DISTRIBUTED TDOA-BASED INDOOR SOURCE LOCALISATION

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ABSTRACT

Indoor localisation is an important research topic with several possible applications. For example, knowing a user's location can be used as navigation aid in hospitals and malls, or for better targeted marketing. In this paper we consider the case where the environment of interest is equipped with several receivers (with known location) from which time-difference-of-arrival (TDOA) measurements are obtained and used to localise the source. We will present a distributed algorithm for localising the source. More specifically, we experimentally show that the distributed algorithm, which only uses time-of-arrival (TOA) measurements obtained from neighbouring receivers to calculate the TDOAs, performs as well as a centralised solution that has access to all TOA measurements in the network. In addition, we propose a method for discarding erroneous TOA measurements which considerably improves the performance in noisy and reverberant environments.

Index Terms— Indoor localisation, time difference of arrival, distributed algorithm, pruning erroneous TOA measurements

1. INTRODUCTION

Localisation techniques are becoming increasingly important in every aspect of daily life. In many applications with multiple sensors, the relative sensor positions are required to be known. For example, in the hospital, the position of patients can be obtained by medical staff if patients take sensors with them. In supermarkets, customers with sensors can be provided with specific sales information based on their current location. In outdoor environments, the global positioning system (GPS) performs well and is widely used. However, GPS does not work well in indoor environments because of many obstacles and line-of-sight is not guaranteed. Instead, alternative techniques for indoor localisation are proposed, such as methods based on received signal strength (RSS), time-of-arrival (TOA), time-difference-of-arrival (TDOA), or angle-of-arrival (AOA).

TOA based techniques estimate the location of sources through the intersection of the range of the receivers. The range information can be obtained from the TOA measurements by multiplying them with the propagation speed of the signals [1]. TOA based techniques can be used in indoor environments because they are robust against multipath effects. However, TOA based techniques have to deal with the unknown onset time, the time that signals are generated, and the unknown internal delay, the time that a receiver uses to register the signal as received after the signal reaches the receiver [2]. In order to obtain accurate TOA measurements, precise synchronisation is required among transmitters and receivers [1]. Different from TOA based techniques, TDOA based techniques use the time differences of arrival between several receivers to locate sources. TDOA often uses the generalised cross correlation of the received signals to compute the TDOAs. In order to compute the generalised cross correlation, receivers must have a data link between each other to share the received signals, which requires large bandwidth and power consumption [3]. Since the difference of arrival time is used for localisation, only the receivers are required to be synchronised and there is no need to eliminate the effect of the unknown onset time. Similar to TOA based techniques, TDOA based techniques are robust against multipath effects [4]. In addition, TDOA based techniques can obtain the same accuracy as TOA based techniques [5]. Alternatively, angle information or received signal strength can be used for localisation. The basic principle of AOA is that the intersection between the angles of received signals can locate the sensors [1]. AOA based techniques are not always considered for localisation because they require large dimensions of directional antennas. The advantage of AOA based techniques is that they are robust to large scale fading [6]. RSS based techniques are always connected with fingerprints [7]. The cost is low and most receivers can estimate the received signal strength. Nevertheless, the accuracy is relatively poor since the strength of the received signals is sensitive to noise and interferers [1].

There are several existing methods for distributed TDOA localisation, for example, [8] estimated the location distributedly with TDOA measurements in OFDM signals, [9] proposed a distributed algorithm based on gossip algorithms, and [10] tracked a moving target with distributed TDOA localisation. Our paper presents a localisation algorithm with TDOA based techniques that can be used in indoor environment for various types of signals and can be solved distributedly using standard solvers. The paper is organised as follows. In Section 2, we formulate the problem. We describe the centralised and distributed TDOA based localisation algorithms in Section 3. In addition, we propose a method to prune out incorrect TOA measurements in Section 4. We present experiment results in Section 5 and finally, we draw conclusions in Section 6.

2. PROBLEM FORMULATION

Consider a scenario with M receivers with known location and N transmitters whose locations are to be estimated using time difference of arrival (TDOA) techniques and the transmitters cannot be disambiguated. Assume the receivers are perfectly synchronised, but not synchronised with the transmitters. We consider sensors located on a plane and the extension to a three dimensional space is straightforward. Let $r_1, r_2, \ldots, r_M$ denote the location of receivers and $s_1, s_2, \ldots, s_N$ denote the location of transmitters. For receiver i from transmitter j, the time of arrival (TOA) information is given by

$$t_{ij} = \|r_i - s_j\| + n_{ij} \tag{1}$$

where c is the propagation speed of the signal, and n_{ij} is the measurement noise of receiver i with respect to transmitter j where we

assume $E[n_{ij}] = 0$ and $\text{Var}[n_{ij}] = \sigma_{ij}^2 < \infty$, with $E[\cdot]$ denotes the expectation and $\text{Var}[\cdot]$ denotes the variance. It is noteworthy that the localisation methods work for various types of signals, but in this paper we will focus on the acoustic scenario in this paper, i.e., the speed of the signal is set as $c = 340$ m/s. Moreover, we assume the measurement noise is uncorrelated across different receivers. TDOA measurements can be obtained by the subtraction of TOA measurements. In our model, each receiver calculates its own TOA measurement and communicates its TOA measurements to neighbours to obtain TDOA measurements. Since we can estimate the location of the transmitters independently in parallel, for simplicity, we use s to denote the location of the transmitter to be estimated and t_j to denote the TOA measurements for receiver j. In a two dimensional space, the coordinates of receivers and the estimated transmitter can be represented by $r_j = (x_j, y_j)$ where $1 \leq j \leq M$ and $s = (x_0, y_0)$. Without loss of generality, we choose that receiver 1 as the reference sensor and assume receiver 1 is at the origin, i.e., $r_1 = (0, 0)$. The network of all the receivers is represented as a graph $G = (V, E)$, where V is the set of vertices and E is the set of edges, which indicate the communication links in the network.

3. LOCALISATION ALGORITHMS

With the position of receivers and the corresponding TOA measurements, we can locate the transmitter with TDOA based techniques. In this section, we present centralised and distributed TDOA based localisation methods.

3.1. Centralised Localisation

We first present a centralised localisation method with a fixed reference sensor, where all TOA measurements are transmitted to a central computer [11]. Receiver 1 is assumed to be the reference sensor to compute TDOAs. Let Δt_{1j} denote the TDOA measurement at receiver j with respect to receiver 1, $2 \leq j \leq M$, which is given by

$$\Delta t_{1j} = t_1 - t_j = \frac{\|s - r_1\|}{c} - \frac{\|s - r_j\|}{c} + n_{1j}, \qquad (2)$$

where $n_{1j} \sim \mathcal{N}(0, \sigma_{1j}^2)$ is Gaussian distributed measurement noise of TDOA measurement between receiver 1 and receiver j. Because of uncorrelated measurements, $\sigma_{1j}^2 = \sigma_1^2 + \sigma_j^2$.

The distance difference between receiver j and receiver 1, say d_{j1}, can be calculated from the TODAs by multiplying it with the propagation speed of the signal, so that

$$d_{j1} = d_j - d_1 = c\Delta t_{1j}, \qquad (3)$$

where $d_j^2 = (x_j - x_0)^2 + (y_j - y_0)^2$ denotes the distance between the transmitter and receiver j. Hence we have

$$(d_{j1} + d_1)^2 = d_j^2 = (x_j - x_0)^2 + (y_j - y_0)^2. \qquad (4)$$

Let $K_j^2 = x_j^2 + y_j^2$, so that we can rewrite (4) as

$$-x_j x_0 - y_j y_0 = d_{j1} d_1 + \frac{1}{2}(d_{j1}^2 - K_j^2). \qquad (5)$$

We write (5) in vector form with $2 \leq j \leq M$ [11],

$$\boldsymbol{A}s = d_1 \boldsymbol{b} + \boldsymbol{c}, \qquad (6)$$

where

$$\boldsymbol{A} = \begin{bmatrix} x_2 & y_2 \\ \vdots & \vdots \\ x_M & y_M \end{bmatrix}, s = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix},$$

$$\boldsymbol{b} = \begin{bmatrix} -d_{21} \\ \vdots \\ -d_{M1} \end{bmatrix}, \boldsymbol{c} = \frac{1}{2} \begin{bmatrix} K_2^2 - d_{21}^2 \\ \vdots \\ K_M^2 - d_{M1}^2 \end{bmatrix}.$$

Since the measurements are noisy, a least-squares location estimate can be found by solving

$$\min_s ||\boldsymbol{A}s - (d_1 \boldsymbol{b} + \boldsymbol{c})||_2^2. \qquad (7)$$

The solution is given by

$$\hat{s} = (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T (d_1 \boldsymbol{b} + \boldsymbol{c}). \qquad (8)$$

Combining the solution with $d_1^2 = x_0^2 + y_0^2$, the location of the transmitter can be estimated [11]. The Cramer-Rao lower bound for centralized localisation is given by [12].

3.2. Centralised Localisation With Arbitrary Reference Sensor

All TOA measurements are transmitted to a central computer. We now extend the approach in Section 3.1 to the case of an arbitrary reference sensor. The benefit of this is that we can improve the estimation accuracy by averaging over all possible estimates obtained by using different reference receivers. Assume that receiver k is chosen as the reference receiver, and the corresponding estimated source location is denoted as s_k. With this, the problem can be formulated as

$$\min_{s_k} ||\boldsymbol{A}_k s_k - (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)||_2^2, \qquad (9)$$

where

$$\boldsymbol{A}_k = \begin{bmatrix} x_1 - x_k & y_1 - y_k \\ \vdots & \vdots \\ x_{k-1} - x_k & y_{k-1} - y_k \\ x_{k+1} - x_k & y_{k+1} - y_k \\ \vdots & \vdots \\ x_M - x_k & y_M - y_k \end{bmatrix}, s_k = \begin{bmatrix} x_{0k} \\ y_{0k} \end{bmatrix},$$

$$\boldsymbol{b}_k = \begin{bmatrix} -d_{1k} \\ \vdots \\ -d_{k-1k} \\ -d_{k+1k} \\ \vdots \\ -d_{Mk} \end{bmatrix}, \boldsymbol{c}_k = \frac{1}{2} \begin{bmatrix} K_1^2 - K_k^2 - d_{1k}^2 \\ \vdots \\ K_{k-1}^2 - K_k^2 - d_{k-1k}^2 \\ K_{k+1}^2 - K_k^2 - d_{k+1k}^2 \\ \vdots \\ K_M^2 - K_k^2 - d_{Mk}^2 \end{bmatrix}.$$

The estimated source location is given by the least-squares solution to problem (9),

$$\hat{s}_k = (\boldsymbol{A}_k^T \boldsymbol{A}_k)^{-1} \boldsymbol{A}_k^T (d_k \boldsymbol{b}_k + \boldsymbol{c}_k). \qquad (10)$$

Given the M estimations, which are computed at M different reference receivers, we can average them to get a more accurate estimate. That is, we compute

$$\hat{s} = \frac{1}{M} \sum_{k=1}^{M} \hat{s}_k. \qquad (11)$$

3.3. Distributed Localisation With Full Information

We would like to decentralize the problem step by step. In contrast to previous methods, we estimate the location of the source locally at each receiver. That is to say, we can get rid of the central computer. Assume each receiver gets access to complete TOA information. Based on received information from neighbours, the receivers can update its local estimate until they reach consensus among the estimated position. With this, our problem can be expressed as

$$\min_{s_k} \sum_{k=1}^{M} ||\boldsymbol{A}_k s_k - (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)||_2^2,$$

$$\text{subject to } s_k = s_j, \forall (k,j) \in E, \qquad (12)$$

which can be solved using standard solvers, like ADMM [13] and PDMM [14].

3.4. Distributed Localisation With Neighbouring Information

In the previous subsection, the information of all receivers in the network was used to locally compute an estimate of the position. However, this implies that the network is fully connected, which is not realistic in many real applications. Considering decentralised localisation, each receiver computes a local estimated transmitter location with the TOA information from its neighbouring nodes only. Assume the neighbouring nodes of receiver k are denoted by $k_1, ..., k_{M_k}$, where M_k is the degree of each receiver. We assume the degree of each receiver is greater than 1, i.e., $M_k \geq 2$. With this, our problem can be expressed as

$$\min_{s_k} \sum_{k=1}^{M_k} ||\boldsymbol{A}_k s_k - (d_k \boldsymbol{b}_k + \boldsymbol{c}_k)||_2^2,$$

$$\text{subject to } s_k = s_j, \forall (k,j) \in E, \qquad (13)$$

where

$$\boldsymbol{A}_k = \begin{bmatrix} x_{k_1} - x_k & y_{k_1} - y_k \\ \vdots & \vdots \\ x_{k_M} - x_k & y_{k_M} - y_k \end{bmatrix}, s_k = \begin{bmatrix} x_{0k} \\ y_{0k} \end{bmatrix},$$

$$\boldsymbol{b}_k = \begin{bmatrix} -d_{k_1 k} \\ \vdots \\ -d_{k_M k} \end{bmatrix}, \boldsymbol{c}_k = \frac{1}{2} \begin{bmatrix} K_{k1}^2 - K_k^2 - d_{k_1 k}^2 \\ \vdots \\ K_{kM}^2 - K_k^2 - d_{k_M k}^2 \end{bmatrix}.$$

Since (13) is of the same form as (12), it can be solved by a standard solver, like ADMM and PDMM.

4. PRUNING OUT ERRONEOUS TOA MEASUREMENTS

As mentioned before, due to measurement noise, we obtain erroneous TOA measurements. Here erroneous refers to an error that is beyond that small error due to sampling of the signal; these are errors due to noise and reverberation and increase with decreasing signal-to-noise ratio (SNR) and increasing reverberation time T_{60}. Even one erroneous time-of-arrival (TOA) measurement may result in a completely wrong estimate of the localisation algorithms. In this section, we describe an algorithm that selects a subset of accurate TOAs from a set of measured TOAs. Without loss of generality, we assume that the propagation speed of the signal is $c = 1$ and that the internal delays and the onset time are compensated [15]. In a

multiple transmitters scenario, the observed TOA at the ith receiver to the jth transmitter is given by (1). For the next step in the derivation we assume that there is no observations noise. Subtracting the square of equation (1) for $i = 1$ and $j = 1$, we obtain

$$-(r_i - r_1)^T(s_j - s_1) = 0.5(t_{ij}^2 - t_{1j}^2 - t_{i1}^2 + t_{11}^2), \qquad (14)$$

for $i = 2, ..., M$ and $j = 2, ..., N$. With this, we can express (14) in the vector form,

$$-\overline{\boldsymbol{L}}\overline{\boldsymbol{S}}^T = \boldsymbol{T}, \qquad (15)$$

where $\overline{\boldsymbol{L}} \in \mathbb{R}^{(M-1)\times 3}$ is the relative receiver location (to r_1) matrix, $\overline{\boldsymbol{S}} \in \mathbb{R}^{(N-1)\times 3}$ is the relative transmitter location (to s_1) matrix and $\boldsymbol{T}_{i-1,j-1} = 0.5(t_{ij}^2 - t_{1j}^2 - t_{i1}^2 + t_{11}^2) \in \mathbb{R}^{(M-1)\times(N-1)}$.

According to (15), if there is no erroneous TOA measurements, matrix $\overline{\boldsymbol{L}}\overline{\boldsymbol{S}}^T$ is at most rank 3. This property can be used to find the correct subset of TOA measurements.

For all M receivers, from N transmitters, the set of TOA measurements is denoted as S_N. The set of all $N - 1$ unique combinations of S_N is then given by

$$U_{N-1} = \binom{S_N}{N-1}. \qquad (16)$$

Take a specific combination u from U_{N-1} to construct $\boldsymbol{T}_u \in \mathbb{R}^{(M-1)\times(N-2)}$ for $j = u$ and compute the error, which is defined as

$$e_u = ||\boldsymbol{T}_u||_F^2 = \sum_{i=1}^{N_r} \lambda_i^2(\boldsymbol{T}_u), \qquad (17)$$

where $N_r = \min(M - 1, N - 2)$ and $\lambda_i(\boldsymbol{T}_u)$ is the singular value of $\boldsymbol{T}_u$. If all TOA measurements are correct, the errors are close to equal and the minimum error can represent the most reliable TOA measurements if they are different. When no TOA measurement is correct, all errors are small but the maximum error is relatively large compared to the error of all correct TOA measurements. As a consequence, the subset of TOA measurements can be selected as

$$S_o = \begin{cases} \arg\min_u \text{Var}\{e_u\} & \text{if } \max e_u < \alpha \\ S_N & \text{otherwise} \end{cases}. \qquad (18)$$

The iterative method to prune all erroneous TOA measurements is summarised in Algorithm 1. The minimum transmitters required for a successful localisation is given by [15]

$$N_{min} = \left\lceil \frac{4M - 7}{M - 4} \right\rceil, \qquad (19)$$

where $\lceil \cdot \rceil$ denotes the ceiling operator.

5. EXPERIMENTS

In this section, we describe how we set up our experiments and discuss the result. We conducted two experiments, one for TDOA localisation and one for pruning out incorrect TOA measurements.

In the first experiment, one transmitter is placed randomly on a 10×10 m plane and eight receivers are placed uniformly on the boundary of the plane. We assume two receivers can communicate with each other if the distance between them is less than 10 m. In addition, the velocity is set to $c = 340$ m/s. We change the variance of TOA measurement and record the variance of localisation error. The result of the simulation is also an average of 1000 independent experiments. The result is shown in Figure 1. From the figure, we

Algorithm 1 Pruning incorrect TOA measurements

1. For $n = 0, 1, N - N_{min}$
2. Generate the set of all possible combinations of the set S_{N-1}

$$U_{N-n+1} = \begin{pmatrix} S_N \\ N - n + 1 \end{pmatrix}$$

3. For each $u \in U_{N-n+1}$, construct $\boldsymbol{T}_u$ and compute the error.
4. Update the best TOA sets,

$$S_{N-n+1} = \begin{cases} \arg\min_s e_u & \text{if } \min e_u / \max e_u < \alpha \\ S_{N-n} & \text{otherwise} \end{cases}$$

5. End if $S_{N-n+1} = S_{N-n}$

can reach three conclusions. Firstly, the variance of localisation increases with the variance of TOA measurements. Secondly, decentralised methods can achieve approximately equal performance as centralised methods. Lastly, accessing neighbouring information only does not decrease accuracy.
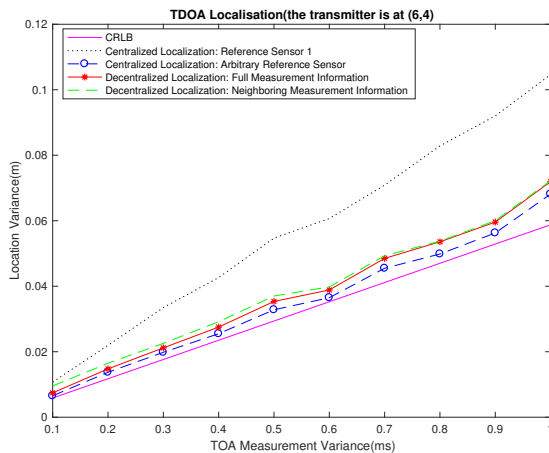

Fig. 1. TDOA Localisation

In the second experiment, we simulated a $6 \times 5 \times 4$ m acoustic room with the source-image method [16]. 8 receivers were positioned randomly within a rectangular space of $2 \times 2 \times 1$ m and 30 transmitters were distributed randomly in a one meter cubic space. The reverberation time T_{60} was varied between 0 s and 0.6 s and SNR was varied between 0 dB and 30 dB. We executed the algorithm and measured hits and false alarms. A hit is defined as an erroneous TOA measurement being detected while a false alarm occurs when a correct TOA measurement is classified as erroneous. We also count a hit when the algorithm has deemed correctly all TOAs to be inadequate for localisation and a false alarm when this decision is made erroneously. The outcome of the experiment is shown in Figure 2. The figure shows that the erroneous TOA measurements have been pruned correctly with only a small number of false alarms.

6. CONCLUSIONS

In this paper, we presented how to determine the location with TDOA measurements. We described centralised and decentralised
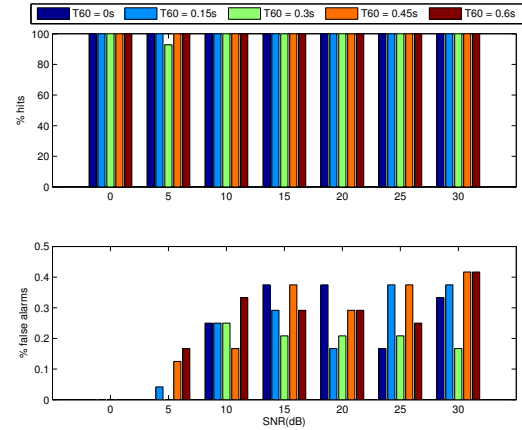
Fig. 2. Accuracy of the algorithm for pruning incorrect TOA measurements.

methods for localisation and compared the accuracy of algorithms with CRLB. From the experiment, we proved that accessing to neighbouring information does not decrease the accuracy. Furthermore, we proposed a method to prune out erroneous TOA measurements, which improves the performance in indoor environments.

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