

# BLIND EQUALIZATION OF SPARSE CHANNELS USING ANTENNAS DIVERSITY

*Amir Leshem and Alle-Jan van der Veen*

Dept. of Electrical Engineering  
Delft University of Technology  
Delft, The Netherlands

## ABSTRACT

In this paper we consider the blind identifiability of multichannel systems when we know a-priori a bound on the number of non-zero coefficients in each channel. This is interesting for specular multi-path channels. We show that in this case, previously derived identifiability conditions are too strong. We demonstrate how to use the a-priori knowledge to weaken these conditions. We also propose a method to estimate the channel responses as well as the signals. Our method is based on a frequency domain LS estimation of the channel parameters combined with conditional MLE for the signals.

**keywords:** Blind channel identification, diversity combining, equalization.

## 1. INTRODUCTION

In many current wireless communication systems, multiple antennas are used to obtain spatial diversity combining of the signals. The combined signal is further processed. The question of equalization of multi-path channels has received considerable interest. Recently many blind methods which do not exploit training data have been developed. Some of these methods exploit high order statistical information [1], while others exploit multichannel structure induced either by multiple antennas or oversampling. Among these we can find [2], [3] and [4]. These methods rely on a linear parameterization of the channel as an FIR filter. Other multichannel methods exploit training data to obtain parametric channel estimates e.g., [5], and [6], in order to construct a rake receiver. Under the narrow-band assumption, propagation delays across the array can be represented as phase shifts. The training is not needed, and the method of [6] can be used to achieve blind equalization [7]. These will not work for wideband signals as is the case with acoustic channels or when the separation between the sensors is large (as in diversity application), and hence the propagation time across the array is large compared to the bandwidth. For a

good overview of the extensive literature on multichannel equalization we refer the reader to [8] and [9].

In this paper we consider the blind identifiability of single input multichannel systems, when we know a-priori a bound on the number of non-zero coefficients in each channel. This is interesting for specular multi-path channels in which we can obtain a-priori upper bounds on the number of propagation paths. We show that in this case the identifiability conditions of [3] are too strong, and prove that a-priori knowledge on the number of propagation paths can be used to weaken these conditions. In our derivation we do not assume narrow-band signal model, and thus we can equalize channels with long delays and wideband input signals. This can be important for underwater acoustic channels [10], as well as diversity reception in wideband cellular communication.

We also propose a method to estimate the channel responses as well as the signals. Our method is based on a LS estimation of the channel parameters combined with conditional MLE for the signals, it can be considered as a frequency domain parametric version of the TS-ML method [8]. This leads to improved diversity combining, which is relevant e.g., to multi-carrier systems. An interesting feature of the solution is the fact that the maximum likelihood signal estimate is achieved by performing maximum ratio combining of each of the frequency channels separately, using the estimated channel response. This natural interpretation also demonstrates the advantage of the method over simple diversity methods, which do not use equalization. The approach presented here has been inspired by previous work [11] on array calibration, and the striking similarity between array calibration and blind channel identification problems.

## 2. PROBLEM FORMULATION

In this section we formulate the problem of blind identification of multi-path channels, using multichannel system.

Assume that we have two channels and let  $y_i(t)$  be the noise free output of the  $i$ 'th channel. It can be described

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The first author was supported by the NOEMI project, contract no. DEL77-476. e-mail:leshem@cas.et.tudelft.nl,allejan@cas.et.tudelft.nl

mathematically as

$$y_i(t) = \sum_{l=1}^{r_i} \rho_{li} s(t - \tau_{li}), \quad i = 1, 2 \quad (1)$$

i.e., each sample is a linear combination of scaled and delayed versions of the input signal. The received signal is further contaminated by noise which we will assume to be white and Gaussian

$$x_i(t) = y_i(t) + n_i(t), \quad i = 1, 2 \quad (2)$$

where  $n_i(t)$  is white Gaussian noise. We assume that the number of paths in the  $i$ 'th channel response is upper bounded by  $r_i$ . Note that we allow overestimation of the number of paths.

The blind identification problem is the following: Given  $N$  observations of the channels output

$$\{x_i(k) : i = 1, \dots, p, k = 1, \dots, N\}$$

determine the channel parameters

$$\{\{\rho_{li}, \tau_{li}\}_{l=1}^{r_i} : i = 1, \dots, p\},$$

and reconstruct the input signal  $s(t)$ . Our analysis holds for arbitrary channels but is very appealing computationally when the channels are sparse, i.e., the number of path's is small while the propagation delays can be large. In that case if we try to model the channel as an FIR of length  $L$  we will obtain that most coefficient will be zero, i.e.  $r_i \ll L$ . The output can be described as a convolution of the input signal with the channel impulse response  $h_i(t)$

$$x_i(t) = h_i(t) * s(t) + n_i(t) \quad (3)$$

where  $h_i(t) = \sum_{l=1}^{r_i} \rho_{li} \delta(t - \tau_{li})$  and  $*$  denotes convolution. By stacking the channels into a vector we obtain that the measurements are given by

$$\mathbf{x}(t) = \mathbf{h} * s(t) + \mathbf{n}(t)$$

Moving to the frequency domain and using the convolution theorem we obtain that

$$\mathbf{x}(\omega) = \mathbf{h}(\omega) s(\omega) + \mathbf{n}(\omega) \quad (4)$$

where

$$h_i(\omega) = \sum_{l=1}^{r_i} \rho_{li} e^{-j\omega \tau_{li}} \quad (5)$$

is the response of the  $i$ 'th channel at frequency  $\omega$  and

$$\mathbf{h}(\omega) = \begin{bmatrix} h_1(\omega) \\ h_2(\omega) \end{bmatrix}. \quad (6)$$

There are two indeterminacies in the above model. First we can exchange a constant between the channel response and

the signal, and second, any delayed version of the signal will be consistent with the appropriate delay of the channel response. To overcome these, we choose  $\rho_{11} = 1$  and  $\tau_{11} = 0$ , i.e., we fix the shortest path of the first channel to a unit response with no delay, and demand that all other delays for that channel are positive.

### 3. MAXIMUM LIKELIHOOD EQUALIZATION

In this section we derive the maximum likelihood estimates for the input signal and channel response. To simplify notation let  $\boldsymbol{\tau}_i = [\tau_{1i}, \dots, \tau_{r_i i}]$  and  $\boldsymbol{\rho}_i = [\rho_{1i}, \dots, \rho_{r_i i}]$ . Also let  $\boldsymbol{\tau} = [\boldsymbol{\tau}_1, \boldsymbol{\tau}_2]$  and  $\boldsymbol{\rho} = [\boldsymbol{\rho}_1, \boldsymbol{\rho}_2]$ .

After some standard algebraic manipulations which includes an estimation of the noise variance  $\sigma^2$  we obtain that the maximum likelihood estimator of the channel parameters and the signals is given by

$$[\tilde{\boldsymbol{\tau}}_1, \tilde{\boldsymbol{\tau}}_2, \tilde{\boldsymbol{\rho}}_1, \tilde{\boldsymbol{\rho}}_2, \tilde{\mathbf{s}}] = \arg \min_{\boldsymbol{\tau}, \boldsymbol{\rho}, \mathbf{s}} \sum_{k=0}^{N-1} \|\mathbf{x}(\omega_k) - \mathbf{h}(\omega_k) s(\omega_k)\|^2. \quad (7)$$

where  $\mathbf{h}$  is defined in (6). Solving for  $\mathbf{s}$  we obtain after some easy manipulations:

$$\tilde{\mathbf{s}}(\omega) = \frac{1}{|h_1(\omega)|^2 + |h_2(\omega)|^2} (h_1(\omega)^* x_1(\omega) + h_2(\omega)^* x_2(\omega)) \quad (8)$$

This expression is very pleasing as it shows that the ML signal estimator consists of averaging each frequency with the relative power in the two channels as is the case in maximum ratio combining. We immediately see that in order that the multichannel be effective it is necessary that the channels would not have common zeros on the unit circle, i.e. no frequency is simultaneously nulled at both channels.

Substituting back into (7) we obtain that the ML channel estimates are given by:

$$[\tilde{\boldsymbol{\tau}}_1, \tilde{\boldsymbol{\tau}}_2, \tilde{\boldsymbol{\rho}}_1, \tilde{\boldsymbol{\rho}}_2] = \arg \min_{\boldsymbol{\tau}, \boldsymbol{\rho}} \sum_{k=0}^{N-1} \|(\mathbf{I} - \mathbf{P}_{\mathbf{h}(\omega_k)}) \mathbf{x}(\omega_k)\|^2. \quad (9)$$

Using Parseval identity it follows that this expression is equivalent to the first step of the two step ML approach of [8], however the frequency domain approach leads to the intuitive interpretation of the signal estimate (8).

### 4. A SIMPLIFIED LS APPROACH

The ML channel estimate (9) is a multidimensional optimization problem which involves both the delays and the powers in a highly-non-linear manner. To reduce the computational complexity we propose to replace the ML channel estimates by a simplified LS estimator, which resembles that of [3]. The two main differences are the use of

frequency domain data and the parametric modeling of the channel. This will enable us to reduce the problem to that of delay estimation, and estimate the powers in a closed form solution using linear least squares solution. To that end note that in the noiseless case for each frequency  $\omega$

$$h_2(\omega)x_1(\omega) = h_1(\omega)x_2(\omega). \quad (10)$$

Thus we will replace (9) with the simplified set of LS equations

$$[\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\rho}_1, \tilde{\rho}_2] = \arg \min_{\tau, \rho} \sum_{k=0}^{N-1} |\Delta_{12}(\tau, \rho, \omega_k)|^2 \quad (11)$$

where  $\Delta_{12}(\tau, \rho, \omega_k) = x_1(\omega_k)h_2(\omega_k) - x_2(\omega_k)h_1(\omega_k)$ , and  $\rho_i, \tau_i$  are related to  $h_i$  by (5). Reformulating the problem and using our choice that  $\tau_{11} = 0, \rho_{11} = 1$  we obtain that (11) can be rewritten in matrix form as

$$[\tilde{\tau}_1, \tilde{\tau}_2, \tilde{\rho}] = \arg \min_{\tau, \rho} \|\mathbf{B}(\tau)\rho' - \mathbf{x}_2\|^2 \quad (12)$$

where  $\mathbf{x}_2 = [x_2(\omega_0), \dots, x_2(\omega_{N-1})]^T$ ,

$$\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2]$$

$$\mathbf{B}_1 = \begin{bmatrix} -x_2(\omega_0)e^{-j\omega_0\tau_{21}} & \dots & -x_2(\omega_0)e^{-j\omega_0\tau_{r11}} \\ \vdots & & \vdots \\ -x_2(\omega_{N-1})e^{-j\omega_{N-1}\tau_{21}} & \dots & -x_2(\omega_{N-1})e^{-j\omega_{N-1}\tau_{r11}} \end{bmatrix}$$

$$\mathbf{B}_2 = \begin{bmatrix} x_1(\omega_0)e^{-j\omega_0\tau_{12}} & \dots & x_1(\omega_0)e^{-j\omega_0\tau_{r22}} \\ \vdots & & \vdots \\ x_1(\omega_{N-1})e^{-j\omega_{N-1}\tau_{12}} & \dots & x_1(\omega_{N-1})e^{-j\omega_{N-1}\tau_{r22}} \end{bmatrix}$$

and  $\rho' = [\rho_{21}, \dots, \rho_{r11}, \rho_{12}, \dots, \rho_{r22}]$ . Now the problem becomes linear in the amplitudes of each path, so we can minimize with respect to  $\rho$  holding  $\tau_1, \tau_2$  fixed and obtain

$$\rho = (\mathbf{B}(\tau_1, \tau_2)^H \mathbf{B}(\tau_1, \tau_2))^{-1} \mathbf{B}(\tau_1, \tau_2)^H \mathbf{x}_2. \quad (13)$$

Substituting into (12) we obtain

$$[\tilde{\tau}_1, \tilde{\tau}_2] = \arg \min_{\tau_1, \tau_2} \|\mathbf{P}_{\mathbf{B}(\tau_1, \tau_2)}^\perp \mathbf{x}_2\|^2 \quad (14)$$

where  $\mathbf{P}_{\mathbf{B}(\tau_1, \tau_2)}^\perp$  is the projection onto the orthogonal complement of the column span of  $\mathbf{B}(\tau_1, \tau_2)$ . Although the optimization of the delays is multidimensional, it has the same functional form as the deterministic maximum likelihood estimate for directions of arrival of multiple waveform. One can use one of the many methods developed for that problem such as [12] to solve (14).

## 5. IDENTIFIABILITY CONDITIONS

In this section we analyze our solution and derive sufficient conditions for identifiability. We will discuss the feasibility of the assumptions and show that in common space diversity systems our assumptions hold.

Our basic assumption was that we know an upper bound on the number of path's  $r_1, r_2$ . Thus in order to have a unique solution for the problem we need that for any two vectors  $\tau'_1, \tau'_2$ , where  $\tau'_1$  has positive entries the system (12) has a solution iff  $\tau'_1 = \tau_1$  and  $\tau'_2 = \tau_2$ . Moreover we would like that when a solution exists it will be unique. This condition will be satisfied iff for every  $\tau'_1, \tau'_2$  the matrix  $[\mathbf{B}(\tau_1, \tau_2) | \mathbf{B}(\tau'_1, \tau'_2)]$  will have full column rank. Rearranging the columns and exploiting our notation it is equivalent to demanding that  $\mathbf{B}(\tau_1, \tau_2)$  is full column rank for any substitution of  $\tau_i \in \mathbb{R}^{\leq 2r_i}, i = 1, 2$ .

To obtain less abstract conditions let

$$\mathbf{\Lambda}_l = \begin{bmatrix} x_l(\omega_0) & & \\ & \ddots & \\ & & x_l(\omega_{N-1}) \end{bmatrix} \quad (15)$$

$$\mathbf{C}(\tau_1, \tau_2) = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{bmatrix} \quad (16)$$

and

$$\mathbf{C}_1 = \begin{bmatrix} e^{-j\omega_0\tau_{21}} & \dots & e^{-j\omega_0\tau_{2r_{1,1}}} \\ \vdots & & \vdots \\ e^{-j\omega_{N-1}\tau_{21}} & \dots & e^{-j\omega_{N-1}\tau_{2r_{1,1}}} \end{bmatrix}$$

$$\mathbf{C}_2 = \begin{bmatrix} e^{-j\omega_0\tau_{12}} & \dots & e^{-j\omega_0\tau_{1r_{2,2}}} \\ \vdots & & \vdots \\ e^{-j\omega_{N-1}\tau_{12}} & \dots & e^{-j\omega_{N-1}\tau_{1r_{2,2}}} \end{bmatrix}.$$

Note that we can factor  $\mathbf{B}$  as

$$\mathbf{B}(\tau_1, \tau_2) = [-\mathbf{\Lambda}_2 | \mathbf{\Lambda}_1] \mathbf{C}(\tau_1, \tau_2) \quad (17)$$

The second matrix is always full column rank due to the Vandermonde structure of each block. Thus the identifiability condition boils down to having the first matrix preserve the column rank. Similarly to the condition in [3] we can now split this condition into two conditions. The first demanding informative signals, and the second is a condition on identifiable channels. Factoring  $\mathbf{\Lambda}_l$  using the frequency domain relation (4) we obtain

$$\mathbf{\Lambda}_l = \begin{bmatrix} h_l(\omega_0) & & \\ & \ddots & \\ & & h_l(\omega_{N-1}) \end{bmatrix} \begin{bmatrix} s(\omega_0) & & \\ & \ddots & \\ & & s(\omega_{N-1}) \end{bmatrix}$$

Thus the following immediately follows:

**Theorem 5.1** Let  $R = 2(r_1 + r_2 - 1)$ . Assume that for every  $0 \leq k \leq R - 1$   $s(\omega_k) \neq 0$ ,  $[\mathbf{h}_1(\omega_1), \dots, \mathbf{h}_1(\omega_R)]^T$ ,  $[\mathbf{h}_2(\omega_1), \dots, \mathbf{h}_2(\omega_R)]^T$  are linearly independent and for each  $l$  there are at least  $r_l$  frequencies among  $\omega_1, \dots, \omega_R$  at which  $\mathbf{h}_l(\omega)$  is not zero, then there is a unique solution to the problem (12).

The proof is straightforward from (17), noting that there are two ways to lose rank. Either one of the columns becomes zero at many points (which means that one channel is not identifiable), or there are  $\tau_{1,l} = \tau_{2,m}$  and the channel responses are linearly dependent (so that the channels are not diverse enough, e.g., one channel is a delayed version of the other channel). By the Vandermonde structure of  $\mathbf{C}(\tau_1, \tau_2)$  other ways are not possible. We have restricted ourselves to an initial segment of the frequency band in order to simplify the notation, and more can be proved.

Note that our conditions depend on the number of paths rather than the channels length. The condition on the informative signal means that we need that the signal should fill at least  $\frac{2(r_1+r_2-1)}{N}$  of the total bandwidth to be equalized. The other conditions demands informative channels, i.e., the channels are sufficiently diverse and each of them carries enough information about the input signal. This would typically hold for diversity since the multi-path response is assumed to be uncorrelated between the two branches. Moreover common nulls of the channels means that certain spectral band is completely filtered out, which means that any diversity combining of the channels would fail, to reconstruct that part of the spectrum.

## 6. SIMULATION RESULTS

To demonstrate the method we have simulated two multipath channels. We have aligned the direct paths of the channels using a cross correlation. The multipath delays were chosen as 7 and 16, so that  $\tau_1 = [0, 7]$ ,  $\tau_2 = [0, 16]$ , and the power of the paths was  $\rho = [1, 0.3e^{0.53j}]$ ,  $\rho_2 = [1.5, 0.2]$ . We have used 256 data samples, and the signal to noise ratio was 20dB. Figure 1 shows the values of the negative of the cost function (14) as a function of the delays. We can clearly see the peak which is located at the correct delays.

## 7. CONCLUSIONS

We have shown that using parametric representation of multipath channels can lead to weaker identifiability conditions for single input multiple output systems. We have also derived a frequency domain parametric identification method. Using a simplified LS approach we have reduced the complexity of the estimation. Since the complexity involved is still large this approach can be used for tracking channel parameters once we have acquired a good initialization using other methods.

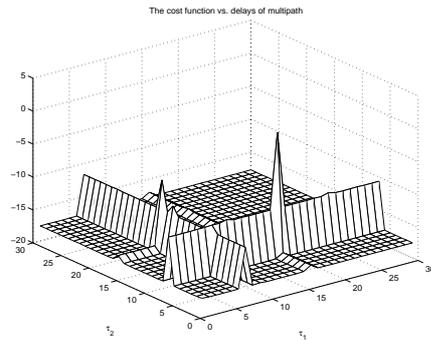


Figure 1: Cost function vs. delays of the multipath's in channels

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