

LOW-COMPLEXITY FREQUENCY-DOMAIN TURBO EQUALIZATION FOR SINGLE-CARRIER TRANSMISSIONS OVER DOUBLY-SELECTIVE CHANNELS

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ABSTRACT

Single-carrier transmissions with frequency-domain equalization have gained much interest due to their comparable complexity and performance to OFDM, which conversely suffers from a high peak-to-average power ratio. In this paper, we develop a new frequency-domain block turbo equalizer for single-carrier (SC) transmissions over doubly-selective channels. The main feature of the proposed equalizer is its low complexity, which is only linear in the block length. A comparison between SC and OFDM systems with channel coding in doubly-selective channels is also given.

Index Terms— Turbo equalization, doubly-selective channels, single-carrier, delay-Doppler diversity

1. INTRODUCTION

It is well known that broadband wireless communication systems require high transmission rates, giving rise to frequency-selectivity due to multipath propagation. In addition, the high-mobility terminals and scatterers induce Doppler shifts that introduce time selectivity, whose effect is relevant especially at high carrier frequencies. Therefore, advanced techniques are needed to accurately model time- and frequency-selective (i.e., doubly-selective) channels and to counteract the related performance degradation.

Long Term Evolution (LTE) is a major 3GPP step in next generation wireless networks. The LTE physical layer relies on a multiple access scheme based on orthogonal frequency-division multiplexing (OFDM) with a cyclic prefix (CP) in the downlink, and on single-carrier frequency-division multiple access (SC-FDMA) with a CP in the uplink [1]. The SC system, which can also be viewed as a fast Fourier transform (FFT) precoded-OFDM system, has a smaller peak-to-average-power ratio than regular OFDM, with complexity and performance comparable to OFDM, and hence leads to more power-efficient terminals suitable for uplink transmission [2]. However, both SC and OFDM suffer from doubly-selective channels, which require appropriate equalization methods.

A possible way to counteract a doubly-selective channel is by means of iterative equalizers. The iterative approach,

inspired by the turbo equalization principle [3], exchanges soft information in an iterative fashion, and greatly improves the system performance. While turbo equalization for SC transmissions over frequency-selective channels has been originally applied in the time domain (see references in [3]), frequency-domain methods have been successively exploited to reduce complexity [4]. For SC systems in doubly-selective channels, a low-complexity iterative equalizer has been proposed in [9], which can be regarded as the time-domain counterpart of the iterative frequency-domain equalizer [5]. However, time-domain iterative equalizers are not suitable for long channels, since their complexity is quadratic in the channel length [9].

In this paper, we develop a new low-complexity frequency-domain turbo equalizer for SC systems in doubly-selective channels. Relying on a block approach, our algorithm equalizes all the subcarriers jointly, differently from previously proposed frequency-domain algorithms that equalize the subcarriers separately [5]. The computational complexity scales linearly with the block length and is independent of the channel length. In the simulation section, we also compare our SC equalization algorithm with its OFDM counterpart [6]. Throughout the paper, the receiver is assumed to have perfect channel state information (CSI), while the transmitter has no access to the CSI.

2. SYSTEM MODEL

We consider a single-user SC system with block transmission, and a channel that is both frequency- and time-selective. The structure of the transmitter and the receiver is shown in Fig. 1. At the transmitter, a sequence of bits is encoded with error correction coding, and the coded bits are interleaved and mapped into N complex symbols, represented by the $N \times 1$ vector \mathbf{s}_t . For simplicity, we consider unit-energy quaternary phase-shift keying (QPSK), and we adopt the common assumption that the maximal channel order is equal to the cyclic prefix (CP) length, both denoted by L . This way, the equalizer can be designed separately for each block, and we can omit the block index from our notation. At the receiver, after removing the CP, the received vector \mathbf{y}_t can be expressed as

$$\mathbf{y}_t = \mathbf{H}_t \mathbf{s}_t + \mathbf{n}_t, \quad (1)$$

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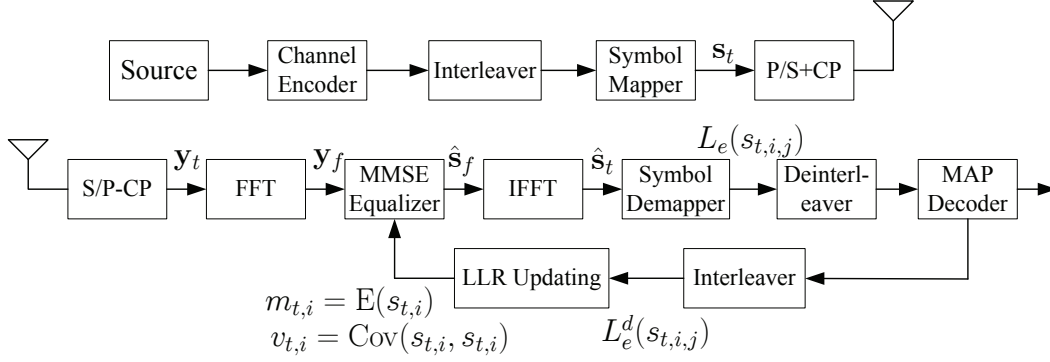


Fig. 1. System model.

where \mathbf{H}_t is the $N \times N$ time-domain channel matrix, and \mathbf{n}_t stands for the $N \times 1$ noise vector. For simplicity, we assume that \mathbf{n}_t is a circularly symmetric complex Gaussian noise vector, with zero mean and covariance matrix $\mathbf{R}_{\mathbf{n}_t} = E(\mathbf{n}_t \mathbf{n}_t^H) = \sigma_n^2 \mathbf{I}_N$.

3. LOW-COMPLEXITY TURBO EQUALIZATION

In this section, we develop a frequency-domain block turbo equalizer for SC transmission over doubly-selective channels. Applying the Fourier transform to \mathbf{y}_t , we get the frequency-domain input-output relationship, which can be written as

$$\mathbf{y}_f = \mathbf{F} \mathbf{H}_t \mathbf{F}^H \mathbf{F} \mathbf{s}_t + \mathbf{F} \mathbf{n}_t = \mathbf{H}_f \mathbf{s}_f + \mathbf{n}_f, \quad (2)$$

where \mathbf{F} denotes the $N \times N$ unitary DFT matrix, $\mathbf{y}_f = \mathbf{F} \mathbf{y}_t$, $\mathbf{s}_f = \mathbf{F} \mathbf{s}_t$, $\mathbf{n}_f = \mathbf{F} \mathbf{n}_t$, and $\mathbf{H}_f = \mathbf{F} \mathbf{H}_t \mathbf{F}^H$.

In a time-varying channel, \mathbf{H}_t is no longer circulant as in the time-invariant case, and \mathbf{H}_f becomes a non-diagonal matrix, giving rise to ICI (intercarrier interference) that corresponds to the non-zero off-diagonal elements of \mathbf{H}_f . However, \mathbf{H}_f is almost banded, with the most significant elements around the main diagonal [5, 6]. Hence, to simplify equalization, the matrix \mathbf{H}_f is further approximated by its banded version $\mathbf{H} = \mathbf{H}_f \circ \mathbf{\Theta}$, where we use the symbol \circ to denote the Hadamard (element-wise) product between matrices, and $\mathbf{\Theta}$ is the $N \times N$ circulant matrix, which has ones on the main diagonal, the B_c super- and B_c sub-diagonals, and zeros on the remaining entries.

Let us define $s_{t,i}$ as the i th QPSK symbol of \mathbf{s}_t , and $(s_{t,i,1}, s_{t,i,2})$ as the related bits. The means and the variances of the time-domain symbols, denoted as $m_{t,i} = E(s_{t,i})$ and $v_{t,i} = \text{Cov}(s_{t,i}, s_{t,i})$, are firstly initialized with zeros and ones, respectively. But in every iteration of the turbo equalizer, they are updated using soft information from the estimated symbols, as will be explained next. Similarly, we define $s_{f,i}$ as the i th symbol of \mathbf{s}_f , $m_{f,i} = E(s_{f,i})$ and $v_{f,i} = \text{Cov}(s_{f,i}, s_{f,i})$ as the means and the variances of the frequency-domain symbols. Stacking different $m_{t,i}$'s and $m_{f,i}$'s, we respectively have $\mathbf{m}_t = [m_{t,1}, \dots, m_{t,N}]^T$ and $\mathbf{m}_f = [m_{f,1}, \dots, m_{f,N}]^T$.

Given $m_{t,i}$ and $v_{t,i}$ as prior information, the equalizer exploits the means and the variances of the frequency-domain symbols. Since $\mathbf{s}_f = \mathbf{F} \mathbf{s}_t$, we have $\mathbf{m}_f = \mathbf{F} \mathbf{m}_t$, and $\text{Cov}(\mathbf{s}_f, \mathbf{s}_f) = \mathbf{F} \text{Cov}(\mathbf{s}_t, \mathbf{s}_t) \mathbf{F}^H$, where $\text{Cov}(\mathbf{s}_t, \mathbf{s}_t) = \text{diag}(v_{t,1}, \dots, v_{t,N})$. In general, $\text{Cov}(\mathbf{s}_f, \mathbf{s}_f)$ is not a diagonal matrix. However, to save complexity, we set its off-diagonal elements to 0, which leads to $\text{Cov}(\mathbf{s}_f, \mathbf{s}_f) = \bar{v}_t \cdot \mathbf{I}_N$ with $\bar{v}_t = \frac{1}{N} \sum_{i=1}^N v_{t,i}$. This approximation, which is used also in [7], basically annihilates the frequency variability of the variance.

With $m_{f,i}$ and $v_{f,i}$, the unbiased frequency-domain linear MMSE equalizer leads to [6]

$$\hat{s}_{f,i} = \mathbf{h}_i^H \mathbf{A}^{-1} (\mathbf{y}_f - \mathbf{H} \mathbf{m}_f) / t_i + m_{f,i}, \quad (3)$$

where \mathbf{h}_i is the i th column of \mathbf{H} , $\mathbf{V}_f = \text{Cov}(\mathbf{s}_f, \mathbf{s}_f)$, $t_i = \mathbf{h}_i^H (\mathbf{H} \mathbf{V}_f \mathbf{H}^H + \mathbf{R}_{\mathbf{n}_f})^{-1} \mathbf{h}_i$, $\mathbf{A} = \mathbf{H} \mathbf{V}_f \mathbf{H}^H + \mathbf{R}_{\mathbf{n}_f}$, and $\mathbf{R}_{\mathbf{n}_f} = E(\mathbf{n}_f \mathbf{n}_f^H)$.

The estimated time-domain transmitted signal can be obtained by $\hat{\mathbf{s}}_t = \mathbf{F}^H \hat{\mathbf{s}}_f$, which leads to

$$\begin{aligned} \hat{s}_{t,i} &= \mathbf{i}_i^H \mathbf{F}^H \hat{\mathbf{s}}_f = \mathbf{i}_i^H \mathbf{F}^H \sum_{k=1}^N \mathbf{i}_k \hat{s}_{f,k} \\ &= \mathbf{i}_i^H \mathbf{F}^H \sum_{k=1}^N \mathbf{i}_k \frac{1}{t_k} \mathbf{h}_k^H \mathbf{A}^{-1} \mathbf{H} \mathbf{F} (\mathbf{s}_t - \mathbf{m}_t) \\ &\quad + \mathbf{i}_i^H \mathbf{F}^H \sum_{k=1}^N \mathbf{i}_k \mathbf{i}_k^H \mathbf{F} \mathbf{m}_t + \mathbf{i}_i^H \mathbf{F}^H \sum_{k=1}^N \mathbf{i}_k \frac{1}{t_k} \mathbf{h}_k^H \mathbf{A}^{-1} \mathbf{F} \mathbf{n}_t \\ &= m_{t,i} + \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{F} (\mathbf{s}_t - \mathbf{m}_t) + \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \mathbf{H}^H \mathbf{A}^{-1} \mathbf{F} \mathbf{n}_t, \end{aligned} \quad (4)$$

where \mathbf{i}_k is an $N \times 1$ indicator function with a 1 on the k th position, $\mathbf{T} = \sum_{k=1}^N \mathbf{i}_k \frac{1}{t_k} \mathbf{i}_k^H$, and $\Sigma = \mathbf{H}^H \mathbf{A}^{-1} \mathbf{H}$.

After the equalization, we need to calculate the extrinsic log-likelihood ratio (LLR), $L_e(s_{t,i,j}) = L(s_{t,i,j} | \hat{s}_{t,i}) - L(s_{t,i,j})$, where $L(s_{t,i,j})$ is the *a priori* LLR and $L(s_{t,i,j} | \hat{s}_{t,i})$ is the *a posteriori* LLR, which can be calculated as in [3, 6]. To perform this calculation, we should derive the probability density function (PDF) $p(\hat{s}_{t,i} | s_{t,i} = s)$, which can be approximated as Gaussian: $p(\hat{s}_{t,i} | s_{t,i} = s) = \frac{1}{\pi \sigma_i^2} \cdot e^{-|\hat{s}_{t,i} - \mu_i|^2 / \sigma_i^2}$.

The mean μ_i and variance σ_i^2 can be calculated from (3) and (4)

$$\begin{aligned}
\mu_i &= \text{E}(\hat{s}_{t,i} | s_{t,i} = s) \\
&= m_{t,i} + \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{F} \mathbf{E}(s_{t,i} - \mathbf{m}_t | s_{t,i} = s) \\
&= m_{t,i} + \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{F} \mathbf{i}_i (s - m_{t,i}), \\
\sigma_i^2 &= \text{Cov}(\hat{s}_{t,i}, \hat{s}_{t,i} | s_{t,i} = s) \\
&= \text{E}(|\hat{s}_{t,i} - \mu_i|^2 | s_{t,i} = s) \\
&= \text{E}(|\mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{F} [s_{t,i} - \mathbf{m}_t - \mathbf{i}_i (s - m_{t,i})] \\
&\quad + \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \mathbf{H}^H \mathbf{A}^{-1} \mathbf{F} \mathbf{n}_t|^2) \\
&= \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{F} (\mathbf{V}_t - \mathbf{i}_i \mathbf{i}_i^H v_{t,i}) \mathbf{F}^H \Sigma^H \mathbf{T}^H \mathbf{F} \mathbf{i}_i \\
&\quad + \sigma_n^2 \mathbf{i}_i^H \mathbf{F}^H \mathbf{T} \Sigma \mathbf{H}^{-1} \mathbf{H}^{-H} \Sigma^H \mathbf{T}^H \mathbf{F} \mathbf{i}_i \\
&\approx \bar{v}_{t,i} + |\bar{\mathbf{h}}|^2,
\end{aligned} \tag{5}$$

where $\mathbf{V}_t = \text{Cov}(s_t, s_t)$, $\bar{v}_{t,i} = \frac{1}{N} (\sum_{k=1}^N v_{t,k} - v_{t,i})$, $|\bar{\mathbf{h}}|^2 = \frac{\sigma_n^2}{N} \sum_{k=1}^N |\mathbf{h}_k^H \mathbf{h}_k|^{-1}$, and we approximate the matrices $\mathbf{F}(\mathbf{V}_t - \mathbf{i}_i \mathbf{i}_i^H v_{t,i}) \mathbf{F}^H$, $\mathbf{H}^H \mathbf{H}$ and Σ to be diagonal matrices by setting the off-diagonal elements to 0, so that $\mathbf{T} \Sigma \approx \mathbf{I}_N$. While the approximation of $\mathbf{F}(\mathbf{V}_t - \mathbf{i}_i \mathbf{i}_i^H v_{t,i}) \mathbf{F}^H$ as diagonal is similar to that of $\text{Cov}(s_f, s_f)$, the diagonal approximations of $\mathbf{H}^H \mathbf{H}$ and Σ can be motivated as follows. First, since \mathbf{H} is banded, the off-diagonal elements of $\mathbf{H}^H \mathbf{H}$ decay to zero very rapidly. Second, $\Sigma = \mathbf{H}^H \mathbf{A}^{-1} \mathbf{H}$ represents the effect of the linear MMSE equalizer $\mathbf{H}^H \mathbf{A}^{-1}$ applied to the channel matrix \mathbf{H} : since the equalizer highly mitigates the cross-interference, Σ is very close to a diagonal matrix.

Therefore, the extrinsic LLR can be expressed as [3, 6]

$$\begin{aligned}
L_e(s_{t,i,1}) &= \ln \frac{p(\hat{s}_{t,i} | s_{t,i} = \alpha_1) P_2(0) + p(\hat{s}_{t,i} | s_{t,i} = \alpha_3) P_2(1)}{p(\hat{s}_{t,i} | s_{t,i} = \alpha_2) P_2(0) + p(\hat{s}_{t,i} | s_{t,i} = \alpha_4) P_2(1)} \\
&= \frac{\sqrt{8} \text{Re}(\hat{s}_{t,i})}{\bar{v}_{t,i} + |\bar{\mathbf{h}}|^2}, \\
L_e(s_{t,i,2}) &= \ln \frac{p(\hat{s}_{t,i} | s_{t,i} = \alpha_1) P_1(0) + p(\hat{s}_{t,i} | s_{t,i} = \alpha_2) P_1(1)}{p(\hat{s}_{t,i} | s_{t,i} = \alpha_3) P_1(0) + p(\hat{s}_{t,i} | s_{t,i} = \alpha_4) P_1(1)} \\
&= \frac{\sqrt{8} \text{Im}(\hat{s}_{t,i})}{\bar{v}_{t,i} + |\bar{\mathbf{h}}|^2},
\end{aligned} \tag{6}$$

where $\alpha_k, k = 1, \dots, 4$ is the QPSK symbol corresponding to the first, second, fourth, and third quadrant, respectively, $P_j(0) = P(s_{t,i,j} = 0)$ and $P_j(1) = P(s_{t,i,j} = 1)$. As shown in Fig. 1, the extrinsic LLR $L_e(s_{t,i,j})$ is passed to the decoder to generate a new extrinsic LLR $L_e^d(s_{t,i,j})$, which is added to the a priori LLR to form the a posteriori LLR or the new version of the a priori LLR, which is used to update the means and the variances of the estimated symbol as in [3, 6]:

$$\begin{aligned}
L_{new}(s_{t,i,j}) &= L(s_{t,i,j}) + L_e^d(s_{t,i,j}), \\
m_{t,i,new} &= \frac{\tanh(\frac{L_{new}(s_{t,i,1})}{2}) + i \cdot \tanh(\frac{L_{new}(s_{t,i,2})}{2})}{\sqrt{2}}, \\
v_{t,i,new} &= 1 - |m_{t,i,new}|^2.
\end{aligned} \tag{7}$$

The whole procedure described in this subsection can then be repeated, depending on the chosen number of iterations.

Similarly to the OFDM equalization algorithm in [6], it can be shown that the proposed block turbo equalization algorithm has linear complexity in the block length N . Specifically, the equalization step in (3) and the calculation of t_i 's have complexity $\mathcal{O}(B_c^2 N)$ [6], while calculating the extrinsic information in (6) has complexity $\mathcal{O}(B_c N)$. Therefore, taking into account FFT operations, the overall complexity per iteration is $\mathcal{O}((B_c^2 + \log(N))N)$, which is independent of the channel order L .

4. SIMULATION RESULTS

We consider an SC system with block length $N = 128$. The channel order and the CP length are the same and equal to $L = 7$. The channel paths, characterized by a U-shaped Doppler spectrum, are assumed to be uncorrelated and Rayleigh distributed with uniform power delay profile. The frequency-domain channel matrix bandwidth parameter is set to $B_c = 3$. A rate 1/2 convolutional code with generator matrix $[1 \ 0 \ 1; 1 \ 1 \ 1]$ and a codeword length of 8192 is used. We employ random interleaving. The decoder employs a linear approximation to the log-MAP decoding algorithm.

Fig. 2 shows the bit-error rate (BER) performance of the proposed frequency-domain turbo equalizer for different numbers of iterations, as a function of the signal-to-noise ratio (SNR), defined as $\frac{1}{\sigma_n^2}$. We consider a medium to high mobility case where the normalized Doppler frequency is $f_d T = 0.15/N$ and $f_d T = 0.25/N$, with f_d the absolute Doppler frequency shift and T the symbol period. In Fig. 2, it is shown that the second iteration produces a significant performance improvement with respect to the first iteration, which corresponds to the output of a non-iterative equalizer. After two iterations, however, the BER improvement converges slowly. Fig. 2 also displays the BER behavior as a function of the channel time selectivity. For low SNR, a Doppler spread increase leads to an improved BER. Therefore, the proposed equalizer is able to properly exploit the time selectivity of the channel. For high SNR, the BER performance gap becomes smaller. This is mainly due to the increased band approximation error at high Doppler spread. Indeed, the increased power of the unmodeled ICI reduces the maximum effective signal-to-interference-and-noise ratio, which limits the BER performance.

It is interesting to compare the SC and OFDM systems for doubly-selective channels. Previous works have shown some performance comparisons for frequency-selective channels [2, 8]. We first consider uncoded systems. For a wireless channel with significant multipath, OFDM systems cannot exploit frequency diversity, since each data symbol is placed on a single frequency, while SC systems can achieve frequency diversity by spreading each data symbol over many frequencies. In a fast time-varying channel, a high Doppler spread

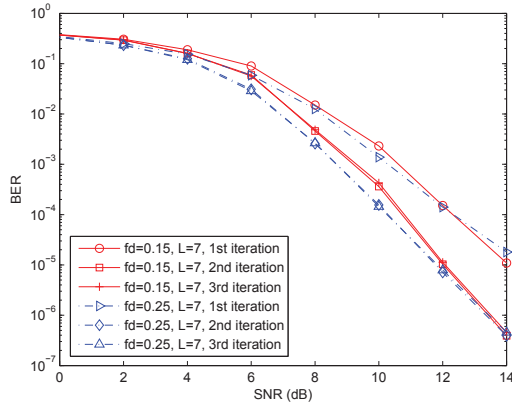


Fig. 2. BER performance of the proposed turbo equalization algorithm for SC systems.

gives time diversity. However, SC systems do not exploit the time diversity, because each data symbol is placed on a single time period, whereas OFDM exploits the time diversity, since it precodes the symbols (by the IFFT) so that each data symbol is spread over the whole OFDM block period. Applying channel coding over OFDM subcarriers offers some frequency diversity, and coding over many OFDM blocks can gain additional time diversity. Similarly, SC system can obtain time diversity by channel coding.

Fig. 3 illustrates the BER performance comparison for the turbo equalizers designed for SC and OFDM systems [6] after three iterations. In the OFDM case, the turbo equalizer includes a time-domain receiver window that reduces the band approximation error, thereby improving the BER at high SNR. The bandwidth parameter $B_c = 3$ is the same for both SC and OFDM systems. The simulation results confirm that both SC and OFDM systems benefit from channel coding, by obtaining time and frequency diversity, respectively, which is not exploited in the uncoded case. However, the achievable diversity gain is difficult to analyze, since the band approximation error impairs the performance at high SNR. In addition, the amount of diversity also depends on the specific code [2, 8]. Anyway, it is interesting to observe that OFDM systems outperform SC systems. This is caused by the larger band approximation error in SC systems with respect to OFDM systems, and by the approximations introduced to simplify the proposed equalizer. In addition, further performance improvements for SC systems could be obtained by employing channel extension techniques [10].

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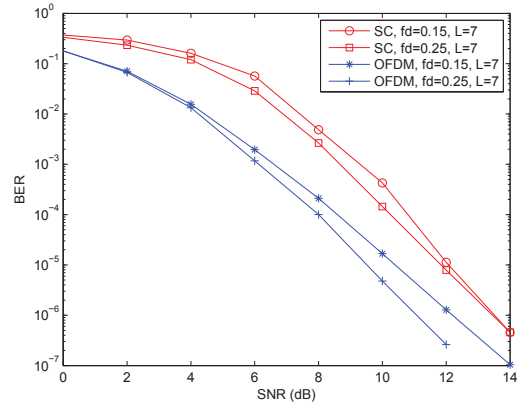


Fig. 3. BER performance comparison of turbo equalization for SC and OFDM systems after three iterations.

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