Chapter C- X-5

Information theoretic waveform design for tracking multiple targets using phased array radars

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1. Introduction

The problem of radar waveform design is of fundamental importance in designing state-of-the-art radar systems. The possibility to vary the transmitted signal on a pulse-by-pulse basis opens the door to great enhancement in estimation and detection capability as well as improved robustness to jamming. Furthermore modern radars can detect and track multiple targets simultaneously. Therefore, designing the transmitted waveforms for detecting and estimating multiple targets becomes a critical issue in radar waveform design.

Most of existing waveform design literature deals with designs for a single target. One of the important tools in such designs is the use of information theoretic techniques, see [1] for a review of early results of Woodward and others. Bell [1] was also the first to propose using the mutual information between a random extended target and the received signal. His optimization led to a water-filling type strategy. In his paper he assumed that the radar signature is a realization of random Gaussian process with a known power spectral density (PSD). However, when considering real-time signal design we can use his approach to enhance the next transmitted waveform based on the *a priori* known signature. Whereas waveform design literature concentrated on the estimation of a single target, modern radars treat multiple targets. Therefore, the development of design techniques for multiple targets is of critical importance to modern radar waveform design.

Recently a great interest has emerged in MIMO radars, where multiple transmit and receive antennas are used with large spatial aperture to overcome target fading, see [2] and the references therein for a review of statistical MIMO radar literature. Much less has been done on MIMO waveform design. Two papers related specifically to waveform design in the MIMO context are [2], [3]. Yang and Blum

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applied MIMO point-to-point communication theory to design radar waveforms by water-filling the power over the spatial modes of the overall radar scene (channel). They also showed that optimizing the non-causal MMSE and optimizing the mutual information leads to identical results. This finding provides another justification for using the maximum mutual information criterion for the radar waveform design problem. Their work is a novel extension of the work of [1]. However, one should note that by water-filling with respect to the spatial modes, higher power is allocated to the stronger targets. This approach is reasonable when using a single target through several remotely located antennas. However, this approach is not always desirable, when tracking multiple targets. De Maio and Lops [3] proposed design criterion for space time codes for MIMO radars based on mutual information. They also analyzed the detection probability of these techniques under the statistical MIMO diversity model that assumes independent scattering towards each of the MIMO systems component and point targets.

The approach proposed in this paper is different. We are interested in reception and transmission for tracking multiple extended targets, by using the insights provided by multi-user information theory instead of the point-to-point MIMO approach. These insights are applied here for the context of coherent phased array receivers that are capable of transmitting independent signals simultaneously, as well as for optimizing the waveforms for extended targets. We assume high range resolution and that the various extended targets are treated as independent signals that need to be estimated. In the optimization process we provide priorities through a set of priority vectors. A linear combination of the mutual information between each radar beam and its respective target is optimized. This leads to a highly complicated optimization problem. However, by assuming *linear* pre- and post-processing and an independent estimation of the targets, we are able to reduce the waveform design problem to a problem similar to that of the centralized dynamic spectrum allocation in communication. Furthermore, recent advances in convex optimization (see [4] and the references therein) open the way to design techniques specifically tailored for radar waveforms that would be suitable for estimating the parameters of multiple targets. While we concentrate on the phased array coherent reception, statistical MIMO modeling might be incorporated into this context where each transmitter optimizes its transmit spectrum towards the various independent targets.

The paper has two main contributions: First we extend Bell's results to the design of multiple transmit waveforms, each optimized towards a specific target where the transmitter employs multiple beamformers as well as receiver. Finally, an optimization algorithm is proposed. We show that using duality theory the problem can be reduced to a search over a single parameter and parallel low-dimensional optimization problems at each frequency. Interestingly even though the proposed design criterion for multiple waveforms is non-convex, strong duality [4] still holds, which allows us to solve the simpler dual problem.

2. Targets Model

In this section we describe the extended targets model. While classical radar target models assume far-field point source targets. This is indeed the case when the radar pulse is relatively narrow-band so that the range span of the target is well within a single range cell. In contrast to these point source models, many modern radars are often capable of transmitting very wide-band pulses or alternatively use very wide-band compressed signals. In this case delays across the target are similar in nature to multipath propagation. This results in a complex target impulse response. Some examples of wide band responses of airplanes and missiles can be found, e.g., in [5]. Under these conditions the targets are called extended targets, which are the focus of the current paper. Models for such targets have been used e.g., in [1]. Extended targets naturally appear in imaging and high range resolution applications [6] where the radar signal bandwidth is sufficiently large so that the target is not contained in a single range cell. Such target models were already described by Van Trees [7] where they are termed range selective targets. Extended targets typically have multiple reflection centers, each with independent statistical behavior. The target impulse response (TIR) is therefore modeled as:

$$\kappa(t) = \sum_{\ell=1}^{L} \kappa_\ell \delta(t - \tau_\ell) \tag{1}$$

where $\tau_\ell < 2d / c$, d is the radial span of the target and c is the speed of light. κ_ℓ are the individual random reflection coefficients. These coefficients can be modeled either deterministically or using the extended Swerling χ^2 models [8]. The temporal variability of the target response is mainly determined by the speed of the target and the carrier frequency. The reflected radar signal is given by:

$$y(t) = \int_0^{\tau_{\max}} \kappa(\tau) s(t - \tau) d\tau \tag{2}$$

where $\kappa(\tau)$ is the TIR and $s(t)$ is the radar signal. Since the targets have non-trivial impulse response, we can consider also the target frequency response (TFR) given by:

$$h(f) = \int_0^{\tau_{\max}} \kappa(\tau) e^{-j2\pi f \tau} d\tau$$

For stochastic target models, we will be interested in the PSD of the TIR which now becomes a stochastic process. Significant amplitude deviations will only appear for extended targets. Typically we will sample the frequency domain and assume that $h(f)$ is given at a set of K equally spaced frequencies $h(f_k) : k = 1, ..., K$.

3. Phased Arrays Transmit And Receive Beamforming

Our main interest in this paper is with phased arrays which use both transmit and receive beamforming. We now provide the basic model for transmit and receive beamforming for multiple targets. We can typically assume that the array manifold is independent of frequency. This holds as long as the transmit signal bandwidth is small relative to the carrier frequency. In this section we will maintain this assumption, but we will use the more general formulation in the following sections. Assume that we have a phased array radar capable of transmitting and receiving simultaneously L beams. Each beam is characterized by transmit beamforming vectors $<\mathbf{u}_m, m=1,\ldots,L>$ and receive vectors $<\mathbf{w}_m, m=1,\ldots,L>$. The baseband signals $s_i(t)$ that are transmitted over the respective beams are multiplied by the transmit beamforming vectors and linearly combined to form the baseband transmit vector

$$\mathbf{t}(t)=\sum_{m=1}^{L}\mathbf{u}_m s_m(t) \tag{3}$$

Let $\mathbf{a}(\theta)$ be the array manifold of the array towards direction θ. The transmitted signal is reflected at a target with direction θ and range R is given by:

$$y(\theta,t)=\frac{1}{R}\int_0^{\tau_{\max}}\kappa(\tau)\mathbf{a}(\theta)^*\mathbf{t}\left(t-\left(\frac{R}{c}+\tau\right)\right)d\tau \tag{4}$$

where we neglect the free space attenuation across the target (since $c\tau_{\max} << R$). The reflection of a target at direction θ is received by the array as $\mathbf{x}(t)=1/R\mathbf{a}(\theta)y(\theta,t)$. Assuming that we have L targets with directions $<\theta_1,\ldots,\theta_L>$ and ranges $<R_\ell : \ell=1,\ldots,L>$ we obtain that the received signal is given by:

$$\mathbf{x}(t)=\sum_{\ell=1}^{L}\mathbf{a}(\theta_\ell)\int_0^{\tau_{\max}^{(\ell)}}\frac{1}{R_\ell^2}\kappa_\ell(\tau)\sum_{m=1}^{L}\mathbf{a}(\theta_\ell)^*\mathbf{u}_m s_m\left(t-\left(\frac{R_\ell}{c}+\tau\right)\right)d\tau \tag{5}$$

To enhance the signal to noise ratio by suppressing directional interference and other targets side-lobes we apply L transmit beamforming vectors $<\mathbf{w}_\ell : \ell=1,\ldots,L>$ to the received signal resulting in

$$z_\ell(t)=\mathbf{w}_\ell^*\mathbf{x}(t) \tag{6}$$

This is the standard way to decouple the estimation between azimuth cell, since it greatly reduces the number of targets that need to be estimated jointly. Using (5) and translating to the frequency domain we now obtain

$$z_\ell(f) = \sum_{\ell=1}^{L}\sum_{m=1}^{L}\frac{1}{R_\ell^2}h_\ell(f)\left(\mathbf{w}_\ell^*\mathbf{a}(\theta_\ell)\right)\left(\mathbf{a}(\theta_\ell)^*\mathbf{u}_m\right)s_m(f) \quad (7)$$

where $h_\ell(f)$ is the ℓ'th target frequency response. To simplify notation from this point on we will assume that $\frac{1}{R_\ell^2}$ is included in the target signature. When the targets are resolved in range or in angle we can separate them in the time domain or using receive beamforming, which means that only certain range cells will include target information. This will imply that each $z_\ell(t)$ is subject to only receiver and clutter noise. When targets are partially overlapping both in range and angle (see e.g., Gini et.al [9]) each beam contains residual interference from other targets. In this case the noise PSD contains contributions from other targets. The next step is a correlation of each $z_\ell(t)$ with $s_\ell(t)$ to obtain the target impulse responses. These impulse responses can be used to enhance the transmitted signal in the next pulse. This can be done by using the targets PSD when the target reflection centers (and therefore the target signature PSD) exhibit pulse to pulse variations as in the Swerling type II models or by using the latest estimate when the variations are sufficiently small. The exact choice of the model depends on the target velocity, radar carrier frequency and PRI or compressed pulse duration.

Since the targets are selective in range, we also obtain that certain frequencies are more reflective. This implies that concentrating the transmitted power according to the target frequency response is beneficial in terms of the information we obtain regarding the target signature.

3.1. Multi-Target Tracking

Finally we discuss the tracking model, and its relationship to the signal design problem. In general multi-target tracking is a well established topic [10]. Our paper is not focused on the tracking itself but rather on the adaptive design of the transmitted waveform, based on the target parameters. Therefore the design will be affected by the following parameters:

- The azimuth and range cells that include each target. These influence the transmit and receive beamforming vectors.
- Target motion during the time interval between pulses relative to the arrier wavelength. This parameter decides the statistical model of choice for estimating the TIR. If the motion is large compared to the wavelength then we can use only target PSD as in the Swerling type II or IV, while if the motion is small so that the local reflection environment can be considered static we can use the previous estimate of the TIR as a predictor for the next realization.

Since our main interest is in adaptive design of the pulse, we shall assume a given estimate for these parameters, assume that the transmit and receive beamforming vectors for each beam are provided by the tracking system, and limit our interest to the radar signal design problem. This is a reasonable approach since the described parameters are provided by existing systems. We will also assume that the radar control provides us priorities with respect to the various targets to be tracked. These priorities are given by a vector of constants. The choice of these constant is important. However, the relative priorities can be determined from the overall SNR estimate of each target as well as its temporal variability, which depends on the target speed. Typically we would like to allocate higher priority to rapidly moving targets or weak targets that are harder to track.

4. Information Theoretic Approach To Waveform Design

In this section we extend the waveform design paradigm of Bell [1] to the case of multiple radar transmitters and receivers. In order to study the trade-off between various radar receivers, we use a linear convex combination of the mutual information between the targets and the received signal at each receiver beam oriented at that specific target.

We begin by revising the received signal model. Assume that an array with p elements simultaneously transmits L waveforms. The transmitted signal at frequency f_k is given by:

$$\mathbf{t}(f_k) = \sum_{l=1}^{L} \mathbf{u}_\ell(f_k) s_\ell(f_k), \qquad k = 1, \ldots, K \tag{8}$$

where $\mathbf{u}_\ell(f_k)$ are the beamformer coefficients for the ℓ'th waveform designed for the ℓ'th target at frequency f_k, and $s_\ell(k)$ is the corresponding waveform at frequency f_k. We assume channel reciprocity; i.e., if the receive steering vector is $\mathbf{a}(\theta_\ell, f_k)$, then the transmitted signal arrives at the target with channels $\mathbf{a}^*(\theta_\ell, f_k)$. The signal reflected from the ℓ'th target having signature $\mathbf{h}_\ell = < h_\ell(f_k), k = 1, \ldots, K >$ is therefore given by:

$$\mathbf{y}_\ell(f_k) = \sum_{m=1}^{L} \left(\mathbf{a}^*(\theta_\ell, f_k)\mathbf{u}_m(f_k)\right) h_\ell(f_k) s_m(f_k) \tag{9}$$

for $k = 1, \ldots, K$ (note that we have used index m to enumerate the transmitted waveforms, $\mathrm{m} = 1, \ldots, \mathrm{L}$, since ℓ is reserved for the target). Hence, the received signal at the array is given by:

$$\mathbf{x}(f_k) = \sum_{m=1}^{L} \mathbf{R}(f_k)\mathbf{u}_m(f_k) s_m(f_k) + \mathbf{v}(f_k)$$

(10)

where $\mathbf{R}(f_k) = \sum_{\ell=1}^{L} \mathbf{R}_\ell(f_k)$. And $\mathbf{R}_\ell$ is the rank-one matrix given by:

$$\mathbf{R}_\ell(f_k) = h_\ell(f_k)\mathbf{a}(\theta_\ell, f_k)\mathbf{a}^*(\theta_\ell, f_k) \quad (11)$$

Assume that a beamformer $\mathbf{w}_\ell(f_k)$ is used to receive the ℓ'th target, resulting in

$$z_\ell(f_k) = \mathbf{w}_\ell^*(f_k)\mathbf{x}(f_k) = \mathbf{w}_\ell^*(f_k)\sum_{m=1}^{L}\mathbf{R}(f_k)\mathbf{u}_m(f_k)s_m(f_k) + \nu'_\ell(f_k) \quad (12)$$

where $\nu'_\ell(f_k) = \mathbf{w}_\ell^*(f_k)\nu(f_k)$ is the received noise and clutter component of the ℓ'th beam. Let $\sigma_{\nu'_\ell}^2(f_k) = E\left|\nu'_\ell(f_k)\right|^2 \Delta f$ be the ℓ'th beam noise power at frequency f_k. After algebraic manipulations we can show that Mutual information the mutual information between the ℓ'th beam and the ℓ'th target at frequency f_k is now given by:

$$I_k\left(h_\ell(f_k), z_\ell(f_k)\right) = \log\left(1 + \frac{|z_\ell^t(f_k)|^2}{|z_\ell^n(f_k)|^2 + \sigma_{\nu'_\ell}^2(f_k)}\right)\Delta f \quad (13)$$

where the signal reflected from the ℓ'th target is denoted by:

$$z_\ell^t(f_k) = \sum_{m=1}^{L}\mathbf{w}_\ell^*(f_k)\mathbf{R}_\ell(f_k)\mathbf{u}_m(f_k)s_m(f_k) \quad (14)$$

while the noise and inter-target interference component at the ℓ'th beam is given by:

$$z_\ell^n(f_k) = \sum_{n\neq l}\sum_{m=1}^{L}\mathbf{w}_\ell^*(f_k)\mathbf{R}_n(f_k)\mathbf{u}_m(f_k)s_m(f_k) + \nu'_\ell(f_k) \quad (15)$$

We assume that the radar allocates one beam towards each target, since non-linear joint processing of all the beams would lead to an infeasible receiver. Therefore, the total mutual information between the ℓ'th beam and the ℓ'th target is given by:

$$I\left(\mathbf{h}_\ell; \mathbf{z}_\ell \mid \mathbf{s}\right) = \sum_{k=1}^{K} I_k\left(h_\ell(f_k); z_\ell(f_k) \mid s(f_k)\right)\Delta f \quad (16)$$

where $\mathbf{z}_\ell = < z_\ell(f_k) : k = 1,...,K >$ and $\mathbf{h}_\ell = < h_\ell(f_k) : k = 1,...,K >$ are the received signals using the ℓ'th received beam and the ℓ'th target signature, respectively.

$\mathbf{s}_m = \left[s_m(f_1),...,s_m(f_K)\right]^T$ are the signal waveform samples directed towards the m'th target,

$$\mathbf{S} = \left[\mathbf{s}_1,...,\mathbf{s}_L\right] \tag{17}$$

is the complete spatio-temporal waveform matrix, and $\mathbf{s} = \mathrm{vec}(\mathbf{S})$. Assuming that the beamforming vectors are known the multiple waveform design problem is now given by:

$$\begin{aligned} &\max_{\mathbf{s}} \sum_{\ell=1}^{L} \alpha_\ell I\left(\mathbf{h}_\ell;\mathbf{z}_\ell \mid \mathbf{s}\right) \\ &\text{subject to } \sum_{\ell=1}^{L}\sum_{k=1}^{K} \left|s_{\ell,k}\right|^2 \le P_{\max} \end{aligned} \tag{18}$$

where $\boldsymbol{\alpha} = [\alpha_1,...,\alpha_L]^T$ is the target priority vector. This problem is highly non-linear in the complex waveforms $\mathbf{S}$. Furthermore, it involves cross-correlations between the waveforms, and therefore phase information plays an important role. Hence we need to design not only the waveform spectrum, but the complete complex envelope. The dependence on the phase will have a secondary drawback, since we will not be able to reduce the peak to average of the overall transmitted waveform by properly choosing the waveform phase. However, we will show that in the typical scenario of multiple beams in a large phased array this problem can be approximated by a simpler spectrum design problem.

5. Waveform Optimization For Multiple Targets

Using certain approximation of the mutual information and relying on the properties of the transmit and receive beamformers we can show that the mutual information (18) can be approximated by:

$$\tilde{\mathrm{I}}\left(\mathbf{h}_\ell;\mathbf{z}_\ell \mid \mathbf{p}\right) = \sum_{k=1}^{K} \log\left(1 + \frac{p_{\ell,k} \mid g_{\ell,\ell}\mid^2}{\sum_{m\neq\ell} g_{\ell,m}(f_k)\, p_{m,k} + \sigma^2_{v'_\ell}(f_k)}\right)\Delta f \tag{19}$$

where $\mathbf{p}_m = \left[p_{m,1},..., p_{m,K}\right]^T$ is the power allocation for the m'th target,

$$\mathbf{P} = \left[\mathbf{p}_1, \ldots, \mathbf{p}_L\right]$$

is the total power allocation matrix, and $\mathbf{p} = \mathrm{vec}(\mathbf{P})$. The constants $g_{\ell,m}$ are defined by:

$$g_{\ell,m}(f_k) = \mathbf{w}_\ell^*(f_k)\mathbf{R}_m(f_k)\mathbf{u}_m(f_k)$$
$$p_{m,k} = |s_m(f_k)|^2 \Delta f$$

and include all the prior information regarding the target signatures and the channels.

The problem (18) can now be simplified to

$$\begin{aligned} &\max_{\mathbf{p}} \sum_{\ell=1}^{L} \alpha_\ell \tilde{\mathrm{I}}\left(\mathbf{h}_\ell; \mathbf{z}_\ell \mid \mathbf{p}\right) \\ &\text{subject to } \sum_{\ell=1}^{L}\sum_{k=1}^{K} p_{\ell,k} \le P_{\max} \end{aligned} \tag{20}$$

To solve the multiple waveform design problem, we should note that (20) is a generalized (non-convex) monotropic optimization problem, since the summands of (19) are not concave functions. While this is a non-convex optimization problem we will show how it can be solved efficiently using duality theory.

Applying duality theory we obtain that the Lagrangian dual function is now given by:

$$L_d(\lambda) = \inf_{\mathbf{p}} -\sum_{k=1}^{K} L_k\left(\mathbf{p}_k, \lambda\right)\Delta f + \lambda P_{\max} \tag{21}$$

where

$$L_k(\mathbf{p}_k, \lambda) = \sum_{\ell=1}^{L} \alpha_\ell \tilde{\mathrm{I}}_k\left(\mathbf{h}_\ell(f_k); \mathbf{z}_\ell(f_k) \mid \mathbf{p}_k\right)\Delta f + \lambda \mathbf{1}^T \mathbf{p}_k \tag{22}$$

The dual problem now becomes

$$\max_{\lambda \ge 0} \left(\sum_{k=1}^{K} \inf_{\mathbf{p}_k} L_k(\mathbf{p}_k, \lambda) - \lambda P_{\max} \right) \tag{23}$$

Note that unlike the case of a single waveform, we will have multi-dimensional parallel optimization problems. However, this problem has two significant simplifications: The dimension L of each problem is much smaller than the typical number of frequency bins. Second, the problem is unconstrained, which is a major simplification in the non-convex problem. We can now solve (23) using bi-section search for λ and solving the parallel problems at each frequency given any specific value of λ. This is done using standard unconstrained optimization tools. While the complexity is still large, it is still linear in the number of frequency bins. Furthermore our functions are smooth, and the gradient and Hessian are rational functions. This can be exploited in solving

$$\hat{\mathbf{p}}_k = \inf_{\mathbf{p}_k} L_k(\mathbf{p}_k, \lambda) \tag{24}$$

In simulations we will show how these problems can be solved efficiently when the total number of variables is much above 100.

6. Simulations

In this section we show some simulated results. we assume that two waveforms are transmitted by an omni-directional equispaced linear phased array with 10 elements ($\frac{\lambda}{2}$ spacing) and received by the same array. The target directions in our simulation were 70^o and 80^o. The number of frequency bins was 100. The receive beamformer used was an MVDR-based beamformer, and the transmit beamformers were classical beamformers directed towards the targets. Target signatures were Gaussians corresponding to targets of length 17m and 10m respectively as shown in *Figure 1*. Waveforms bandwidth was 80MHz.The priorities used in the first simulation were $\alpha_1 = 0.4$, $\alpha_2 = 0.6$ with the higher priority given to the weaker target.

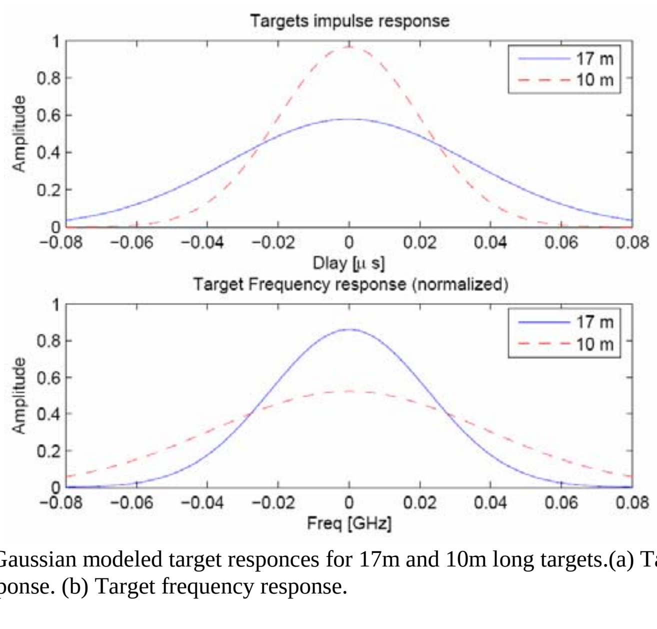
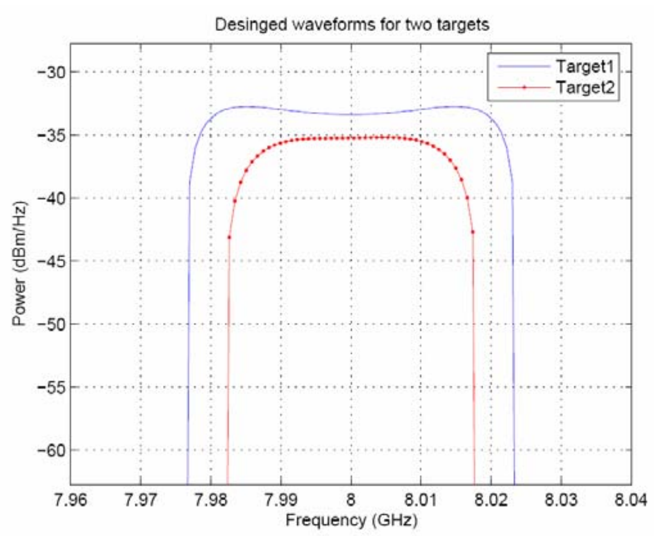


Figure 1. Gaussian modeled target responces for 17m and 10m long targets.(a) Target impulse response. (b) Target frequency response.

the two targets were chosen to be spatially separated. *Figure 2* shows the transmitted PSD for both targets.



Fiure 2 . Transmitted waveforms towards the targets.

In this example we can see that the algorithm transmits for both targets with a very large frequency overlap. This could be explained by the received PSD shown in *Figure 3*.

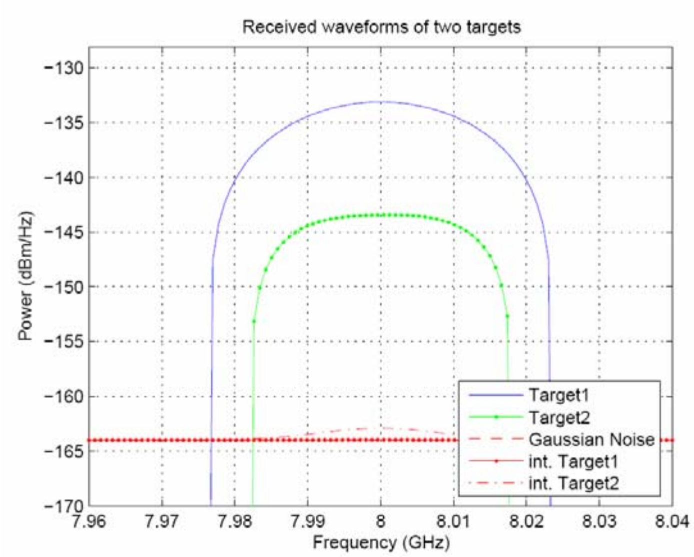


Figure 3. Received waveforms, after beamforming.

The targets do not have strong interference on each other and the interference noise is flat over all frequencies. This implies that the algorithm can transmit for both targets in the frequencies where their SINR is high even if the frequencies are overlapping, without losing information due to interference between the targets.

Finally we have studied the information region of the two targets and compared to the case where no spectral shaping is applied to the transmitted pulse. The results are presented in *Figure 4*. Using flat spectrum causes a loss of 100% for the weak target compared to the case where the design is according to the weak target profile. However choosing $\alpha = 0.5$ leads to performance enhancement of 33% for both targets.

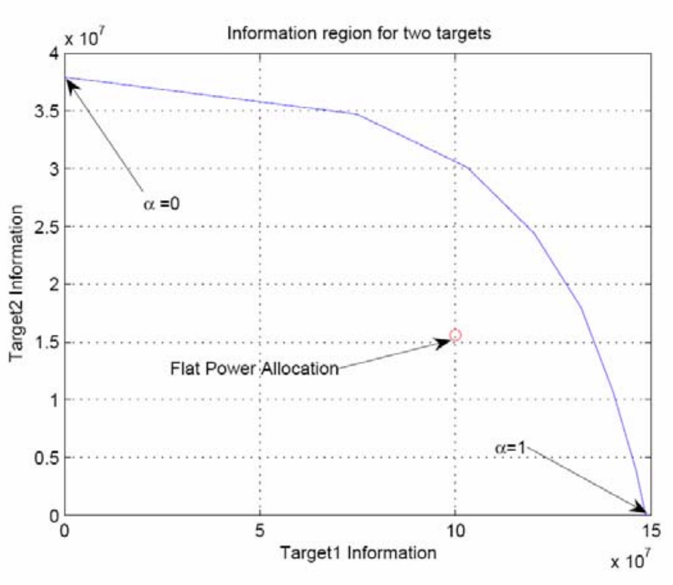


Figure 4. Information region, for two extended targets

In the next set of simulations we assumed that three targets are present and designed three waveforms transmitted by an omni-directional equispaced linear phased array with 10 elements ($\lambda / 2$ spacing) and received by the same array. The target directions were $90^o, 160^0$ and 20^o, respectively. The number of frequency bins was 100. The receive beamformer used was an MVDR beamformer, and the transmit beamformers were classical beamformers directed towards the targets. Target signatures were Gaussians corresponding to target sizes of 17, 10 and 13 meter respectively. The priority vector was $\alpha = [1,10,1]/12$. The transmit-power-to-receive noise ratio was 20 dB, and the targets were centered at 8 GHz.

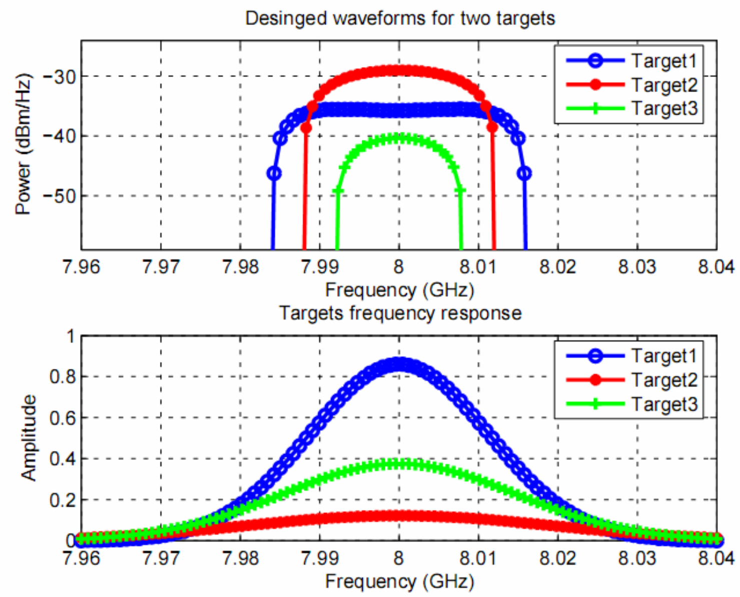


Figure 5. (a) Designed waveforms (up); (b) Targets' signatures (bottom)}

We can clearly see that the algorithm designed waveforms centered around 8 GHz with respect to their weights and sizes. In the next experiment, we tested the sensitivity to spatial resolution of the targets. We have used the same target sizes and the same target weights as before. The direction-of-arrival was changed to be 70^{o}, 70.5^{o} and 71^{o} respectively, these directions were chosen in order to make a strong interference between targets.

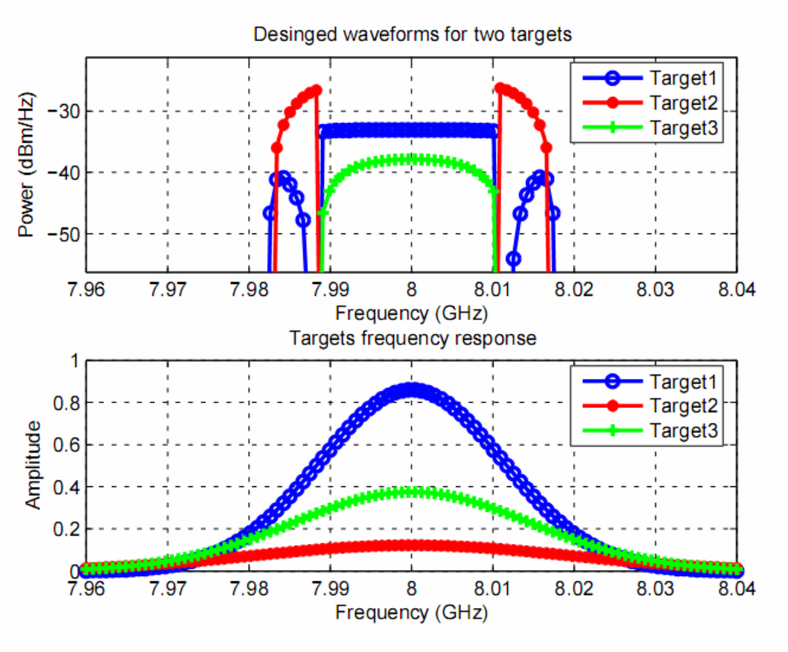


Figure 6. (a) Designed waveforms. Spatially unresolved targets (solid). Spatially resolved targets (dashed); (b) Targets' signatures (bottom)

In the previous experiment where the targets were spatially resolved can see a large spectral overlap of the designed waveforms. This overlap is caused by the fact that the spatial resolution enables the array to suppress reflections from the other targets therefore allowing better utilization of the frequency domain for both targets. When the targets become close the design criteria reduces the inter-target interference through spectral separation.

7. Conclusions

In this paper we discuss the optimization of multiple waveforms for multiple targets under joint power constraint. This type of waveform design is suitable for unresolved extended targets. We have derived computationally efficient algorithm and presented the result of the optimization in simulations. Further results as well as design of a single waveform optimized for multiple targets can be found in [11]. The combination of waveform design and direction-of-arrival estimation is discussed in [12].

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