Direct Semi-Blind Design of Serial Linear Equalizers for Doubly-Selective Channels

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***Abstract*— Recently, serial linear equalizers (SLEs) and serial decision feedback equalizers (SDFEs) have been proposed to mitigate doubly-selective channel effects. To design the SLE/SDFE and to model the doubly-selective channel, a so-called finite impulse response basis expansion model (FIR-BEM) is used. Initially, the FIR-BEM coefficients of the SLE/SDFE were designed based on the exact knowledge of the FIR-BEM coefficients of the doubly-selective channel. In practice, we can use a direct SLE/SDFE design procedure, which avoids an intermediate channel estimation step. In this paper, we describe this idea for the SLE and focus on direct semi-blind design of the FIR-BEM coefficients of the SLE. Simulation results demonstrate the validity of the proposed approach.**

I. INTRODUCTION

The quest for high data rates and high mobility in future mobile wireless systems comes with the burden of distortive time- and frequency-selective (doubly-selective) channel effects. To mitigate these effects, serial linear equalizers (SLEs) and serial decision feedback equalizers (SDFEs) have recently been proposed to equalize doubly-selective channels [1]–[3]. A so-called finite impulse response basis expansion model (FIR-BEM) [4]–[6] is used to design the SLE/SDFE and to model the doubly-selective channel. Note that these SLEs and SDFEs differ from the ones proposed in [7], [8], in the fact that they fully exploit the FIR-BEM structure of the channel and do not view it as a frequency-selective channel with multiple inputs.

Many possibilities exist to design the FIR-BEM coefficients of the SLE/SDFE. First of all, we can assume exact knowledge of the FIR-BEM coefficients of the doubly-selective channel to design the FIR-BEM coefficients of the SLE/SDFE, as done in [1]–[3], which is of course not very realistic. In practice, we can use training-based [9], blind [10], or even a combination of both, labeled semi-blind, channel estimation to estimate the FIR-BEM coefficients of the doubly-selective channel, which can then be used to design the FIR-BEM coefficients of the SLE/SDFE. However, we can also avoid this intermediate channel estimation step and directly design the FIR-BEM coefficients of the SLE/SDFE in a training-based, blind, or semi-blind fashion. In this paper, we illustrate this procedure for the SLE and focus on the semi-blind method, which encompasses the training-based and blind method as special cases.

Notation: We use upper (lower) bold face letters to denote matrices (column vectors). Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian, respectively. Further, $\star$ denotes the convolution and $\otimes$ the Kronecker product. We represent the Dirac delta by $\delta(t)$ and the Kronecker delta by $\delta[n]$. We write the $N \times N$ identity matrix as $\mathbf{I}_N$, the $M \times N$ all-zero matrix as $\mathbf{0}_{M\times N}$, and the $M \times N$ all-one matrix as $\mathbf{1}_{M\times N}$. Finally, diag$\{\mathbf{x}\}$ represents the diagonal matrix with $\mathbf{x}$ on the diagonal.

II. CHANNEL MODEL

We consider a baseband description of a wireless system with 1 transmit and M receive antennas. For the mth receive antenna, the symbol sequence $x[n]$ is filtered by the transmit filter $g_{\text{tr}}(t)$, distorted by the physical channel $g_{\text{ch}}^{(m)}(t;\tau)$, corrupted by additive noise $v^{(m)}(t)$, and finally filtered by the receive filter $g_{\text{rec}}(t)$. With a symbol period of T, the received signal at the mth receive antenna $y^{(m)}(t)$ can then be written as

$$y^{(m)}(t) = \sum_{n=-\infty}^{\infty} g^{(m)}(t; t-nT)x[n] + w^{(m)}(t),$$

where $w^{(m)}(t) := g_{\text{rec}}(t) \star v^{(m)}(t)$ and $g^{(m)}(t;\tau) := g_{\text{tr}}(\tau) \star g_{\text{rec}}(\tau) \star g_{\text{ch}}^{(m)}(t;\tau)$ (if the variation of $g_{\text{ch}}^{(m)}(t;\tau)$ over the span of $g_{\text{rec}}(t)$ is negligible).

Sampling the mth receive antenna at rate S/T with $S \geq 1$, we obtain a rate-S/T received sequence, which can be split into S rate-$1/T$ received sequences. The sth rate-$1/T$ received sequence at the mth receive antenna $y^{(mS+s)}[n] := y^{(m)}((nS+s)T/S)$ can be written as

$$y^{(mS+s)}[n] := \sum_{\nu=-\infty}^{\infty} g^{(mS+s)}[n;\nu]x[n-\nu] + w^{(mS+s)}[n],$$

where $w^{(mS+s)}[n] := w^{(m)}((nS+s)T/S)$ and $g^{(mS+s)}[n;\nu] := g^{(m)}((nS+s)T/S; (\nu S+s)T/S)$. Hence, we obtain a symbol rate single-input multiple-output (SIMO) system with $A = MS$ outputs, which are obtained by multiple receive antennas and/or fractional sampling.

To find a simplified model for the channel $g^{(a)}[n;\nu]$ ($a \in \{0, 1 \ldots, A-1\}$), we will look at a limited time window $t \in [0, NT)$, which corresponds to $n \in \{0, 1, \ldots, N-1\}$. Assuming $g^{(m)}(t;\tau) = 0$ for $\tau \notin [0, (L+1)T)$, the channel

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$g^{(a)}[n;\nu]$ can be modeled for $n \in \{0,1,\ldots,N-1\}$ by a so-called FIR-BEM:

$$h^{(a)}[n;\nu] = \sum_{l=0}^{L} \delta[\nu - l] \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} e^{j2\pi qn/K}, \qquad (1)$$

which represents a serial filter designed to have $L+1$ time-varying taps, where the time-variation of each tap is modeled by $Q+1$ complex exponentials. In this model, Q and K should be selected such that $Q/(2KT) \approx f_{\max}$, with $f_{\max}$ the overall Doppler spread of all M channels. In addition, we need $K \geq N$, since otherwise the FIR-BEM $h^{(a)}[n;\nu]$ will be periodic over $n \in \{0,1,\ldots,N-1\}$ with period K. Note that when NT is smaller than $1/(2f_{\max})$, a good fit can generally be obtained with $Q=2$.

The FIR-BEM input-output relation for $n \in \{0,1,\ldots,N-1\}$ can finally be written as

$$y^{(a)}[n] = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} e^{j2\pi qn/K} x[n-l] + w^{(a)}[n]. \quad (2)$$

III. System Model

In this section, we rewrite (2) on a block level, which will turn out to be useful at a later stage. Defining the $(N+L)\times 1$ data symbol block $\mathbf{x} := [x[-L],\ldots,x[N-1]]^T$, the $N \times 1$ received sample block at the ath output $\mathbf{y}^{(a)} := [y^{(a)}[0],\ldots,y^{(a)}[N-1]]^T$ can be written as

$$\mathbf{y}^{(a)} = \mathbf{H}^{(a)}\mathbf{x} + \mathbf{w}^{(a)}, \qquad (3)$$

where $\mathbf{w}^{(a)}$ is similarly defined as $\mathbf{y}^{(a)}$, and $\mathbf{H}^{(a)}$ is the $N \times (N+L)$ matrix given by

$$\mathbf{H}^{(a)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} \mathbf{D}_q \mathbf{Z}_l, \qquad (4)$$

where $\mathbf{D}_q := \mathrm{diag}\{[1, e^{j2\pi q/K}, \ldots, e^{j2\pi q(N-1)/K}]^T\}$ and $\mathbf{Z}_l := [\mathbf{0}_{N\times(L-l)}, \mathbf{I}_N, \mathbf{0}_{N\times l}]$. Substituting (4) in (3), we can write

$$\mathbf{y}^{(a)} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l}^{(a)} \mathbf{D}_q \mathbf{Z}_l \mathbf{x} + \mathbf{w}^{(a)}. \qquad (5)$$

Defining $\mathbf{y} := [\mathbf{y}^{(0)T},\ldots,\mathbf{y}^{(A-1)T}]^T$, we finally obtain

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \qquad (6)$$

where $\mathbf{w}$ is similarly defined as $\mathbf{y}$, and $\mathbf{H}$ is the $AN \times (N+L)$ matrix given by $\mathbf{H} := [\mathbf{H}^{(0)T},\ldots,\mathbf{H}^{(A-1)T}]^T$.

Based on (6), we can apply block linear equalization to recover $\mathbf{x}$ from $\mathbf{y}$. However, the complexity of such an approach depends on the block size N, which can often be very large. In this paper, we will therefore focus on serial linear equalization, for which the complexity is basically independent of the block size N. We focus on a non-precoded transmission, i.e., we assume that all entries of $\mathbf{x}$ contain raw data symbols. However, we will not estimate the edges of $\mathbf{x}$ and only estimate the middle part of $\mathbf{x}$ (denoted as $\mathbf{x}_\star$). The edges are either estimated in a previous step (top entries of $\mathbf{x}$) or will be estimated in a next step (bottom entries of $\mathbf{x}$).

IV. Serial Linear Equalization

We adopt a Serial Linear Equalizer (SLE), consisting of a serial filter $f^{(a)}[n;\nu]$ for the ath output, in order to find an estimate of $x[n-d]$ (see Figure 1):

$$\hat{x}[n-d] = \sum_{a=0}^{A-1} \sum_{\nu=-\infty}^{\infty} f^{(a)}[n;\nu] y^{(a)}[n-\nu],$$

where d represents the synchronization delay. Since for the doubly-selective channel, the FIR-BEM of (1) was applied, it is also convenient to use a FIR-BEM for the serial filter $f^{(a)}[n;\nu]$. In other words, we design each serial filter $f^{(a)}[n;\nu]$ to have $L'+1$ time-varying taps, where the time-variation of each tap is modeled by $Q'+1$ complex exponentials with frequencies on the same grid as the one for the channel:

$$f^{(a)}[n;\nu] = \sum_{l'=0}^{L'} \delta[\nu - l'] \sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/K} f_{q',l'}^{(a)}.$$

An estimate of $x[n-d]$ is then computed as

$$\hat{x}[n-d] = \sum_{a=0}^{A-1} \sum_{l'=0}^{L'} \sum_{q'=-Q'/2}^{Q'/2} e^{j2\pi q'n/K} f_{q',l'}^{(a)} y^{(a)}[n-l']. \quad (7)$$

Again, it will be more convenient to formulate (7) on a block level. Defining the q'th frequency-shifted and l'th time-shifted received sequence related to the ath output as

$$\mathbf{y}_{q',l'}^{(a)} := \bar{\mathbf{D}}_{q'} \bar{\mathbf{Z}}_{l'} \mathbf{y}^{(a)},$$

where $\bar{\mathbf{D}}_{q'} := \mathrm{diag}\{[1, e^{j2\pi q'/K}, \ldots, e^{j2\pi q(N-L'-1)/K}]^T\}$ and $\bar{\mathbf{Z}}_{l'} := [\mathbf{0}_{(N-L')\times(L'-l')}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times l'}]$, and introducing

$$\mathbf{x}_\star := [x[L'-d],\ldots,x[N-d-1]]^T,$$

an estimate of $\mathbf{x}_\star$ is obtained as

$$\hat{\mathbf{x}}_\star^T = \sum_{a=0}^{A-1} \mathbf{f}^{(a)T} \mathbf{Y}^{(a)}, \qquad (8)$$

where $\mathbf{f}^{(a)}$ is the $(Q'+1)(L'+1)\times 1$ vector given by $\mathbf{f}^{(a)} := [f_{Q'/2,L'}^{(a)},\ldots,f_{Q'/2,0}^{(a)},\ldots,f_{-Q'/2,0}^{(a)}]^T$, and $\mathbf{Y}^{(a)}$ is the $(Q'+1)(L'+1)\times(N-L')$ matrix given by $\mathbf{Y}^{(a)} := [\mathbf{y}_{Q'/2,L'}^{(a)},\ldots,\mathbf{y}_{Q'/2,0}^{(a)},\ldots,\mathbf{y}_{-Q'/2,0}^{(a)}]^T$.

Let us now express $\mathbf{Y}^{(a)}$ as a function of the FIR-BEM coefficients of the doubly-selective channel and the data symbols. Using the property $\bar{\mathbf{Z}}_{l'}\mathbf{D}_q = e^{j2\pi q(L'-l')/K} \bar{\mathbf{D}}_q \bar{\mathbf{Z}}_{l'}$, the q'th frequency-shifted and l'th time-shifted received sequence related to the ath output can be written as

$$\begin{aligned}
\mathbf{y}_{q',l'}^{(a)} &:= \bar{\mathbf{D}}_{q'} \bar{\mathbf{Z}}_{l'} \mathbf{y}^{(a)} \\
&= \sum_{l=0}^{L} \sum_{q=0}^{Q} h_{q,l}^{(a)} e^{j2\pi q(L'-l')/K} \bar{\mathbf{D}}_{q'} \bar{\mathbf{D}}_q \bar{\mathbf{Z}}_{l'} \mathbf{Z}_l \mathbf{x} + \mathbf{w}_{q',l'}^{(a)} \\
&= \sum_{l=0}^{L} \sum_{q=0}^{Q} e^{j2\pi q(L'-l')/K} h_{q,l}^{(a)} \bar{\mathbf{D}}_{q+q'} \tilde{\mathbf{Z}}_{l+l'} \mathbf{x} + \mathbf{w}_{q',l'}^{(a)},
\end{aligned}$$

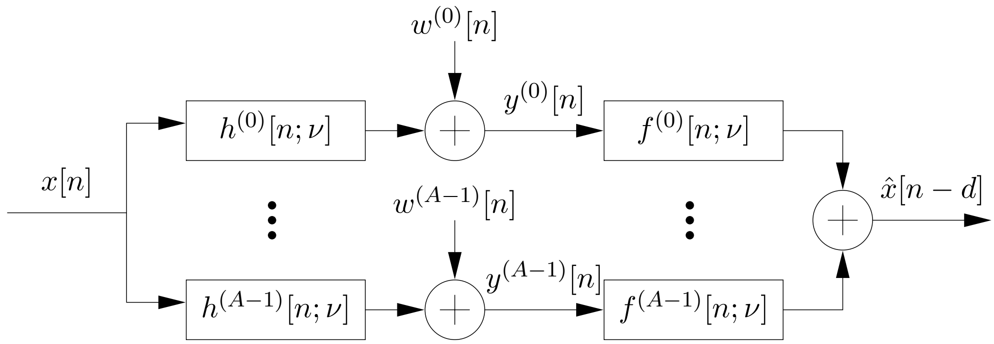

Fig. 1. Serial linear equalization.

where $\mathbf{w}_{q',l'}^{(a)}$ is similarly defined as $\mathbf{y}_{q',l'}^{(a)}$ and $\tilde{\mathbf{Z}}_k := [\mathbf{0}_{(N-L')\times(L+L'-k)}, \mathbf{I}_{N-L'}, \mathbf{0}_{(N-L')\times k}]$. Introducing $k := l + l'$ and $p := q + q'$, and defining $\mathbf{x}_{p,k} := \bar{\mathbf{D}}_p \tilde{\mathbf{Z}}_k \mathbf{x}$ (note that $\mathbf{x}_\star = \mathbf{x}_{0,d}$), we can also write this as

$$\mathbf{y}_{q',l'}^{(a)} = \sum_{k=0}^{L+L'} \sum_{p=-(Q+Q')/2}^{(Q+Q')/2} e^{j2\pi(p-q')(L'-l')/K} h_{p-q',k-l'}^{(a)} \mathbf{x}_{p,k} + \mathbf{w}_{q',l'}^{(a)}.$$

Then, defining $\mathbf{X} := [\mathbf{x}_{Q/2+Q'/2,L+L'}, \dots, \mathbf{x}_{Q/2+Q'/2,0}, \dots, \mathbf{x}_{-Q/2-Q'/2,0}]^T$, $\mathbf{Y}^{(a)}$ can be expressed as

$$\mathbf{Y}^{(a)} = \boldsymbol{\mathcal{H}}^{(a)} \mathbf{X} + \mathbf{W}^{(a)},$$

where $\mathbf{W}^{(a)}$ is similarly defined as $\mathbf{Y}^{(a)}$ and $\boldsymbol{\mathcal{H}}^{(a)}$ is the $(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ matrix given by

$$\boldsymbol{\mathcal{H}}^{(a)} := \begin{bmatrix} \boldsymbol{\Omega}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(a)} \dots \boldsymbol{\Omega}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(a)} & & \mathbf{0} \\ & \ddots & \ddots & \\ \mathbf{0} & & \boldsymbol{\Omega}^{Q/2}\boldsymbol{\mathcal{H}}_{Q/2}^{(a)} \dots \boldsymbol{\Omega}^{-Q/2}\boldsymbol{\mathcal{H}}_{-Q/2}^{(a)} \end{bmatrix},$$

with $\boldsymbol{\mathcal{H}}_q^{(a)}$ the $(L'+1) \times (L+L'+1)$ Toeplitz matrix given by

$$\boldsymbol{\mathcal{H}}_q^{(a)} := \begin{bmatrix} h_{q,L}^{(a)} \dots h_{q,0}^{(a)} & & 0 \\ & \ddots & \ddots & \\ 0 & & h_{q,L}^{(a)} \dots h_{q,0}^{(a)} \end{bmatrix},$$

and $\boldsymbol{\Omega} := \mathrm{diag}\{[1, e^{j2\pi/K}, \dots, e^{j2\pi L'/K}]^T\}$. Defining $\mathbf{Y} := [\mathbf{Y}^{(0)T}, \dots, \mathbf{Y}^{(A-1)T}]^T$, we then obtain

$$\mathbf{Y} = \boldsymbol{\mathcal{H}}\mathbf{X} + \mathbf{W}, \tag{9}$$

where $\mathbf{W}$ is similarly defined as $\mathbf{Y}$ and $\boldsymbol{\mathcal{H}}$ is the $A(Q'+1)(L'+1) \times (Q+Q'+1)(L+L'+1)$ matrix given by $\boldsymbol{\mathcal{H}} := [\boldsymbol{\mathcal{H}}^{(0)T}, \dots, \boldsymbol{\mathcal{H}}^{(A-1)T}]^T$. Hence, (8) can be rewritten as

$$\hat{\mathbf{x}}_\star^T = \sum_{a=0}^{A-1} \mathbf{f}^{(a)T} \mathbf{Y}^{(a)} = \mathbf{f}^T \mathbf{Y} = \mathbf{f}^T \boldsymbol{\mathcal{H}} \mathbf{X} + \mathbf{f}^T \mathbf{W}, \tag{10}$$

where $\mathbf{f}$ is the $A(L'+1)(Q'+1) \times 1$ vector given by $\mathbf{f} := [\mathbf{f}^{(0)T}, \dots, \mathbf{f}^{(A-1)T}]^T$.

V. Direct Semi-Blind Equalizer Design

We can design the BEM-FIR coefficients of the SLE based on the exact knowledge of the BEM-FIR coefficients of the doubly-selective channel. Focusing on the MMSE SLE this results into

$$\mathbf{f}_{MMSE}^T = \mathbf{e}^T (\boldsymbol{\mathcal{H}}^H \mathbf{R}_W^{-1} \boldsymbol{\mathcal{H}} + \mathbf{R}_X^{-1})^{-1} \boldsymbol{\mathcal{H}}^H \mathbf{R}_W^{-1}, \tag{11}$$

where $\mathbf{R}_X := \mathrm{E}\{\mathbf{X}\mathbf{X}^H\}$ is the data covariance matrix, $\mathbf{R}_W = \mathrm{E}\{\mathbf{W}\mathbf{W}^H\}$ is the noise covariance matrix, and $\mathbf{e}$ is the $(Q+Q'+1)(L+L'+1) \times 1$ unit vector with a 1 in position $(Q+Q')(L+L'+1)/2 + d + 1$.

Assuming the data sequence and the additive noises are mutually uncorrelated and white with variance σ_x^2 and σ_v^2, respectively, the data and noise covariance matrices are given by

$$\mathbf{R}_X = \sigma_x^2 \mathbf{J}_{Q+Q'+1} \otimes \mathbf{I}_{L+L'+1},$$
$$\mathbf{R}_W = \sigma_v^2 \mathbf{I}_M \otimes \begin{bmatrix} \mathbf{J}_{Q'+1} \otimes \boldsymbol{\Phi}_{L'+1,0} & \cdots \mathbf{J}_{Q'+1} \otimes \boldsymbol{\Phi}_{L'+1,P-1} \\ \vdots & \vdots \\ \mathbf{J}_{Q'+1} \otimes \boldsymbol{\Phi}_{L'+1,-P+1} \cdots & \mathbf{J}_{Q'+1} \otimes \boldsymbol{\Phi}_{L'+1,0} \end{bmatrix},$$

where $\mathbf{J}_I$ is the $I \times I$ matrix defined as

$$[\mathbf{J}_I]_{i,i'} = \sum_{n=0}^{N-L'-1} e^{j2\pi(i-i')n/K},$$

and $\boldsymbol{\Phi}_{I,p}$ is the $I \times I$ matrix defined as

$$[\boldsymbol{\Phi}_{I,p}]_{i,i'} := \int_{-\infty}^{\infty} g_{\mathrm{rec}}(\tau) g_{\mathrm{rec}}(\tau + (i'-i)T + pT/P) d\tau.$$

In this paper, however, we aim at the direct semi-blind design of the FIR-BEM coefficients of the SLE, thereby avoiding the intermediate channel estimation step. The proposed approach consists of a combination of the training-based least-squares (LS) method [11] and the blind mutually referenced equalizers (MRE) method [12], both well-known for frequency-selective channels, but here applied to doubly-selective channels. The basic idea is that we consider different SLEs that detect different time- and frequency-shifted versions of the transmitted sequence. While during training periods, the training symbols are used to train all equalizers, during data transmission periods, each equalizer output is used to train the other equalizers.

In the noiseless case, suppose that $\mathbf{f}_{p,k}$ collects the FIR-BEM coefficients of the SLE aiming at the pth frequency-shift and kth time-shift of $\mathbf{x}_\star$, where $p \in \{-P/2, \dots, P/2\}$ with $P \le Q + Q'$, and $k \in \{-K_1, \dots, K_2\}$ with $K_1 \le d$ and $K_2 \le L + L' - d$ (note that $\mathbf{f}_{0,0}$ corresponds to the $\mathbf{f}$ we used before). We can then write

$$\mathbf{f}_{p,k}^T \mathbf{Y} \breve{\mathbf{Z}}_{-k}^T \breve{\mathbf{D}}_{-p}^T e^{-j2\pi p(K_1+k)/K} = \mathbf{x}_\bullet^T,$$

where $\breve{\mathbf{Z}}_k = [\mathbf{0}_{(N-L'-K_1-K_2)\times(K_1-k)}, \mathbf{I}_{N-L'-K_1-K_2}, \mathbf{0}_{(N-L'-K_1-K_2)\times(K_2+k)}]$, and $\breve{\mathbf{D}}_p = \text{diag}\{[1, e^{j2\pi p/K}, \dots, e^{j2\pi p(N-L'-K_1-K_2-1)/K}]^T\}$ and $\mathbf{x}_\bullet = \breve{\mathbf{Z}}_0 \mathbf{x}_\star = [x[L' - d + K_1], \dots, x[N - d - 1 - K_2]]^T$. Defining $\mathbf{Y}_{p,k} = \mathbf{Y}\breve{\mathbf{Z}}_{-k}^T \breve{\mathbf{D}}_{-p}^T e^{-j2\pi p(K_1+k)/K}$, we thus obtain

$$\mathbf{f}_{p,k}^T \mathbf{Y}_{p,k} = \mathbf{x}_\bullet^T. \tag{12}$$

Suppose now that N_t symbols in $\mathbf{x}_\bullet$ are training symbols and the remaining $N_d = N - L' - K_1 - K_2 - N_t$ symbols in $\mathbf{x}_\bullet$ are data symbols. Let us then collect the training symbols of $\mathbf{x}_\bullet$ in $\mathbf{x}^{(t)}$ and the data symbols of $\mathbf{x}_\bullet$ in $\mathbf{x}^{(d)}$. Let us further collect the corresponding columns of $\mathbf{Y}_{p,k}$ in $\mathbf{Y}_{p,k}^{(t)}$ and $\mathbf{Y}_{p,k}^{(d)}$, respectively. Splitting (12) into its training and data part, stacking the results for $p \in \{-P/2, \dots, P/2\}$ and $k \in \{-K_1, \dots, K_2\}$, and defining $R = (P+1)(K_1+K_2+1)$ as the total number of time- and frequency-shifts taken into account, we then obtain

$$\underline{\mathbf{f}}^T[\underline{\mathbf{Y}}^{(t)}, \underline{\mathbf{Y}}^{(d)}] = [\mathbf{x}^{(t)T}\underline{\mathbf{I}}_{N_t}, \mathbf{x}^{(d)T}\underline{\mathbf{I}}_{N_t}],$$

where $\underline{\mathbf{f}} = [\mathbf{f}_{-P/2,-K_1}^T, \dots, \mathbf{f}_{-P/2,K_2}^T, \dots, \mathbf{f}_{P/2,K_2}^T]^T$,

$$\underline{\mathbf{Y}}^{(t)} = \begin{bmatrix} \mathbf{Y}_{-P/2,-K_1}^{(t)} & & & & \\ & \ddots & & & \\ & & \mathbf{Y}_{-P/2,K_2}^{(t)} & & \\ & & & \ddots & \\ & & & & \mathbf{Y}_{P/2,K_2}^{(t)} \end{bmatrix},$$

$$\underline{\mathbf{Y}}^{(d)} = \begin{bmatrix} \mathbf{Y}_{-P/2,-K_1}^{(d)} & & & & \\ & \ddots & & & \\ & & \mathbf{Y}_{-P/2,K_2}^{(d)} & & \\ & & & \ddots & \\ & & & & \mathbf{Y}_{P/2,K_2}^{(d)} \end{bmatrix},$$

and

$$\underline{\mathbf{I}}_{N_t} = \mathbf{1}_{1\times R} \otimes \mathbf{I}_{N_t},$$
$$\underline{\mathbf{I}}_{N_d} = \mathbf{1}_{1\times R} \otimes \mathbf{I}_{N_d}.$$

In the noisy case, we then have to solve

$$\min_{\underline{\mathbf{f}}, \mathbf{x}^{(d)}} \{\|\underline{\mathbf{f}}^T[\underline{\mathbf{Y}}^{(t)}, \underline{\mathbf{Y}}^{(d)}] - [\mathbf{x}^{(t)T}\underline{\mathbf{I}}_{N_t}, \mathbf{x}^{(d)T}\underline{\mathbf{I}}_{N_d}]\|^2\}. \tag{13}$$

The solution for $\mathbf{x}^{(d)}$ is given by

$$\hat{\mathbf{x}}^{(d)T} = \underline{\mathbf{f}}^T \underline{\mathbf{Y}}^{(d)} R^{-1} \underline{\mathbf{I}}_{N_d}^T. \tag{14}$$

Substituting (14) into (13), we obtain

$$\min_{\underline{\mathbf{f}}} \{\|\underline{\mathbf{f}}^T[\underline{\mathbf{Y}}^{(t)}, \underline{\mathbf{Z}}^{(d)}] - [\mathbf{x}^{(t)T}\underline{\mathbf{I}}_{N_t}, \mathbf{0}_{1\times N_d R}]\|^2\}, \tag{15}$$

where $\underline{\mathbf{Z}}^{(d)}$ is given in (16) on the top of the next page. In this equation, the left and right part respectively correspond to the training-based LS method [11] and the blind MRE method [12], applied to doubly-selective channels.

Note that we could use (14) to find an estimate of the unknown data symbols. However, in this case, the weak performing equalizers $\mathbf{f}_{p,k}$ contained in $\underline{\mathbf{f}}$ might pull down the overall performance. A better approach is to select the best performing equalizer $\mathbf{f}_{p,k}$ contained in $\underline{\mathbf{f}}$. However, this will be computationally expensive. We therefore simply select the equalizer $\mathbf{f}_{0,0}$ from $\underline{\mathbf{f}}$ (remember that $\mathbf{f}_{0,0}$ corresponds to the $\mathbf{f}$ we used before) and apply this to $\mathbf{Y}$ in order to find an estimate of $\mathbf{x}_\star$. Identifiability results can be derived along the lines of those for purely frequency-selective channels, and will be presented elsewhere.

VI. Simulation Results

In this section, we illustrate the proposed approach with some simulation results. We generate $M = 2$ channels consisting of 5 unit-power clusters with delays 0, $T/2$, T, $3T/2$, and $2T$, all modeled using Jakes' model with a Doppler spread of $f_{\max} = 1/(400T)$. Assuming that $g_{\text{tr}}(t)$ and $g_{\text{rec}}(t)$ are rectangular functions over $[0, T)$ with height $1/T$, we can take $L = 3$. We further consider fractional sampling with a factor of $S = 2$. Hence, we obtain a SIMO system with $A = MS = 4$ outputs. The modulation we use is QPSK. We assume the data sequence and the additive noises are mutually uncorrelated and white. The SNR is defined as $SNR = 5\sigma_x^2/\sigma_v^2$, where σ_x^2 and σ_v^2 are the variances of the data sequence and the additive noises, respectively. The factor 5 is due to the fact that we consider 5 unit-power clusters.

We consider a time-window of $NT = 200T$. As already mentioned, when $NT \le 1/(2f_{\max})$, which is the case here, an accurate channel model can be obtained by taking $Q = 2$. To satisfy $Q/(2KT) \approx f_{\max} = 1/(400T)$, we then take $K = 400$. Although optimal training for channel estimation has been developed for the case $K = N$ [9], it is not easy to extend it to the case $K > N$. Moreover, optimal training for channel estimation probably does not correspond to optimal training for direct equalizer design, which is a much more complicated problem. For all these reasons, we simply use pilot symbol assisted modulation (PSAM) [13] in this work and insert a pilot symbol after every three data symbols. We consider three SLE designs: the ideal design (see below), the direct training-based design, and the direct semi-blind design. For all designs, we assume $Q' = 2$, $L' = 3$, and $d = (L + L')/2 = 3$. For the ideal design, we first fit a FIR-BEM to the true doubly-selective channel over the time window of $NT = 200T$, and use the obtained FIR-BEM coefficients to design the FIR-BEM

$$\underline{\mathbf{Z}}^{(d)} = R^{-1} \begin{bmatrix} (R-1)\mathbf{Y}^{(d)}_{-P/2,-K_1} \cdots & -\mathbf{Y}^{(d)}_{-P/2,K_2} & \cdots & -\mathbf{Y}^{(d)}_{P/2,K_2} \\ \vdots & \vdots & & \vdots \\ -\mathbf{Y}^{(d)}_{-P/2,-K_1} & \cdots (R-1)\mathbf{Y}^{(d)}_{-P/2,K_2} \cdots & & -\mathbf{Y}^{(d)}_{P/2,K_2} \\ \vdots & \vdots & & \vdots \\ -\mathbf{Y}^{(d)}_{-P/2,-K_1} & \cdots \quad -\mathbf{Y}^{(d)}_{-P/2,K_2} & & \cdots (R-1)\mathbf{Y}^{(d)}_{P/2,K_2} \end{bmatrix} \tag{16}$$

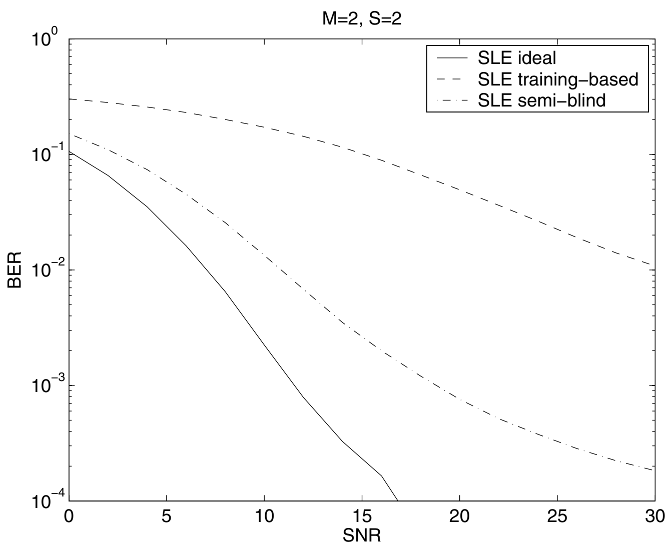

Fig. 2. Comparison of different SLE designs for doubly-selective channels.

coefficients of the SLE as in (11). For the direct training-based design, we consider the proposed approach with $P = K_1 = K_2 = 0$. For the direct semi-blind design, we consider the proposed approach with $P = 2$ and $K_1 = K_2 = 1$. From Figure 2, we can observe that the direct semi-blind design clearly outperforms the direct training-based design, and is not too far from the performance of the ideal design.

VII. CONCLUSIONS

In this paper, we have focused on equalizing a doubly-selective channel by means of an SLE, where both the SLE and the doubly-selective channel are modeled by an FIR-BEM. We have derived a direct semi-blind design method for the FIR-BEM coefficients of the SLE, thereby avoiding the intermediate step of estimating the FIR-BEM coefficients of the doubly-selective channel. Simulation results have shown that the direct semi-blind design outperforms the direct training-based design, and approaches the performance of the ideal design using exact channel knowledge.

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