Nonlinear Equalization for Frame-Differential IR-UWB Receivers

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***Abstract*— This paper shows equalization approaches for high-data-rate transmitted-reference (TR) IR-UWB systems employing an autocorrelation receiver front-end. Using a maximum likelihood sequence detector (MLSD) with decision feedback (in the back-end of the TR-receiver), a significant improvement of the receiver performance is possible. To avoid the high complexity of the MLSD detector, alternative equalizer structures are evaluated. If the parameters of the channel are not known a priori, an equalizer has to be adapted during the transmission of training data. Such an adaptive equalizer is presented as a reference. Furthermore, we study the design of Minimum Mean Square Error (MMSE) equalizers assuming knowledge of the channel. A linear MMSE equalizer is designed using the linear and nonlinear channel coefficients. Then the concept of the linear equalizer is extended to a nonlinear Volterra equalizer which further improves the performance of the IR-UWB receiver structure. All proposed methods are discussed and the effects are shown with computer simulations.**

I. Introduction

Autocorrelation Receivers (AcR) are widely used in Transmitted Reference (TR) Ultra-wide Band (UWB) communications. The authors in [1] have shown that the Inter-symbol Interference (ISI) of a frame-differential AcR and the wireless multipath propagation channel can be modelled as a second order Volterra system [2]. Thus, the output of the AcR depends non-linearly on the input data $d[i] \in \{-1, +1\}$ where i denotes the symbol index. The decision variable without noise $z[i]$ is given by

$$z[i] = h_0 + \mathbf{d}[i]^\mathsf{T}\mathbf{h}_1 + \mathbf{d}[i]^\mathsf{T}\mathbf{H}_2\mathbf{d}[i], \quad (1)$$

where the vector $\mathbf{d}[i]$ consists of the data symbols up to a finite memory depth $L = \eta + 2$, i.e.,

$$\mathbf{d}[i] = [d[i-\eta], d[i-\eta+1], \ldots, d[i+1]]^\mathsf{T} \quad (2)$$

and $h_0, \mathbf{h}_1$ and $\mathbf{H}_2$ are the zeroth, first and second order kernels of the Volterra system, respectively. η denotes the number of past interfering symbols and depends on the excess delay of the channel impulse response. If a low-data-rate scenario with a data rate up to about one-fifth of the reciprocal of the channel RMS delay spread (τ_{rms}) is considered, (e.g. 10 Mbps [1] at an RMS delay spread of τ_{rms}=10ns) little ISI occurs and the conventional slicer is capable of retreiving the data without equalization. If the data rate is increased further (e.g. 125 Mbps) equalization has to be investigated for the receiver.

To equalize this structure, one can use a Maximum Likelihood Sequence Detector with decision feedback (MLSD-DF), where the Euclidean distance of the received sequence and all possible symbol states is minimized to find the currently transmitted data symbol. Hence, the complexity of an MLSD can become very large. Differences between a nonlinear and linear distance calculation heavily affect the performance of the receiver structure [1].

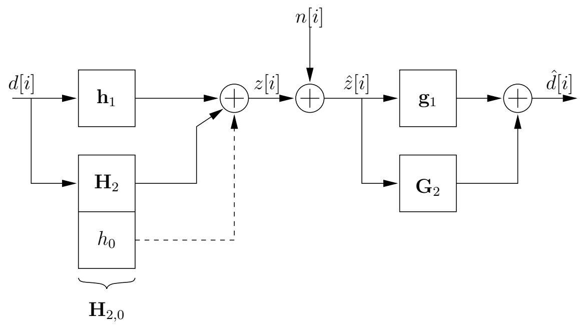

Fig. 1. Equivalent system model and proposed nonlinear equalization.

To avoid the high computational complexity, adaptive linear or polynomial equalizers, i.e., Volterra equalizers, can be used to improve the performance of the system. Usually the coefficients of the kernels are not available at the receiver side which means the kernels of the equalizer have to be determined at the beginning of a transmission using a training sequence. This consumes part of the available bandwith. With the estimated coefficients of the system model, the MLSD-DF may work properly.

In this paper linear Minimum Mean Square Error (MMSE) equalizers are designed to compensate the introduced distortions. As shown in Fig. 1 the linear equalizer coefficients are denoted as $\mathbf{g}_1$ and are optimized under a MMSE criterion.

Furthermore, it is possible to design a nonlinear equalizer where the determined linear equalizer coefficients are kept constant and one tries to improve the performance by finding a second order Volterra kernel $\mathbf{G}_2$, shown in Fig.1, to compensate the nonlinear distortions and to recover the transmitted data symbols $d[i]$. These approaches are taken under the assumption of knowledge of the system parameters $\mathbf{h}_1$ and $\mathbf{H}_2$.

The paper is organized as follows. A short summary of the specific properties of this second order equivalent system model is presented in Section II. The performance of an

MLSD-DF detector with linear and nonlinear distance calculation is reviewed in Section III. Adaptive approaches, using training data to adapt polynomial filters for equalization are found in Section IV. These results serve as a benchmark for the equalizers derived in this paper. In Section V, the design of a linear equalizer is shown, where the coefficients are optimized in a MMSE sense. This concept is extended in Section VI, where higher order kernels of the equalizer are designed. All approaches are compared in their performance by computer simulations. The results are shown in Section VII and finally conclusions are presented in Section VIII.

II. Special properties of the Volterra model

This work is based on the derivation of an equivalent nonlinear system model for TR-IR-UWB receivers. *Witrisal et. al.* have shown in [1], that the inter-symbol interference (ISI) can be modeled as a second order Volterra system which has several properties used throughout this paper. Since $d^2[i] \equiv 1$, all the main-diagonal elements of the second order kernel matrix $\mathbf{H}_2$ can be summed up in the bias term h_0 because the data dependency vanishes for these elements. Hence, the output of the second order Volterra system can be written as in (1) or, when including the bias term and distributing it equally on the main-diagonal of a matrix $\mathbf{H}_{2,0}$ (cf. Fig. 1) as

$$z[i] = \mathbf{d}[i]^\mathsf{T}\mathbf{h}_1 + \mathbf{d}[i]^\mathsf{T}\mathbf{H}_{2,0}\mathbf{d}[i]. \quad (3)$$

One other specific property of the model assumed in [1] is that the second order kernel matrix $\mathbf{H}_2$ is represented by an upper triangular matrix. Furthermore, it can be easily shown that a symmetric version of this matrix can be computed [3].

III. MLSD with decision feedback

The MLSD detector is used to serve as a reference for further results in this paper. Also for the nonlinear case the MLSD is an optimal detector since we assume that the parameters of the equivalent system model are known. In the noisy case, the output of the equivalent nonlinear system model is written as

$$\hat{z}[i] = h_0 + \mathbf{h}_1^\mathsf{T}\mathbf{d}[i] + \mathbf{d}^\mathsf{T}[i]\mathbf{H}_2\mathbf{d}[i] + n[i], \quad (4)$$

where $n[i]$ is a zero-mean Gaussian process which has, as a first approximation, independent samples and a constant variance σ_n^2 for all data symbols. This assumption has been validated by computer simulations in [1]. To detect the sequence of transmitted symbols $\{d[i]\}$, the MLSD with decision feedback [4] applies the Viterbi algorithm to a reduced trellis with $2^{\eta-K+1}$ states. For this trellis, the branch metric m_1 from state $\mathbf{q}_1[i] = [d[i-\eta+K], d[i-\eta+K+1], \dots, d[i]]^\mathsf{T}$ to state $\mathbf{q}_2[i] = [d[i-\eta+K+1], d[i-\eta+K+2], \dots, d[i+1]]^\mathsf{T}$ for the reduced complexity MLSD-DF-NLIN with decision feedback is given by

$$m_1(\mathbf{q}_1[i], \mathbf{q}_2[i]) = \|\hat{z}[i] - h_0 - \mathbf{h}_1^\mathsf{T}\hat{\mathbf{d}}[i] - \hat{\mathbf{d}}[i]^\mathsf{T}\mathbf{H}_2\hat{\mathbf{d}}[i]\|^2. \quad (5)$$

Conversely, the MLSD-DF-LIN with linear distance calculation has a branch metric m_2 given by

$$m_2(\mathbf{q}_1[i], \mathbf{q}_2[i]) = \|\hat{z}[i] - h_0 - \mathbf{h}_1^\mathsf{T}\hat{\mathbf{d}}[i]\|^2. \quad (6)$$

In both cases, $\hat{\mathbf{d}}[i] = [\hat{\mathbf{d}}^\mathsf{T}(\mathbf{q}_1[i]), d[i-\eta+K], d[i-\eta+K+1], \dots, d[i+1]]^\mathsf{T}$ denotes the vector for computing the distances, where $\hat{\mathbf{d}}(\mathbf{q}_1[i])$ denotes the $K \times 1$ vector containing feedback decisions which depend on the state $\mathbf{q}_1[i]$ determined by the path histories. Thus, the branch metric, computed in (5) is only dependending on the noise because all other quantities can be computed deterministically resulting in the optimality of the detector even in the nonlinear case.

The achieved performance of linearly and nonlinearly computed branch metrics is evaluated and compared to the proposed equalization methods in Section VII, where for the feedback vector, a length of $K = \eta - 1$ is chosen leading to a trellis of only 4 states..

IV. Adaptive polynomial filters for channel equalization

Usually the parameters of the channel are not known a priori on the receiver side to compute the best equalizer for a given channel. Therefore these equalizer coefficients have to be estimated during transmission of training data $\{d_t[i]\}_{i=0}^{N_t-1}$, where N_t denotes the length of the training sequence. Conventional least mean square (LMS) adaptive linear and adaptive Volterra filters [5] are used to adapt an equalizer at the receiver side. Thus, the coefficient vector of the adaptive polynomial filter can be written as

$$\mathbf{g} = [g_0^{(1)}, g_1^{(1)}, \dots, g_{L-1}^{(1)}, g_{0,0}^{(2)}, g_{0,1}^{(2)}, \dots, g_{L-1,L-1}^{(2)}], \quad (7)$$

where the superscript denotes the order of the kernel. For updating the coefficients, a conventional LMS adaptive algorithm with the update equation

$$\mathbf{g}[k] = \mathbf{g}[k-1] + \mu\hat{\mathbf{z}}e[k], \quad (8)$$

is used. In the update equation $e[k]$ denotes the error at output of the adaptive filter, i.e.

$$e[k] = d_t[k] - \hat{d}[k]. \quad (9)$$

$\hat{\mathbf{z}}$ is a vector containing the received symbols, and in the nonlinear case, products of the received symbols and μ is the stepsize of the algorithm, which can be chosen differently for the linear and nonlinear part [5]. To achieve reliable convergence of the adaptive filters, the length of the training sequence has to be long enough to estimate equalizer parameters properly. In our simulations the convergence of the filters was achieved with sufficiently long training sequences and the steady state solution of the equalizer was used for equalization.

V. Linear MMSE equalizer

Considering the rather high complexity of the MLSD, a linear equalizer can help to equalize the channel distortions and detecting the symbols with a very simple conventional threshold detector. The complexity of the linear equalizer is rather low which justifies the design and reduces implementation costs. The linear MMSE approach for designing an equalizer tries to minimize the variance of the error $\sigma_{e_{lin}}^2$, which is the difference between the desired response and the given response, at the detector. This variance $\sigma_{e_{lin}}^2$ for the linear term is given by

$$\sigma_{e_{lin}}^2 = (\mathbf{f}_1 - \tilde{\mathbf{H}}_1\mathbf{g}_1)^\mathsf{T}(\mathbf{f}_1 - \tilde{\mathbf{H}}_1\mathbf{g}_1)\sigma_d^2 + \mathbf{g}_1^\mathsf{T}\mathbf{g}_1\sigma_n^2, \quad (10)$$

where $\mathbf{g}_1 = [\, g_1[0] \dots g_1[L-1] \,]^\mathsf{T}$ are the equalizer coefficients, $\mathbf{f}_1 = [\, 0 \dots 0\ 1\ 0 \dots 0 \,]^\mathsf{T}$ is the desired impulse response from the interconnection of the two systems, i.e. a δ-pulse with a certain delay, and $\tilde{\mathbf{H}}_1$ is a $2L-1 \times L$ Toeplitz matrix consisting of the channel coefficients $\mathbf{h}_1$,

$$\tilde{\mathbf{H}}_1 = \begin{bmatrix} h_1[0] & 0 & \cdots & 0 & 0 \\ h_1[1] & h_1[0] & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & h_1[L-1] & h_1[L-2] \\ 0 & 0 & \cdots & 0 & h_1[L-1] \end{bmatrix}. \tag{11}$$

The solution to the regularized least squares problem (10) is given by

$$\mathbf{g}_1 = (\tilde{\mathbf{H}}_1^\mathsf{T}\tilde{\mathbf{H}}_1 + \frac{\sigma_n^2}{\sigma_d^2}\mathbf{I})^{-1}\tilde{\mathbf{H}}_1^\mathsf{T}\mathbf{f}_1, \tag{12}$$

where the solution with the lowest noise gain, i.e. lowest amplification of the noise σ_n^2 at the equalizer input, is found and $\mathbf{I}$ denotes the identity matrix. Assuming that the AWGN $n[i]$ in Fig. 1 is zero, the pseudo-inverse solution yields for the equalizer coefficients

$$\mathbf{g}_1^\dagger = (\tilde{\mathbf{H}}_1^\mathsf{T}\tilde{\mathbf{H}}_1)^{-1}\tilde{\mathbf{H}}_1^\mathsf{T}\mathbf{f}_1. \tag{13}$$

Furthermore, it is possible to compute a linear MMSE equalizer with consideration of the nonlinear channel model, i.e. considering the nonlinear channel coefficients in the design of the linear equalizer. In this case the MSE is given as

$$\begin{aligned}\sigma^2_{e_{lin2}} =& (\mathbf{f}_1 - \tilde{\mathbf{H}}_1\mathbf{g}_1)^\mathsf{T}(\mathbf{f}_1 - \tilde{\mathbf{H}}_1\mathbf{g}_1)\sigma_d^2 + \\ & (\mathbf{f}_2 - \mathbf{H}_2'\mathbf{g}_1)^\mathsf{T}(\mathbf{f}_2 - \mathbf{H}_2'\mathbf{g}_1)\sigma_d^4 + \mathbf{g}_1^\mathsf{T}\mathbf{g}_1\sigma_n^2,\end{aligned} \tag{14}$$

where $\mathbf{H'}_2$ is a matrix consisting of the coefficients of the second order kernel and $\mathbf{f}_2$ is the vector representing the second order kernel of the cascade, which should be ideally zero. The variance of the error in (14) is rewritten as a combined error equation yielding

$$\begin{aligned}\sigma^2_{e_{lin2}} =& \left(\begin{bmatrix}\mathbf{f}_1 \\ \sigma_d\mathbf{f}_2\end{bmatrix} - \begin{bmatrix}\tilde{\mathbf{H}}_1 \\ \sigma_d\mathbf{H}_2'\end{bmatrix}\mathbf{g}_{1,2}\right)^\mathsf{T} \\ & \times \left(\begin{bmatrix}\mathbf{f}_1 \\ \sigma_d\mathbf{f}_2\end{bmatrix} - \begin{bmatrix}\tilde{\mathbf{H}}_1 \\ \sigma_d\mathbf{H}_2'\end{bmatrix}\mathbf{g}_{1,2}\right)\sigma_d^2 + \mathbf{g}_{1,2}^\mathsf{T}\mathbf{g}_{1,2}\sigma_n^2.\end{aligned} \tag{15}$$

Since the variance and the squared variance $\sigma_d^2 = \sigma_d^4 = 1$ the additional data variance vanishes in front of $\mathbf{f}_2$ and $\mathbf{H}_2'$. For that reason the matrix $\tilde{\mathbf{H}}_1$ has to be extended by the nonlinear channel coefficients $\mathbf{H}_2$. Similarly to the linear equalizer the nonlinear channel coefficients are collected in a matrix $\mathbf{H}_2'$ in a regular Toeplitz structure,

$$\mathbf{H}_2' = \begin{bmatrix} H_2(1,0) & 0 & \cdots & 0 \\ H_2(1,1) & H_2(1,0) & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & H_2(1,L-2) & H_2(1,L-3) \\ 0 & 0 & \cdots & H_2(1,L-2) \\ H_2(L-1,1) & \cdots & 0 & 0 \\ 0 & H_2(L-1,1) & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & H_2(L-1,1) \end{bmatrix} \tag{16}$$

where the notation $H_2(\alpha, \beta)$ simply maps the off-diagonals to a vector notation. The first index α denotes the off-diagonal of the matrix $\mathbf{H}_2$ and the second index β denotes the element index within the off-diagonal. Consequently, the solution for the "improved" linear equalizer coefficients is given in the same manner as before by

$$\mathbf{g}_{1,2} = (\tilde{\mathbf{H}}_{1,2}^\mathsf{T}\tilde{\mathbf{H}}_{1,2} + \frac{\sigma_n^2}{\sigma_d^2}\mathbf{I})^{-1}\tilde{\mathbf{H}}_{1,2}^\mathsf{T}\mathbf{f}_{1,2}, \tag{17}$$

where $\tilde{\mathbf{H}}_{1,2}$ and $\mathbf{f}_{1,2}$ denote the concatenation of the linear and the nonlinear channel coefficients and desired impulse responses as seen in (15), respectively. Note that these derivatives assume that the bias term has been subtracted before.

VI. Second order MMSE equalizer

Similarly to the linear equalizer design approach it is possible to design a nonlinear, i.e., second order Volterra equalizer. This second order kernel ($\mathbf{G}_2$ in Fig.1) should further reduce the introduced distortions from the receiver front-end. To design a second order equalizer one possible criterion is, to furthermore claim that the second order kernel of the interconnection is forced to zero, i.e.,

$$\mathbf{f}_1 = [\,0\ \ldots\ 0\ 1\ 0\ \ldots\ 0\,]^\mathsf{T}, \quad \mathbf{F}_2 = \mathbf{0}, \tag{18}$$

where $\mathbf{F}_2$ is the second order kernel matrix of the interconnection of two nonlinear Volterra systems. Generally, the size of $\mathbf{F}_2$ is determined by the length of the two interconnected systems $\{H_j\}$ and $\{G_j\}$, where $\{\cdot\}$ denotes a set of homogeneous Volterra kernels. In the proposed equalizers we assume that channel and equalizer have the same memory length, either linear or nonlinear, which means that $\mathbf{f}_1$ is a $2L-1 \times 1$ vector and $\mathbf{F}_2$ is a $2L-1 \times 2L-1$ matrix with their desired responses as shown in (18).

Of course, this introduces distortions of order 3 and 4 which are not negligible. However, in weakly nonlinear systems the contributions of these kernels might be very small compared to the first and second order distortions but it is not clear whether this is the case in the studied system. Nevertheless, the first approach for the computation of the equalizer coefficients does not consider these higher order terms at all. A second approach tries to model the effects of the third and fourth order term approximately. Furthermore, it is assumed that the coefficients of the linear equalizer are designed properly with the approach shown in Section V and a second order equalizer is designed to improve the performance.

The resulting second order kernel $F_2[m_1, m_2]$ of the interconnection of two nonlinear systems is given as

$$\begin{aligned} F_2[m_1, m_2] &= \sum_{l=0}^{L-1} g_1[l] h_2[m_1 - l, m_2 - l] \\ &+ \sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} g_2[l_1, l_2] h_1[m_1 - l_1] h_1[m_2 - l_2]. \end{aligned} \tag{19}$$

To have a more compact expression for optimizing the Volterra equalizer, the convolution in (19) is rewritten as a matrix multiplication. For that reason we have to compose new matrices $\tilde{\mathbf{G}}_1$ and $\tilde{\mathbf{H}}_2$ which map the two-dimensional convolution on a matrix multiplication. Note that if we assume the same length L for the channel and for the equalizer, the second order kernel of the interconnection has to be $2L-1$ taps long, i.e. a kernel with the dimensions $2L-1 \times 2L-1$. If this dimension problem is considered properly, the convolution is given as

$$\mathbf{F}_2 = \tilde{\mathbf{G}}_1 \tilde{\mathbf{H}}_2 + \tilde{\mathbf{H}}_1 \mathbf{G}_2 \tilde{\mathbf{H}}_1^{\mathsf{T}} \tag{20}$$

where the matrices are composed as follows

$$\tilde{\mathbf{G}}_1 = \begin{bmatrix} \begin{bmatrix} \mathbf{G}_{1,0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} & \mathbf{G}_{1,1} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \\ \vdots \quad \vdots \quad \vdots \quad \ddots \quad \vdots \\ \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{G}_{1,L-1} \end{bmatrix} \end{bmatrix}^{\mathsf{T}}, \tag{21}$$

where the matrix $\mathbf{G}_{1,j}$ is defined as

$$\mathbf{G}_{1,j} = g_{1,2}[j] \mathbf{I}_L \quad j = 0 \ldots L-1, \tag{22}$$

and

$$\tilde{\mathbf{H}}_2 = \begin{bmatrix} \begin{bmatrix} \mathbf{H}_2 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \\ \begin{bmatrix} \mathbf{0} & \mathbf{H}_2 & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \\ \vdots \quad \vdots \quad \vdots \quad \ddots \quad \ddots \quad \vdots \\ \begin{bmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{H}_2 \end{bmatrix} \end{bmatrix}, \tag{23}$$

and $\tilde{\mathbf{H}}_1$ is the previously defined Toeplitz matrix consisting of the linear channel coefficients. To minimize the mean squared error (MSE) of all the coefficients in the resulting kernel matrix $\mathbf{F}_2$, we have to minimize the MSE given by

$$\begin{aligned} \sigma^2_{e_{non}} &= \left\| \left(\mathbf{0} - (\tilde{\mathbf{G}}_1 \tilde{\mathbf{H}}_2 + \tilde{\mathbf{H}}_1 \mathbf{G}_2 \tilde{\mathbf{H}}_1^{\mathsf{T}}) \right) \right\|_F^2 \sigma_d^4 \\ &+ 2\sigma_n^4 \mathrm{Tr}(\mathbf{G}_2^{\mathsf{T}} \mathbf{G}_2), \end{aligned} \tag{24}$$

where one can show that white Gaussian noise with $\mathcal{N}(0, \sigma_n^2)$ is amplified over a nonlinear homogeneous second order

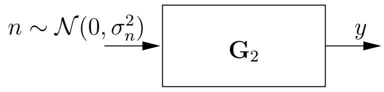

Fig. 2. Noise amplification over a homogeneous second order Volterra kernel.

Volterra system $\mathbf{G}_2$ as follows. The output of the system is denoted as y (cf. Fig 2) and has a mean given as [6]

$$\mu_y = \sum_{l=0}^{L-1} G_2[l, l] \sigma_n^2 \tag{25}$$

and a variance

$$\sigma_y^2 = 2\sigma_n^4 \mathrm{Tr}(\mathbf{G}_2^{\mathsf{T}} \mathbf{G}_2), \tag{26}$$

where $\mathrm{Tr}(\cdot)$ denotes the trace operator.

Finding a minimum for the variance of the error can be done by computing the first derivative of the expression w.r.t. $\mathbf{G}_2$ and setting it to zero. Neglecting the noise gives

$$\mathbf{G}_2 = -(\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1)^{-1} \tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{G}}_1 \tilde{\mathbf{H}}_2 \tilde{\mathbf{H}}_1 (\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1)^{-1}. \tag{27}$$

Considering the noise term in (24) and computing the first derivative w.r.t. $\mathbf{G}_2$ yields the expression

$$\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1 \mathbf{G}_2 \tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1 + 2\frac{\sigma_n^4}{\sigma_d^4} \mathbf{G}_2 = -\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{G}}_1 \tilde{\mathbf{H}}_2 \tilde{\mathbf{H}}_1. \tag{28}$$

Equation (28) is a matrix equation of the form

$$\mathbf{A}\mathbf{X}\mathbf{A}^{\mathsf{T}} + \mathbf{X} \cdot c + \mathbf{B} = \mathbf{0}, \tag{29}$$

which is very similar to the well-known Lyapunov equation [7]. Rewriting (29) as a linear equation system, the solution is given by

$$\mathrm{vec}(\mathbf{X}) = -(\mathbf{A} \otimes \mathbf{A} + \mathbf{I}c)^{-1} \mathrm{vec}(\mathbf{B}), \tag{30}$$

where $\otimes$ denotes the Kronecker product of two matrices and $\mathbf{I}$ is an identity matrix. Then we obtain for the equalizer coefficients

$$\begin{aligned} \mathrm{vec}(\mathbf{G}_2) = &- (\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1 \otimes \tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{H}}_1 + 2\frac{\sigma_n^4}{\sigma_d^4} \mathbf{I})^{-1} \\ &\mathrm{vec}(\tilde{\mathbf{H}}_1^{\mathsf{T}} \tilde{\mathbf{G}}_1 \tilde{\mathbf{H}}_2 \tilde{\mathbf{H}}_1). \end{aligned} \tag{31}$$

Remembering that the cascade connection of two second order systems results in a fourth order system, the nonlinear equalizer coefficients should account for the third and fourth order kernel of the interconnection. Similarly to the first nonlinear equalizer design, the calculation of these equalizer coefficients is done by computing the first order kernel of the equalizer with the conventional equations given in [8] or Section V and finding the optimal second order equalizer for the determined linear equalizer coefficients. Consequently, the third and fourth order kernel of the cascade are given as [9],[10]

$$\begin{aligned} &F_3[m_1, m_2, m_3] = \\ &\sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} g_2[l_1, l_2] h_1[m_1 - l_1] h_2[m_2 - l_2, m_3 - l_2] + \\ &\sum_{l_1=0}^{L-1} \sum_{l_2=0}^{L-1} g_2[l_1, l_2] h_2[m_1 - l_1, m_2 - l_2] h_1[m_3 - l_2] \end{aligned} \tag{32}$$

and

$$F_4[m_1,m_2,m_3,m_4] = \sum_{l_1=0}^{L-1}\sum_{l_2=0}^{L-1} g_2[l_1,l_2]h_2[m_1-l_1,m_2-l_1]h_2[m_3-l_2,m_4-l_2], \quad (33)$$

where many contributions of these kernels are already zero due to the limited order of $\{H_j\}$ and $\{G_j\}$ and only the remaining parts of the kernels are shown. It is seen in (32) and (33), that the linear equalizer coefficients $\mathbf{g}_1$ do not contribute to the third and fourth order kernel of the interconnection. Furthermore it is seen, that the third order kernel consists of combinations of $\mathbf{h}_1$ and $\mathbf{H}_2$ and the fourth order kernel has contributions proportional to the squared and mixed products of the kernel matrix $\mathbf{H}_2$. If the third and fourth order kernel is included in the computation of the second order kernel $\mathbf{G}_2$ many terms in the resulting equation can be immediately skipped because their influence is rather weak. In fact it turns out, that as a first approximation, only one single term has the main contribution to the overall equalizer coefficients. Using this term in the equation for the MSE we obtain

$$\mathrm{vec}(\mathbf{G}_2) = -\,(2\tilde{\mathbf{H}}_1^{\mathsf{T}}\tilde{\mathbf{H}}_1 \otimes \tilde{\mathbf{H}}_1^{\mathsf{T}}\tilde{\mathbf{H}}_1 + 2\frac{\sigma_n^4}{\sigma_d^4}\mathbf{I})^{-1} \mathrm{vec}(\tilde{\mathbf{H}}_1^{\mathsf{T}}\tilde{\mathbf{G}}_1\tilde{\mathbf{H}}_2\tilde{\mathbf{H}}_1), \quad (34)$$

for the "improved" second order equalizer kernel. To model the third and fourth order term in the MSE equation is not easy. It is not clear how the third and fourth dimension of these kernels are mapped to a two-dimensional system. However, the approximate consideration in computing the equalizer coefficients shows that this rather crude consideration gives a slight improvement.

VII. Performance Comparison

For evaluating the performance of the different equalization methods, 1000 different channels, i.e. nonlinear equivalent system models like shown in (3) have been used to transmit data sequences and evaluate the bit error rate (BER). These models correspond to a high-data-rate scenario with 125Mbps and an RMS delay spread of τ_{rms}=10ns to demonstrate the variations of individual performance results for different channels. Additionally the median, 10% and 90% quantiles of the BERs are shown.

It is seen in Fig. 3 that the lower complexity approach, with calculating the linear distance in the MLSD-DF-LIN, achieves only limited reduction of the introduced distortions. Conversely, the full knowledge of the channel parameters is exploited in the MLSD-DF-NLIN with a nonlinear distance calculation and a very good performance is achieved.

The adaptive approaches with an adaptive linear and an adaptive polynomial, i.e. second order Volterra filter, are shown in Fig. 4. It is seen that also for the linear equalizer most of the channels can be equalized referring to the median of the BER curves. A significant improvement is made when the nonlinear Volterra equalizer is used.

However, the computed linear MMSE equalizer, shown in Section V achieves slightly better performance than the linear adaptive approach because at high SNR the bias term heavily

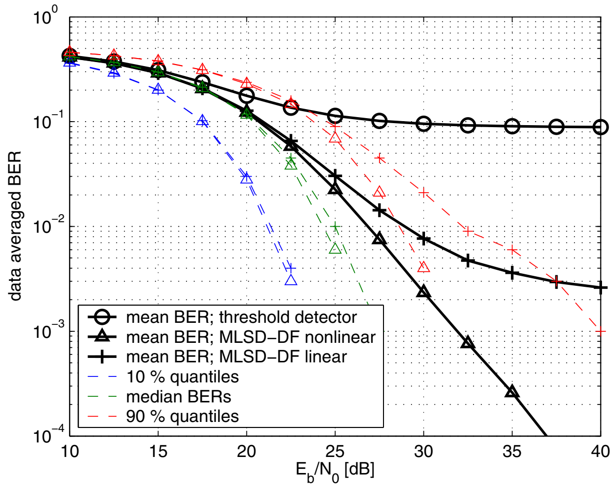

Fig. 3. Comparison of linear and nonlinear distance calculation in the MLSD-DF algorithm.

affects the result. For the non-adaptive approaches the bias is known and subtracted. Furthermore, the consideration of the nonlinear coefficients for the design of an improved linear equalizer shows, that a better performance could be achieved with a consideration of nonlinear terms for computing the linear equalizer. If the 90% quantiles of the approaches are compared, the improved linear equalizer shows room for further gain.

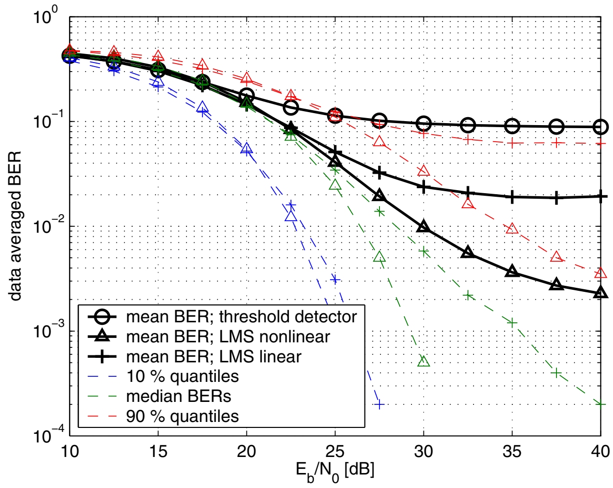

Fig. 4. Equalization performance of linear and nonlinear adaptive equalizers.

A similarly designed nonlinear equalizer which tries to compensate the nonlinear distortions was used to compare the linear to the nonlinear equalizer performance. One can easily see that this results in similar mean BER performance. Comparing the 90% quantiles again, a performance increase to the linear equalizer is achieved. Furthermore it is assumed that if all channel parameters are available to design the equalizer, the bias term h_0 is easily subtracted from the signals provided to the computed equalizers. Otherwise an ambiguity like described in [11] may occur, which heavily

TABLE I

COMPARISON OF THE ALGORITHMS

	Complexity #add = #mul	Channel knowledge -
MLSD-DF-LIN	$L \cdot 2^{(\eta - K + 2)}$	full
MLSD-DF-NLIN	$L + (L(L+1)/2) \cdot 2^{(\eta - K + 2)}$	full
Adaptive linear	$2L$	none
Adaptive nonlinear	$2L + L(L+1)$	none
MMSE linear	L	full
MMSE nonlinear	$L + L(L+1)/2$	full

affects the performance of the proposed equalizer. Similarly to the p-th order inverse approach described in [3] this approach may handle weak nonlinearities very well but has a lack in performance when equalizing severe nonlinearities.

Results of the computer simulation are depicted in Fig. 5. As mentioned in Section VI the third and fourth order term are only approximately included in the computation of the second order kernel matrix $\mathbf{G}_2$. This presumably causes the observed deviation in terms of BER (cf. Fig. 4 and Fig. 5) from the adapted equalizer.

A comparison with respect to algorithm complexity and prior channel knowledge is shown in Table I. It is obvious that the MLSD-DF has the highest complexity since it tries to compute all possible outcomes of the receiver. Due to the reduction in complexity by using decision feedback from past detected symbols, the complexity can be reduced for the state computation but the complete information about the channel parameters has to be available to the algorithm. The two adaptive approaches are twice as complex as the non-adaptive approaches also shown in this paper. This results of the additional update equation (8) which has similar complexity as the convolution and thus depends on the filter length.

However, the adaptive equalizers have the additional advantage that no a priori information about the channel has to be available. The proposed MMSE equalizers assume knowledge of the channel but stay rather low in terms of complexity. Compared to the adaptive approach it just has to be computed once and remains constant for the transmission of a data burst, thus resulting in an "online" comlexity of a non-adaptive linear and second order Volterra filter.

VIII. CONCLUSION

It is shown in this paper that the nonlinearity represented by the second order equivalent system model of a TR-IR-UWB system has severe impact on the ISI introduced. For a reduced state MLSD-DF detector the performance is heavily deteriorated when the branch metric is computed linearly while a nonlinear version yields nearly-optimal detection. For a huge range of possible channels also linear equalizers may equalize the distortions very well. Adaptive linear approaches only achieve limited equalization performance. For a further performance improvement a nonlinear adaptive equalizer can be used.

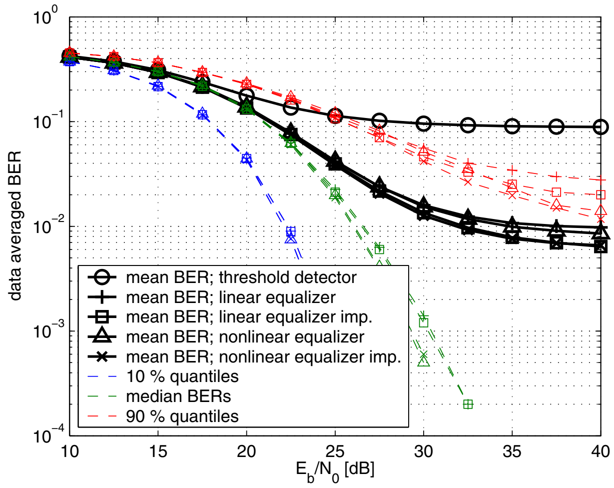

Fig. 5. Equalizers with manually determined first and second order kernels.

We have shown how to compute linear MMSE equalizer coefficients. Taking into consideration the second order kernel of the equivalent system model different linear equalizer coefficients are computed which further improve the performance of the equalizer. To compensate for the nonlinear distortions, the computation of the coefficients for a second order MMSE equalizer was derived. It is furthermore shown that the third and fourth order terms should be considered in the computation. An exact consideration of these terms not easily possible due to the multi-dimensionality of the kernels, however.

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