Sparse Multi-Target Localization Using Cooperative Access Points

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***Abstract*—In this paper, a novel multi-target sparse localization (SL) algorithm based on compressive sampling (CS) is proposed. Different from the existing literature for target counting and localization where signal/received-signal-strength (RSS) readings at different access points (APs) are used separately, we propose to reformulate the SL problem so that we can make use of the cross-correlations of the signal readings at different APs. We analytically show that this new framework can provide a considerable amount of extra information compared to classical SL algorithms. We further highlight that in some cases this extra information converts the under-determined problem of SL into an over-determined problem for which we can use ordinary least-squares (LS) to efficiently recover the target vector even if it is not sparse. Our simulation results illustrate that compared to classical SL this extra information leads to a considerable improvement in terms of number of localizable targets as well as localization accuracy.**

I. Introduction

Precise localization of multiple targets in a covered area is a fundamental problem which has received a lot of attention recently [1]. Many different approaches have been proposed in the literature to recover the location of the targets based on time-of-flight (ToF), time-difference-of-arrival (TDOA) or received-signal-strength (RSS) measurements. A traditional approach in RSS-based localization tries to extract distance information from the RSS measurements. However, this approach fails to provide accurate location estimates due to the complexity and unpredictability of the wireless channel. The other category of RSS-based positioning, called location fingerprinting, discretizes the physical space into grid points and creates a map representing the space by assigning to every grid point a location-dependent RSS parameter. The location of the target is then estimated by comparing real-time measurements with the fingerprinting map at the target or access points (APs). The algorithms built on this approach (including the K-nearest neighbors (KNN) [2] and the Bayesian classification [3]) proved themselves as low-complexity and cost-efficient methods which can also attain an acceptable accuracy as long as the size of the grid is large. However, many of these fingerprinting approaches are costly as they require a large amount of data exchange between transmitter-receiver pairs.

A deeper look at the grid-based fingerprinting localization problem reveals that the target location is unique in the spatial domain, and can thus be represented by a 1-sparse vector. This motivated the use of compressive sampling (CS) [4] to recover the location of the target using far fewer measurements by solving an ℓ_1-norm minimization problem. This approach illustrated promising results for the first time in [5] as well as in the following works, e.g., [6], [7], [8]. In [7], it is proposed to use a joint distributed CS (JDCS) in a practical localization scenario in order to exploit the common sparse structure of the received measurements to localize one mobile target. In [8], a greedy matching pursuit algorithm is proposed for target counting and localization with high accuracy.

Existing sparse localization (SL) algorithms only make use of the signal/RSS readings at different receivers (or APs) separately. However, there is potential information in the cross-correlations of these received signals at different APs which has not been exploited yet. In this paper, we propose to use cooperation between the APs by exchanging the signal readings among them. In this way, we reformulate the SL problem so that we can also make use of the cross-correlations of the received signals at different APs. To be able to do this, we propose to construct a new fingerprinting map, during the *training* phase, which includes the cross-correlation information between the APs. Next, in the *runtime* phase, the locations of the targets are simultaneously recovered using the signal readings at different APs. We illustrate that our proposed SL framework provides an extra amount of information for the same number of APs compared to the classical SL in literature. This in turn leads to the possibility of recovering a larger number of targets simultaneously as well as of obtaining a higher accuracy. It is also worth mentioning that cases can occur where this extra information converts the under-determined SL problem into an over-determined problem for which we can use ordinary least-squares (LS) to efficiently recover the target vector even if it is not sparse. The rest of the paper is organized as follows. In Section II the classical SL problem is explained and then the new SL algorithm based on the cooperation of APs is introduced and analyzed. Section III illustrates the proposed method by simulation results to validate the improvement gained by the proposed SL in terms of localization accuracy and number of localizable targets. Finally, the paper is wrapped up in Section IV.

II. Sparse Target Localization

Consider that we have M access points (APs) distributed over a square area which is discretized into N square cells represented by their central grid point. Note that the APs can be located anywhere, not necessarily on the grid points. We consider K target nodes which can be located randomly on any of these grid points. Suppose that the APs are connected to each other in a wireless or wired fashion so that they can cooperate by exchanging their signal readings. Now, if the k-th target broadcasts a time domain signal $s_k(t)$, the M received signals at the APs stacked in a vector $\mathbf{s}_k^r(t)$ can be expressed by $\mathbf{s}_k^r(t) = \left[h_{1,k}s_k(t-\tau_{1,k}), \cdots, h_{M,k}s_k(t-\tau_{M,k})\right]^T$, where in

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general $h_{i,k}$ is the random channel coefficient from the k-th target to the i-th AP which is considered to be known from the physics of propagation or acquired through training. Further, the $h_{i,k}$s are considered to be fixed during the localization process. Correspondingly, $\tau_{i,k}$ represents the related time-delay. The signals $s_k(t)$ are considered to be wide sense stationary mutually uncorrelated sequences which is a typical assumption in the context of multi-target localization. The total received signal is then given by

$$\mathbf{s}^r(t) = \sum_{k=1}^{K} \mathbf{s}_k^r(t). \tag{1}$$

A. Classical Sparse Localization

Now considering that the targets can only be located on a finite set of positions determined by the N grid points, the total received signal corrupted by additive white Gaussian noise (AWGN) can be expressed as

$$\mathbf{s}^r(t) = \mathbf{\Gamma}(t)\boldsymbol{\theta} + \boldsymbol{\epsilon}(t), \tag{2}$$

where $\mathbf{\Gamma}(t)$ is an $M \times N$ matrix of the form

$$\mathbf{\Gamma}(t) = \begin{bmatrix} h_{1,1}x_1(t-\tau_{1,1}) & \cdots & h_{1,N}x_N(t-\tau_{1,N}) \\ h_{2,1}x_1(t-\tau_{2,1}) & \cdots & h_{2,N}x_N(t-\tau_{2,N}) \\ \vdots & \ddots & \vdots \\ h_{M,1}x_1(t-\tau_{M,1}) & \cdots & h_{M,N}x_N(t-\tau_{M,N}) \end{bmatrix}, \tag{3}$$

where $x_j(t)$ is the transmitted signal from the j-th grid point[1], and $h_{i,j}$ and $\tau_{i,j}$ respectively represent the channel coefficient and time-delay observed by the signal transmitted from a target on the j-th grid point to the i-th AP. $\boldsymbol{\theta}$ is a $N \times 1$ vector with all elements equal to zero except for those K elements corresponding to the locations of the targets which are equal to one. $\boldsymbol{\epsilon}(t)$ stacks the AWGN at the different APs. One way to create the SL fingerprinting map is to compute the RSS by taking the autocorrelation of the received time-domain signals at the APs as

$$\begin{aligned} \mathbf{y} &= \mathbb{E}\{\mathbf{s}^r(t) \odot \mathbf{s}^{r*}(t)\} \\ &= \mathbb{E}\{(\mathbf{\Gamma}(t)\boldsymbol{\theta} + \boldsymbol{\epsilon}(t)) \odot (\mathbf{\Gamma}(t)\boldsymbol{\theta} + \boldsymbol{\epsilon}(t))^*\} \\ &= \mathbb{E}\{\mathbf{\Gamma}(t) \odot \mathbf{\Gamma}^*(t)\}\boldsymbol{\theta} + \mathbb{E}\{\boldsymbol{\epsilon}(t) \odot \boldsymbol{\epsilon}^*(t)\} \\ &= \mathbf{\Psi}\boldsymbol{\theta} + \sigma_n^2 \mathbf{1}_M, \end{aligned} \tag{4}$$

where $\odot$ stands for the Hadamard product, $(.)^*$ means the complex-conjugate operation, and $\mathbf{1}_M$ is the $M \times 1$ vector of all ones. Note that to derive the third equality in (4) we assume that the transmitted signals from different targets are stationary and mutually uncorrelated. Moreover, we also assume that the transmitted signals are also uncorrelated with the AWGN and σ_n^2 denotes the noise variance at the APs. As is shown in (4), $\mathbf{y}$ is the K-sparse RSS characterized by the fingerprinting map $\mathbf{\Psi}$ and the ultimate goal is to recover $\boldsymbol{\theta}$ only by determining the index of its K non-zero elements. From (4), $\mathbf{\Psi}$ will have the following form

$$\mathbf{\Psi} = \begin{bmatrix} |h_{1,1}|^2 & |h_{1,2}|^2 & \cdots & |h_{1,N}|^2 \\ |h_{2,1}|^2 & |h_{2,2}|^2 & \cdots & |h_{2,N}|^2 \\ \vdots & \vdots & \ddots & \vdots \\ |h_{M,1}|^2 & |h_{M,1}|^2 & \cdots & |h_{M,1}|^2 \end{bmatrix}. \tag{5}$$

[1] $x_j(t) = s_k(t)$ if k-th target is at the j-th grid point, otherwise it is zero.

Now if $K \ll N$, compressive sampling aims to reconstruct $\boldsymbol{\theta}$ by only taking M measurements from the M APs [6]. It is worth mentioning that here we have a natural compression in the problem in the sense that the number of measurements is limited to the number of APs (M) which in many practical scenarios is much less than the number grid points (N). Therefore, as long as $M \geq c.K.log(N/K)$ with c some positive constant and $\mathbf{\Psi}$ holds the restricted isometry property (RIP), $\boldsymbol{\theta}$ can be well-recovered by solving the following ℓ_1-norm minimization program

$$\hat{\boldsymbol{\theta}} = \text{argmin} \left\|\boldsymbol{\theta}\right\|_1 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{\Psi}\boldsymbol{\theta}. \tag{6}$$

To solve the above problem we can use the basis pursuit (BP) algorithm. Notably, $\mathbf{\Psi}$ obeys the RIP for $\delta \in (0, 1)$ if

$$1 - \delta \leq \frac{\left\|\mathbf{\Psi}\boldsymbol{\theta}\right\|_2^2}{\left\|\boldsymbol{\theta}\right\|_2^2} \leq 1 + \delta \tag{7}$$

For the current $\mathbf{\Psi}$, by simulations, it can be shown to meet the RIP condition as δ is not too close to 1. A computationally intensive approach is proposed in [6] which helps to slightly improve this condition.

B. Sparse Localization using Cooperative APs

As explained in the previous subsection, the existing SL algorithms only make use of the autocorrelation (signal strength) of the signals received at each AP separately and ignore the potential information present in the cross-correlation of this information. We propose to reformulate the problem so that we can exploit this extra information with the help of a cooperation among the APs. This new model requires the construction of a new fingerprinting map as will be explained subsequently. Let us reconsider (2) and define the new RSS measurements as

$$\begin{aligned} \mathbf{Y} &= \mathbb{E}\{\mathbf{s}^r(t)\mathbf{s}^{rH}(t)\} \\ &= \mathbb{E}\{(\mathbf{\Gamma}(t)\boldsymbol{\theta} + \boldsymbol{\epsilon}(t))(\mathbf{\Gamma}(t)\boldsymbol{\theta} + \boldsymbol{\epsilon}(t))^H\} \\ &= \mathbb{E}\{\mathbf{\Gamma}(t)\boldsymbol{\theta}\boldsymbol{\theta}^H\mathbf{\Gamma}^H(t)\} + \mathbb{E}\{\boldsymbol{\epsilon}(t)\boldsymbol{\epsilon}^H(t)\}, \end{aligned} \tag{8}$$

where $(.)^H$ means conjugate-transpose. According to (2), $\mathbf{s}^r(t)$ is the sum of the received signals from the different targets. Now, if we consider a target on the j-th grid point, $\mathbf{Y}$ is given by

$$\mathbf{Y} = \begin{bmatrix} |h_{1,j}|^2 & h_{1,j}^*h_{2,j} & \cdots & h_{1,j}^*h_{M,j} \\ h_{2,j}^*h_{1,j} & |h_{2,j}|^2 & \cdots & h_{2,j}^*h_{M,j} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M,j}^*h_{1,j} & h_{M,j}^*h_{2,j} & \cdots & |h_{M,j}|^2 \end{bmatrix} + \sigma_n^2\mathbf{I}_M, \tag{9}$$

which is a symmetric $M \times M$ matrix and where $\mathbf{I}_M$ stands for the $M \times M$ identity matrix. Correspondingly, the overall $\mathbf{Y}$ can be calculated be adding the signals received from the K different targets. To end up with a similar expression like (4) we vectorize both sides of (8) as follows

$$\begin{aligned} \text{vec}(\mathbf{Y}) &= \text{vec}(\mathbb{E}\{\mathbf{\Gamma}(t)\boldsymbol{\theta}\boldsymbol{\theta}^H\mathbf{\Gamma}^H(t)\}) + \text{vec}(\boldsymbol{\epsilon}(t)\boldsymbol{\epsilon}^H(t)) \\ &= \mathbb{E}\{(\mathbf{\Gamma}^*(t) \otimes \mathbf{\Gamma}(t))\}\text{vec}(\boldsymbol{\theta}\boldsymbol{\theta}^H) + \text{vec}(\boldsymbol{\epsilon}(t)\boldsymbol{\epsilon}^H(t)) \\ &= \mathbb{E}\{(\mathbf{\Gamma}^*(t) \circ \mathbf{\Gamma}(t))\}\boldsymbol{\theta} + \text{vec}(\sigma_n^2\mathbf{I}_M) \end{aligned} \tag{10}$$

where $\otimes$ represents Kronecker product, and $\circ$ stands for the Khatri-Rao product. Note that to derive the third equality in (10) we assume that the transmitted signals from different targets are stationary and mutually uncorrelated. Obviously,

(10) automatically accommodates the signals received from the K targets as $\boldsymbol{\theta}$ has K non-zero elements. Note that considering (9) and (10), there are M^2 or $M(M+1)/2$ (if $\mathbf{Y}$ is symmetric) different elements in (9) and correspondingly the same number of independent linear equations in (10). To formulate the problem in a more general fashion and without losing generality, we consider $\mathbf{Y}$ to be symmetric. To select only those independent equations, we define a selection operator $\mathbf{\Phi}$ which only selects the rows corresponding to the M diagonal and $M(M-1)/2$ upper-diagonal elements of $\mathbf{Y}$. The indices of the selected rows are contained in $\Omega = \{(i-1)N+1, \cdots, (i-1)N+i \mid i = 1, \cdots, N\}$. Thus,

$$\begin{aligned}\tilde{\mathbf{y}} &= \mathbf{\Phi}\text{vec}(\mathbf{Y}) \\ &= \mathbf{\Phi}\mathbb{E}\{(\mathbf{\Gamma}^* \circ \mathbf{\Gamma})\}\boldsymbol{\theta} + \mathbf{\Phi}\text{vec}(\sigma_n^2 \mathbf{I}_M) \\ &= \tilde{\mathbf{\Psi}}\boldsymbol{\theta} + \tilde{\mathbf{p}}_n, \end{aligned} \tag{11}$$

where $\tilde{\mathbf{\Psi}}$ is the new RSS fingerprinting map of size $M(M+1)/2 \times N$ that should be calculated during a *training* phase and $\tilde{\mathbf{p}}_n$ is the corresponding noise vector of size $M(M+1)/2 \times 1$ ($\tilde{(.)}$ denotes the proposed cooperation among the APs). Following the aforementioned explanation, $\tilde{\mathbf{\Psi}}$ will have the following form

$$\tilde{\mathbf{\Psi}}^T = \begin{bmatrix} |h_{1,1}|^2, & h_{1,1}^* h_{2,1}, & |h_{2,1}|^2, & \cdots, & h_{1,1}^* h_{M,1}, & \cdots, & |h_{M,1}|^2 \\ |h_{1,2}|^2, & h_{1,2}^* h_{2,2}, & |h_{2,2}|^2, & \cdots, & h_{1,2}^* h_{M,2}, & \cdots, & |h_{M,2}|^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ |h_{1,N}|^2, & h_{1,N}^* h_{2,N}, & |h_{2,N}|^2, & \cdots, & h_{1,N}^* h_{M,N}, & \cdots, & |h_{M,N}|^2 \end{bmatrix}.$$

It is noteworthy that in our simulations, since we choose random channel effects (particularly a Rician fading channel) both $\mathbf{\Psi}$ and $\tilde{\mathbf{\Psi}}$ will satisfy the RIP. As can be seen, the newly proposed SL model in (11) provides us with a set of $M(M+1)/2$ linear equations instead of only M as in (4). This added information ($M(M+1)/2 - M$ extra equations), obtained by taking the cross-correlations of the received signals at different APs into account, makes it possible for the system to localize a larger number of targets with a fixed number of APs which becomes even more important when the physical conditions of the covered area limit the number of possible APs. This means that we only need $M' \ll M$ APs with

$$\begin{aligned}&\frac{M'(M'+1)}{2} \geq c.K.log(N/K) \\ &\Leftrightarrow M' \geq \frac{-1+\sqrt{1+8.c.K.log(N/K)}}{2}. \end{aligned} \tag{12}$$

For $c = 3$ (just a typical number), Fig. 1 illustrates the minimum number of APs required to localize K targets simultaneously. As can be seen, the proposed SL is theoretically capable of localizing the same number of targets with much fewer APs. Next, the new SL problem in (11) can be solved by considering the following two possible cases.

– **Case I:** $N > M(M+1)/2$; In this case, by considering the sparse structure of $\boldsymbol{\theta}$, the extra information enables us to locate more targets by solving the following ℓ_1-norm minimization problem (again using BP)

$$\hat{\boldsymbol{\theta}} = \text{argmin} \|\boldsymbol{\theta}\|_1 \quad \text{s.t.} \quad \tilde{\mathbf{y}} = \tilde{\mathbf{\Psi}}\boldsymbol{\theta}. \tag{13}$$

– **Case II:** $N \leq M(M+1)/2$; Since $\tilde{\mathbf{\Psi}}$ has generally full column rank in this case, no matter what the structure of $\boldsymbol{\theta}$ might be, even if it is not sparse, it can be efficiently recovered by ordinary LS as

$$\hat{\boldsymbol{\theta}} = \tilde{\mathbf{\Psi}}^{\dagger}\tilde{\mathbf{y}}, \tag{14}$$

where $(.)^{\dagger}$ represents the pseudo-inverse.

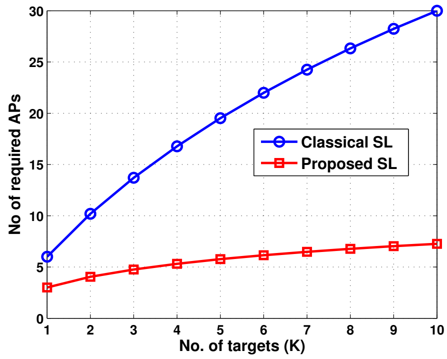

Fig. 1: Reduction of number of required APs

III. Numerical Results

In this section, we investigate the performance of the proposed SL compared to the existing classical approach in terms of the localization accuracy and the number of localizable targets. To this aim, we consider a wireless network of size 10×10 m^2 divided into $N = 100$ grid points and we consider 14 APs covering the whole area. We also consider up to $K = 10$ targets to be simultaneously localized. In our simulations, we consider a Rician fading channel defined by $h_{i,j} \sim \text{Rice}(\nu, \sigma)$ with $\nu = 1$ and $\sigma = 1$ (corresponding to $K = \nu^2/(2\sigma^2) = 0.5$) which indicates equal power in the direct path and other paths. We define the SNR as the ratio of the transmit power (P_t, assumed to be equal for all targets) to the noise power at the receiver σ_n^2.

The computations of the autocorrelations as well as the cross-correlations cannot be exact as in the derivations of Section II. Therefore, in practice, in the *runtime* phase we always compute estimates of $\mathbf{y}$ and $\tilde{\mathbf{y}}$ as expressed by (4) and (11). However, since $\mathbf{s}^r(t)$ signals are stationary, meaning that the statistical expectation can be calculated using time-domain averaging, it is sufficient to record a time-slot of length T from $\mathbf{s}^r(t)$ which can even be short in order to provide an estimation of the autocorrelations and cross-correlations with small error. Here, we consider a baseband BPSK signal with bandwidth $B = 10$kHz and compute the autocorrelations and cross-correlations during a time-slot of length $T = 0.1$s. This is equal to sending $T \times B = 1000$ BPSK symbols for our computations. Hence, for moving targets with low dynamics, which is a realistic assumption for the network under consideration, the length of the time-slot ($T = 0.1$s) will not put a large constraint on the dynamics of the targets. Further, note that we do not have to estimate the time delays ($\tau_{i,j}$) since we only require the peak values of the autocorrelations and cross-correlations.

To be able to quantitatively compare the performances of the algorithms under consideration, we consider the positioning root mean squared error (PRMSE) defined by

$$\text{PRMSE} = \sqrt{\frac{\sum_{m=1}^{M}\sum_{k=1}^{K} e_{k,m}^2}{M}}, \tag{15}$$

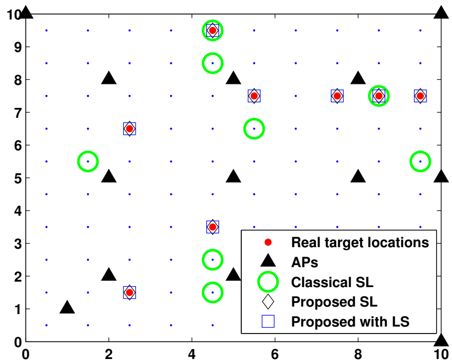

Fig. 2: Multi-target (K = 8) localization with 14 APs

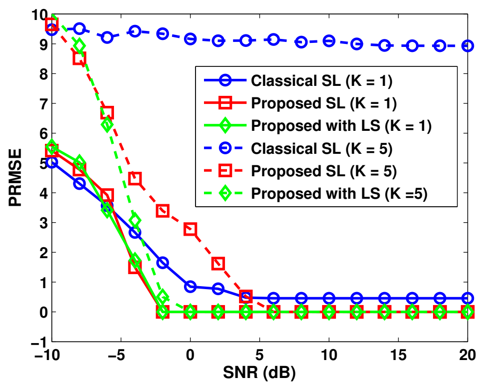

Fig. 4: PRMSE vs. SNR for $K = 1$ and 5

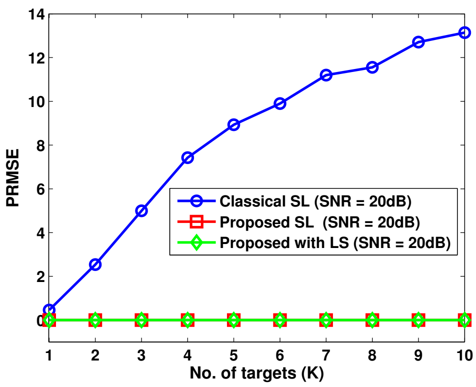

Fig. 3: PRMSE vs. No of targets for SNR = 20dB

where $e_{k,m}$ represents the distance between the real location of the k-th target and its estimated location at the m-th Monte Carlo (MC) trial. All simulations are averaged over $M = 100$ independent MC realizations of the Rician fading channel where in each run the targets are deployed on different random locations. In the first simulation, as shown by Fig. 2, we consider $K = 8$ targets randomly deployed over the covered area. As mentioned earlier, we have $M = 14$ APs which can be deployed uniformly at random. The SNR is set to 20dB. Considering the definition of the channel in the simulations, the cooperation among the APs provides us with $(14 \times 15)/2 = 105 > N$ independent measurements compared to only $M = 14$ in the classical SL. Now, referring to (12) as well as Fig. 1, we expect that up to approximately $K = 3$ targets can be localized using the classical SL while the proposed approach using either ℓ_1-norm minimization or LS can accurately localize all the targets simultaneously.

To further investigate this improvement in terms of the number or localizable targets, we illustrate the PRMSE of localization vs. the number of targets increasing up to $K = 10$. The simulation parameters are the same as in Fig. 2. As can be seen from the figure, by increasing the number of targets, the PRMSE of localization in the classical SL increases sharply while the proposed approach can handle all the targets simultaneously with the minimum error. Note that we do not plot the results for $K > 10$ targets since for those cases $\boldsymbol{\theta}$ is not really sparse ($K \ll N$) anymore. Even though ℓ_1-norm minimization might not be applicable in those cases, as long as the condition in Case II holds, i.e., $M(M+1)/2 > N$, LS can efficiently recover $\boldsymbol{\theta}$.

In order to better investigate the localization accuracy, we also depict the PRMSE vs. SNR for the number of targets $K = 1$ and 5 in Fig. 4. As is clear from the figure, the classical SL in the best case with only $K = 1$ target attains its minimum PRMSE. Besides, further increasing the number of targets will degrade the performance of the classical SL. However, the proposed approach can attain the minimum PRMSE even with $K = 5$ targets for SNR values larger than 5dB. It is noteworthy that for SNR values smaller than 5dB the proposed approach degrades gradually by increasing the number of targets. We also highlight that we do not compare our results with the KNN or the Bayesian classification algorithms because the superiority of classical SL compared to those two is already shown in [6], [7].

IV. Conclusions

In principle, there is potential information within the cross-correlations of the received signals at different APs of a wireless networks which is not exploited in the existing SL algorithms. To exploit this information, we have proposed to construct a new fingerprinting map to include these cross-correlations and we have shown that this new framework leads to obtaining an improved performance of SL in terms of accuracy and number of localizable targets. Future work is conducted on extending the proposed approach for off-grid targets by using grid mismatch concepts.

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