of 30 Mbps.

different setups, a near-far downstream scenario with two different distances and an upstream scenario with three distances, were simulated. Discrete Multi Tone modulation (DMT) was used. The total power was 14.5 dBm and the AWGN had PSD of -140 dBm/Hz. These parameters are based on the VDSL and ADSL standards G.993.2 and G.992.1. Standard VDSL bandplan 998 was used.

The first simulation tested the near-far scenario. The network setup is depicted in Fig. 1 where $N_0 = N_1 = 3$, and $N_2 = 0$. In this setup three users were served from the CO located 1500 m away, and the other three users were served from a RT with a loop length of 600m. The cables overlap was 300m, where $l_0 = 1200$ m, $l_{0,1} + l_{0,1,2} = 300$m, and $l_{1,2} = 300$m. All users were VDSL and transmitted in the downstream direction. The RT served users shaped their spectrum using the IPP algorithm. The price functions $c_i(k)$ of the RT users are given in (15) and fit their channel transfer functions. In order to obtain a rate region, the CO (weak) users performed RA-IWF to reach their maximal rate. The rate region is compared to the FM-IWF algorithm in which the RT and the CO users performed FM-IWF and RA-IWF respectively. The results are presented in Fig. 2 and compared to the rate of the optimal spectrum balancing (OSB) [17] algorithm which serves as a bound. It can be seen that IPP outperforms IWF "at all rates" and achieves performance very close to that of OSB.

In the second simulation, a three-distance upstream scenario was tested. The network setup is depicted in Fig. 1 where, $N_0 = N_1 = N_2 = 3$, and $l_0 = l_{0,1} = 0$ which means than the CO, RT_1, and RT_2 are co-located. Users (4-6) have a loop length of 600 m and transmit at a constant rate of 30 Mbps. Users (1-3) have a loop length of 300 m, and users (7-9) have a loop length of 1000 m. Users (1-6) used the IPP algorithm with power price function given in (15) where the parameters were chosen to fit each user's transfer function. In order to obtain the rate region, users (7-9) (weak users) performed RA-IWF. The rate region is presented in Fig. 3 and compared to the rate region of the FM-IWF and to the rate obtained by the iterative spectrum balancing (ISB) [18] which serves as a bound.

V. Conclusions

In this paper we have proposed a novel distributed dynamic spectrum management technique for DSL networks. The method operates with a fixed rate and fixed margin which is important for fixed rate agreement scenarios. The unique property of the proposed technique is its capability to self-correct overly ambitious pricing functions. The proposed method was demonstrated on diverse DSL scenarios using measured transfer functions and achieved near optimal performance.

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