Energy Estimation Technique Utilizing Timing Information for TOF-PET Application

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I. Abstract

This paper proposes an energy estimation technique utilizing only timing information from a Multi-channel Digital a SiPM (MD-SiPM) and presents a statistical analysis of the proposed approach for TOF PET applications with a LYSO scintillator. By utilizing only timing information for estimating energy of gamma photons, circuitry in the MD-SiPM is minimized so as to increase fill-factor and the dead time is reduced dramatically. The statistical analysis of the proposed estimation techniques show that the expected error of the estimate and its uncertainty are a function of the number of primary photons triggering the detector cells that, in turn, is a function of the energy of the gamma photon. The maximum uncertainty (FWHM) acquired by the random simulation is 345 photons in the range of 60–2000 primary photons by utilizing multiple photon timestamps arriving within only 4 nanosecond after the detection of the first photon.

II. Introduction

SiPMs are an alternative to photomultiplier tubes because of their robustness to magnetic fields, compactness, and low bias voltage. Fig. 1 (a) and (b) show the structure of conventional analog SiPMs (A-SiPMs) [1]–[8] and digital SiPMs (D-SiPMs) [8]–[12]. Recently, an approach has been proposed based, called multi-channel D-SiPM (MD-SiPM), which advocates the use of multiple measurements or timestamps in each event to improve the statistical characterization of it and ultimately the accuracy of its timing [13], [14]. Fig. 1 (c) and (d) show two extremes of the MD-SiPM architecture. Multiple timestamps within a single event can be collected in A-SiPMs as well, using multiple thresholds. However this techniques is generally not used due to the complexity of its implementation and its stability [15]. In a MD-SiPM, every sensitive cell measures the time-of-arrival (TOA) of detected photons independently. This capability ensures one to approach the Crámer-Rao lowerbound on timing resolution [14], [16], [17].

In this paper, an energy estimation technique based on multiple timestamps is presented. By estimating energy utilizing only timestamps, the fill factor can be increased by minimizing the circuitry in the cells and the dead time can be reduced by skipping the procedure of accessing the cells to check the photon detection result. In conventional D-SiPMs, this procedure requires from several hundreds of nanosecond to several microseconds.

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III. Energy estimation model

For the emitted photons from a LYSO scintillator, we can assume that a gamma photon is absorbed at time, θ. The generated photons follow a probability density function (pdf), which is modeled as a single-exponential with decay time τ_d, $f(t|\theta) = 1/\tau_d \exp(-\frac{t-\theta}{\tau_d})$ when $t > \theta$, otherwise, $f(t) = 0$. The k-th primary photon's generation time's pdf with n primary photons, is calculated using cumulative density function, $F(t)$, as $f_{k:n}(t) = n\binom{n-1}{k-1}f(t)F(t)^{k-1}(1-F(t))^{n-k}$. Assuming that a detector is ideal, the photon's arrival pdf is $f_{k:n}(t)$. The most likely arrival time (at the highest probability density), t_k, is calculated by solving $df_{k:n}(t)/dt = 0$, resulting in $t_k - \theta = -\tau_d \times log(1-(k-1)/n)$. Especially, this can be approximated when $(k-1) << n$ as,

$$t_k - \theta = \tau_d \frac{k-1}{n}. \tag{1}$$

As one can see the eq. 1, each photon's timestamp is proportional to $\tau_d(k-1)/n$. τ_d/n is estimated from multi timestamps, $(k, t_k) = (1, t_1), (2, t_2), ..., (m, t_m)$ by least-squares minimization. The expected value of τ_d/n, $\tau_d/\hat{n}$, is calculated to be $\Sigma_{k=1}^{m} t_k(k-\overline{k})/\Sigma_{k=1}^{m}(k-\overline{k})^2$. Therefore, the expected estimated number of photons, $\hat{n}$, and error of the estimate, $\epsilon_{\hat{n}}$, are calculated as,

$$\hat{n} = \tau_d \frac{\Sigma_{k=1}^{m}(k-\overline{k})^2}{\Sigma_{k=1}^{m} t_k(k-\overline{k})} \tag{2}$$

$$\epsilon_{\hat{n}} = n - \hat{n}. \tag{3}$$

IV. Simulations

We assumed that scintillations in LYSO follow a double-exponential with the 100 ps and 40 ns rise and decay times, respectively, while the number of primary photons is varied from 20 to 2000, the temporal jitter of the detector cells is swept from 28.3 ps to 113.1 ps in σ, and dark count rate (DCR) is varied from 1 Hz to 20 MHz. Fig. 2 shows the pdf of k_{th} primary photon and TOA with highest pdf for k_{th} photon. Eq. 1 is available only when $(k-1) \ll n$, thus limiting useful timestamps to the first 4 ns after the first primary photon's TOA, which we call linear region.

Fig. 3 shows the effect on our proposed estimation technique by the number of timestamps. $\hat{n}$ approaches n in the entire range and $\Delta\epsilon_{\hat{n}}$ dramatically improves by increasing the number of timestamps, as shown in Fig. 3 (a) and (b). The maximum ϵ_n and $\Delta\epsilon_{\hat{n}}$ is 120 and 345 photons in the range of 20–2000 primary photons utilizing at most first 200 timestamps in 4 ns linear region, while 100 and 600 photons at most first 50 timestamps. Fig. 4 shows the effect by the temporal jitter using 200 timestamps in the linear region. The temporal jitter affects only the first few primary photons time uncertainty, while higher rank's photons timing uncertainty is dominated by the time uncertainty derived by the scintillator pdf. Fig. 5 shows the effect of dark count noise using the mixed pdf approach [14]. The time uncertainty for the first few photons dramatically increases after 10 MHz DCR, resulting in high uncertainty at fewer primary photons. However, the effect is small when the number of primary photons is large.

V. Conclusion

From the estimation theory and the simulation we performed it can be concluded that the availability of a large number of timestamps not only helps improve the CRT eventually leading to approach the Cramer-Rao limit, but it also help reduce the energy estimation uncertainty, while the effects of detection cell jitter and dark noise becomes less and less relevant. The energy estimation model proposed in this paper will be improved by utilizing various weighted-average methods utilizing covariance of timestamps.

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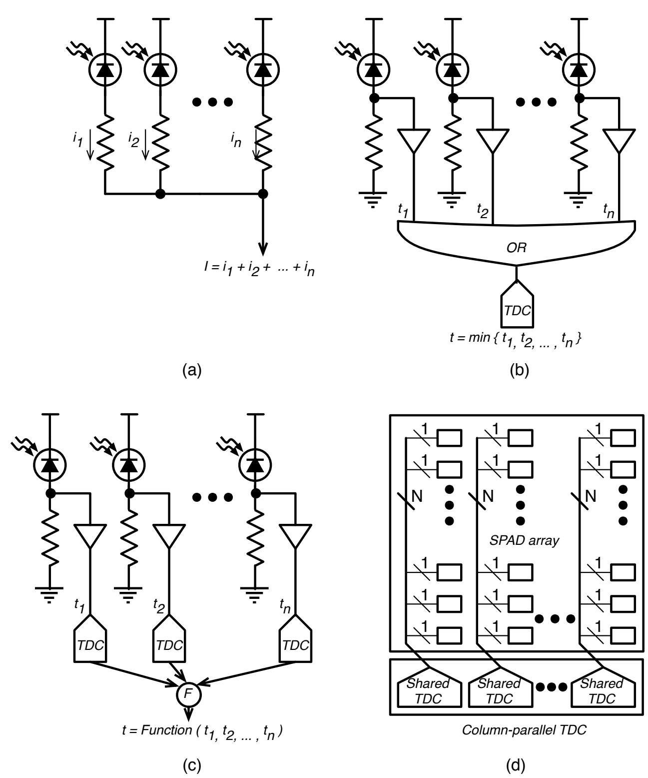

Fig. 1. The concept of (a)Analog SiPM, (b)Conventional Digital SiPM with a integrated time-to-digital converter (TDC), (c)Ideal multi-channel digital SiPM with on-pixel TDCs and (d) Multi-Digital SiPM with shared TDCs.

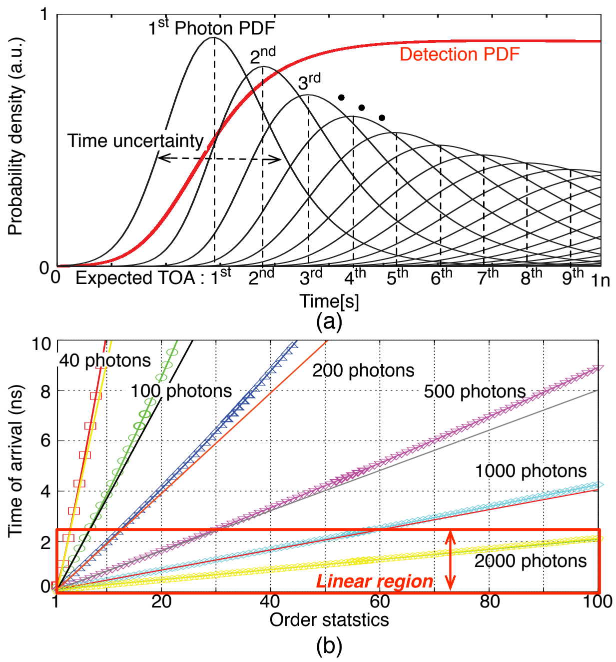

Fig. 2. (a) Probability density function (pdf) of the k-th primary photon detected after the first detected photon. (b) Time of arrival at the peak of the pdf for the k-th photon.

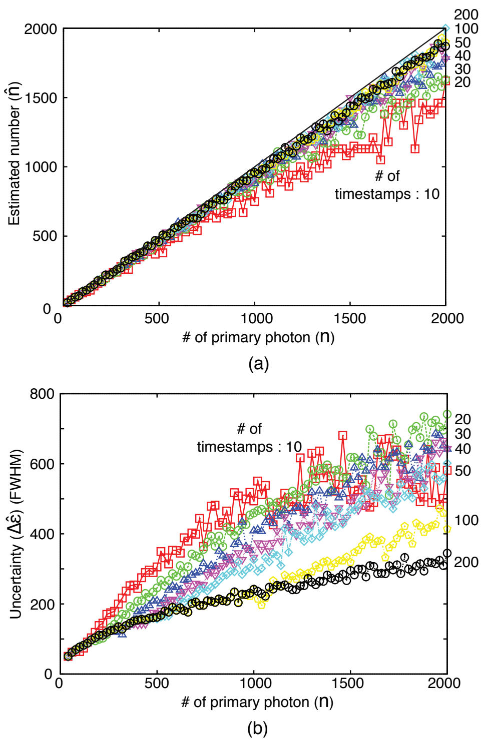

Fig. 3. Estimated number of photons $\hat{n}$ and uncertainty of the estimate $\Delta\epsilon_{\hat{n}}$ as a function of the actual number of photons for various timestamp population sizes. Zero jitter and no DCR are assumed.

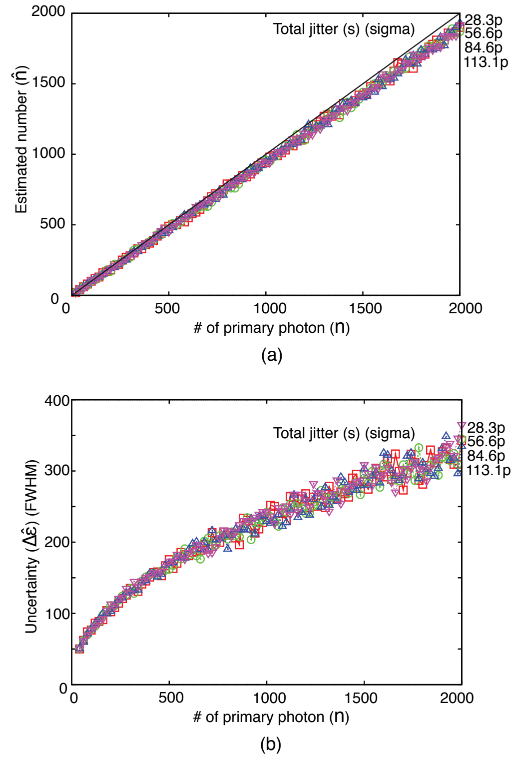

Fig. 4. Estimated number of photons $\hat{n}$ and uncertainty of the estimate $\Delta\epsilon_{\hat{n}}$ as a function of the actual number of photons for various detection jitter values. 50 timestamps and no DCR are assumed.

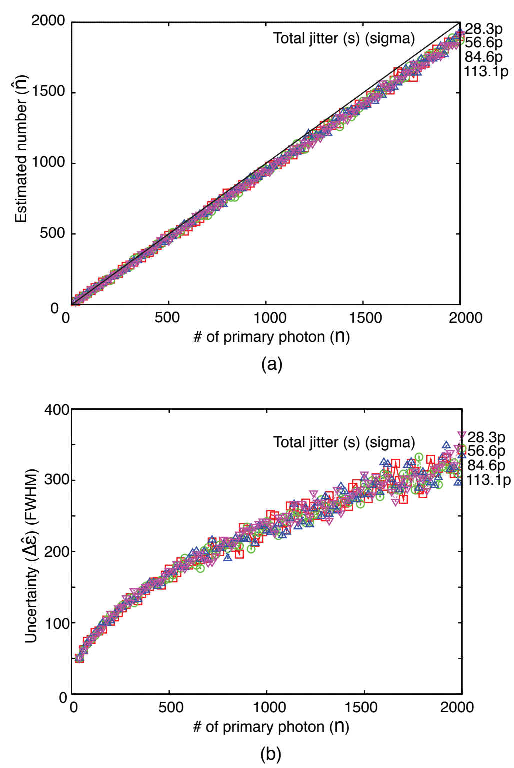

Fig. 5. Estimated number of photons $\hat{n}$ and uncertainty of the estimate $\Delta\epsilon_{\hat{n}}$ as a function of the actual number of photons for DCR levels. 50 timestamps and zero jitter are assumed.