A COMPUTATIONALLY EFFICIENT BLIND ESTIMATOR OF POLYNOMIAL PHASE SIGNALS OBSERVED BY A SENSOR ARRAY

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ABSTRACT

Consider estimating the parameters of polynomial phase signals observed by an antenna array given that the array manifold is unknown (e.g., uncalibrated array). To date, only an approximated maximum likelihood estimator (AMLE) was suggested, however, it involves a multidimensional search over the entire coefficient space. Instead, we propose a two-step estimation approach, termed as SEparate-EStimate (SEES): First, the signals are separated with a blind source separation technique by exploiting the constant modulus property; Then, the coefficients of each polynomial are estimated using a least squares method from the unwrapped phase of the estimated signal. This estimator does not involve any search in the coefficient spaces and its computational complexity increases linearly with respect to the polynomial order, whereas that of the AMLE increases exponentially. Simulations show that the proposed estimator achieves the Cramér-Rao lower bound (CRLB) at moderate or high signal to noise ratio (SNR).

1. INTRODUCTION

Polynomial phase signals (PPSs) are commonly used in synthetic aperture radar (SAR) imaging and radio communications applications (such as the 3GPP standard). For example, in SAR surveillance systems [1] the challenge is to separate the moving targets and then to estimate the motion parameters of each target (velocity and acceleration) which are embedded in the coefficients of each polynomial.

The problem of estimating the coefficients of the polynomial phases has been researched for signals observed by a single sensor [2–4] and by a sensor array [5–9]. The techniques in [5–8] assume that the array is perfectly calibrated. In practice, it is difficult to maintain a precisely calibrated array due to pressure, humidity, and mechanical vibrations. Furthermore, calibration is an expensive process. Alternatively, the spatial signature of the array is then assumed to be completely unknown, and blind estimation techniques are applied [10]. To date, the paper of Zeira and Friedlander [9] is the only published work on blind estimation of the polynomial coefficients of each of the signals impinging on a sensor array.

The exact maximum likelihood estimator (MLE) of the coefficients of each of the polynomials requires a PQ-dimensional search in the polynomial coefficients space, where P is the order of each polynomial and Q is the number of PPSs. Instead, Zeira and Friedlander suggested to estimate the coefficients with an AMLE [9], which reduces the computation of the exact MLE to a single P-dimensional search in the polynomial coefficients space. The AMLE is based on the assumption that PPSs tend to be orthogonal to each other. Then, the AMLE is obtained by decoupling the estimation of one PPS from the others. Given the observations, we evaluate the cost function of the AMLE for each point in the space by performing a P-dimensional search in the polynomial coefficients space. The PPSs are estimated as the Q local peaks of the cost function.

Herein, we suggest a two-step estimator termed SEES. First, we exploit the fact that the waveforms of the signals are constant modulus (CM), and separate the signals using the zero-forcing algebraic constant modulus algorithm (ZF-ACMA) [11] which estimates all signals simultaneously using linear algebraic operations. Then, we estimate the polynomial coefficients of each signal from the wrapped phase of the estimated signal. We suggest to first perform phase unwrapping [3] and then estimate the polynomial coefficients given the unwrapped phases using a least squares (LS) method. The last step can be implemented with other approaches [12]. Due to space limitations, we deferred some of the analysis to [15] where we also: i) derive the complexity of the proposed SEES technique showing that it increases linearly with respect to (w.r.t.) the polynomial order and the number of samples, whereas that of the AMLE increases exponentially; ii) show that the estimates are asymptotically unbiased, and derive explicit expressions for their asymptotic covariance matrix; iii) obtain closed-form expressions of the estimate variances in case of two signals with closely spaced directions of arrival (DOAs).

The root mean square error (RMSE) performance of the SEES estimator is demonstrated in simulations where we compare it with the AMLE and the CRLB [9]. It is shown that:1) The RMSE of the SEES estimator achieves the CRLB at moderate or high SNR; 2) The RMSE of the SEES estimator is sensitive to the small separation between the DOAs, while for large separation, the RMSEs of the AMLE and the SEES method are similar. In [15] we further demonstrate that the AMLE and the SEES are not sensitive to the separation between the initial frequencies or between the frequency rates of the signals, and show that the processing time of the SEES algorithm is much smaller than that of the AMLE.

2. PROBLEM FORMULATION

Consider a sensor array composed of M sensors, and Q transmitting sources. Each source transmits a narrowband PPS. The $M \times 1$ noisy sampled signal vector at the array output is given by [9]

$$\begin{aligned} \mathbf{x}(n) &= \sum_{q=1}^{Q} \alpha_q \mathbf{a}(\theta_q) s_q(n) + \mathbf{e}(n) \\ &= \mathbf{A}\mathbf{s}(n) + \mathbf{e}(n) \quad , n = 0, \ldots, N-1 \end{aligned} \tag{1}$$

where N is the number of samples, the $M \times 1$ vector $\mathbf{x}(n)$ containing the outputs of the array elements is defined as,

$$\mathbf{x}(n) \triangleq [x_1(n), \ldots, x_M(n)]^T \tag{2}$$

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The $M \times Q$ array response matrix $\mathbf{A}$ is given by,

$$\mathbf{A} \triangleq [\alpha_1 \mathbf{a}(\theta_1), \cdots, \alpha_Q \mathbf{a}(\theta_Q)] \tag{3}$$

where α_q is the unknown complex amplitude of the qth PPS, and the $M \times 1$ vector $\mathbf{a}(\theta_q)$ is the array response to the qth signal transmitted from DOA, denoted by θ_q. The $Q \times 1$ vector $\mathbf{s}(n)$, containing the PPSs $s_1(n), \ldots, s_Q(n)$, is defined as,

$$\mathbf{s}(n) \triangleq [s_1(n), \ldots, s_Q(n)]^T \tag{4}$$

Finally, the $M \times 1$ vector $\mathbf{e}(n)$ containing the noises is,

$$\mathbf{e}(n) \triangleq [e_1(n), \ldots, e_M(n)]^T \tag{5}$$

We assume that $\mathbf{a}(\theta)$ (the array manifold) is unknown as occurs, for example, when the array is uncalibrated [9, 10]. As a result, the unknown complex amplitude of the qth signal, α_q, is absorbed in the array response of the qth signal, $\mathbf{a}(\theta_q)$.

The q-th PPS is given as

$$s_q(n) \triangleq e^{j\phi_q(n)} \tag{6}$$

where the instantaneous phase of the signal is,

$$\phi_q(n) \triangleq \mathbf{u}^T(n)\mathbf{b}_q \tag{7}$$

and the $P \times 1$ vector $\mathbf{b}_q$, containing the polynomial coefficients of the q-th PPS, and the $P \times 1$ vector $\mathbf{u}(n)$ are defined as

$$\mathbf{b}_q \triangleq [b_{q,1}, \ldots, b_{q,P}]^T \tag{8}$$

$$\mathbf{u}(n) \triangleq [nT_s, \ldots, (nT_s)^P]^T \tag{9}$$

where T_s is the sampling time, and P is the known order of the polynomial. The vector $\mathbf{e}(n)$ is assumed to be spatially and temporally white Gaussian complex random vector with zero mean and covariance matrix $\sigma_n^2 \mathbf{I}_M$, where σ_n^2 is unknown and $\mathbf{I}_M$ is the $M \times M$ identity matrix.

The problem discussed is: Given the observations $\{\mathbf{x}(n)\}_{n=0}^{N-1}$, estimate the polynomial coefficients $\{\mathbf{b}_q\}_{q=1}^{Q}$, assuming that the array manifold is unknown.

3. THE PROPOSED SEES METHOD

The proposed SEES method to estimate the polynomial coefficients of each signal is based on two main steps:

1. Separate the signals using a blind source separation technique. We consider the ZF-ACMA [11] which exploits the CM property of the signals and not their polynomial phase structure.
2. Estimate the polynomial coefficients of the qth signal given the unwrapped phases of the qth estimated signal.

A similar approach is suggested in [13] for estimating DOAs of CM signals using a sensor array with perfectly known array response.

Based on these two steps we develop a model which linearly depends on the polynomial coefficients of the qth signal with the presence of small additive noises. This model will be used to obtain a LS estimate of the polynomial coefficients. According to [11] the output of the ZF-ACMA in the presence of asymptotically small measurement noises or large number of samples is

$$\hat{\mathbf{s}}(n) = \mathbf{W}^H \mathbf{x}(n) \tag{10}$$

where $\mathbf{W}$ is the $M \times Q$ zero-forcing beamforming matrix,

$$\mathbf{W} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \tag{11}$$

Due to the phase ambiguity of the ZF-ACMA, the phase of each signal at the output of the ZF-ACMA is obtained up to an unknown constant phase shift, represented by c_q, $q = 1, \ldots, Q$ where $c_q \in [-\pi, \pi)$. We collect these phase shifts in a $Q \times Q$ diagonal matrix $\mathbf{D} \triangleq \mathrm{diag}(e^{jc_1}, \ldots, e^{jc_Q})$. The output of the ZF-ACMA is then

$$\hat{\mathbf{s}}(n) = \mathbf{D}\mathbf{s}(n) + \boldsymbol{\varepsilon}(n) \tag{12}$$

where $\boldsymbol{\varepsilon}(n) \triangleq [\varepsilon_1(n), \ldots, \varepsilon_Q(n)]^T$ is the $Q \times 1$ vector defined as,

$$\boldsymbol{\varepsilon}(n) = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{e}(n) \tag{13}$$

Observe that the qth signal at the output of the ZF-ACMA is

$$\hat{s}_q(n) = |\hat{s}_q(n)| e^{j\hat{\phi}_q(n)} \tag{14}$$

Following (12) we show in [15] that the magnitude $|\hat{s}_q(n)|$ approximately equals to unity assuming small errors, that is, $|\varepsilon_q(n)| \ll 1$, $q = 1, \ldots, Q$. This means that under the assumption of small errors, each signal at the output of the ZF-ACMA is (approximately) a CM signal. Consider next the phase $\hat{\phi}_q(n)$. By assuming that $\Re\{\varepsilon_q(n)\}$ and $\Im\{\varepsilon_q(n)\}$ are small, we can use a first order Taylor series to approximate this phase as

$$\hat{\phi}_q(n) \cong \phi_q(n) + c_q + \eta_q \tag{15}$$

where

$$\eta_q \triangleq \cos(\phi_q(n)+c_q)\Im\{\varepsilon_q(n)\} - \sin(\phi_q(n)+c_q)\Re\{\varepsilon_q(n)\} \tag{16}$$

The information on the polynomial coefficients is hidden in the phases $\{\hat{\phi}_q(n)\}_{q=1,n=0}^{Q,N-1}$. The wrapped phases are simply computed by considering the arguments of $\{\hat{s}_q(n)\}_{n=0,q=1}^{N-1,Q}$. However, to estimate the polynomial coefficients we are interested in the unwrapped version of the phase, which is obtained from $\{\hat{s}_q(n)\}_{n=0,q=1}^{N-1,Q}$ with an unwrapping procedure. There are several techniques to perform phase unwrapping for PPSs (e.g., [3, 12]). Herein, we use the procedure presented in [3]. The unwrapped phases of $\{\hat{\phi}_q(n)\}_{n=0}^{N-1}$, denoted by $\{\tilde{\phi}_q(n)\}_{n=0}^{N-1}$, are given by (see [3, p. 2120] and also [8])

$$\begin{aligned} \tilde{\phi}_q(0) &= \hat{\phi}_q(0) \bmod 2\pi \\ \tilde{\phi}_q(1) &= ((\hat{\phi}_q(1) - \hat{\phi}_q(0)) \bmod 2\pi) + \tilde{\phi}_q(0) \\ \tilde{\phi}_q(n) &= \delta_q(n) + 2\tilde{\phi}_q(n-1) - \tilde{\phi}_q(n-2)\ , n = 2, \ldots, N-1 \end{aligned} \tag{17}$$

and

$$\delta_q(n) \triangleq (\hat{\phi}_q(n) - 2\hat{\phi}_q(n-1) + \hat{\phi}_q(n-2)) \bmod 2\pi \tag{18}$$

We further assume that the noises are small enough such that they do not cause any 2π jumps in the unwrapping procedure. Thus, the unwrapped phase at the end of this step is

$$\tilde{\phi}_q(n) = \hat{\phi}_q(n) \cong c_q + \sum_{q=1}^{Q} n^p b_{q,p} + \eta_q \tag{19}$$

Consider the noise term η_q, defined in (16). Since $\Re\{\boldsymbol{\varepsilon}(n)\}$ and $\Im\{\boldsymbol{\varepsilon}(n)\}$ are zero mean Gaussian random vectors, this means that η_q is also a zero mean Gaussian random variable. This serves as a motivation for applying the LS method. By neglecting the additive noise part we obtain an approximated linear model for the parameters of interest (polynomial coefficients) given the measurements (unwrapped phase). Define the $N \times 1$ vector

$$\tilde{\boldsymbol{\phi}}_q \triangleq [\tilde{\phi}_q(0), \ldots, \tilde{\phi}_q(N-1)]^T \tag{20}$$

We then rewrite (19) in a vector form as

$$\tilde{\boldsymbol{\phi}}_q \cong c_q \mathbf{1}_N + \mathbf{U}^T \mathbf{b}_q = \tilde{\mathbf{U}}^T \mathbf{h}_q \tag{21}$$

where $\mathbf{1}_N$ is a $N \times 1$ vector of ones, $\mathbf{h}_q \triangleq [c_q, \mathbf{b}_q^T]^T$ is a $(P+1) \times 1$ vector, and $\tilde{\mathbf{U}}$ is the $(P+1) \times N$ Vandermonde matrix defined as,

$$\tilde{\mathbf{U}} \triangleq [\mathbf{1}_N \ \mathbf{U}^T]^T \tag{22}$$

$$\mathbf{U} \triangleq [\mathbf{u}(0), \cdots, \mathbf{u}(N-1)] \tag{23}$$

The unknown parameters $\hat{c}_q, \hat{\mathbf{b}}_q$ are estimated using a LS method as follows (a similar LS estimation was used in [8] without considering the estimation of the arbitrary initial phase),

$$[\hat{c}_q, \hat{\mathbf{b}}_q^T] = \underset{\mathbf{h}_q}{\operatorname{argmin}} \left\| \tilde{\boldsymbol{\phi}}_q - \tilde{\mathbf{U}}^T \mathbf{h}_q \right\|^2 = (\tilde{\mathbf{U}}\tilde{\mathbf{U}}^T)^{-1} \tilde{\mathbf{U}} \tilde{\boldsymbol{\phi}}_q \tag{24}$$

Note that the first entry of the estimated vector is the estimate of the nuisance parameter c_q. The estimate of the polynomial coefficients vector is given after few mathematical steps as [15]

$$\hat{\mathbf{b}}_q = (\mathbf{U}\mathbf{U}^T)^{-1} \mathbf{U} \left(\mathbf{I}_N - \frac{1}{\kappa} \mathbf{1}_N \mathbf{1}_N^T \left(\mathbf{I}_N - \mathbf{U}(\mathbf{U}\mathbf{U}^T)^{-1}\mathbf{U} \right) \right) \tilde{\boldsymbol{\phi}}_q \tag{25}$$

where $\kappa \triangleq N - \mathbf{1}_N^T \mathbf{U}^T (\mathbf{U}\mathbf{U}^T)^{-1} \mathbf{U} \mathbf{1}_N$. Using the matrix inversion lemma we obtain that (25) can be written more compactly as,

$$\hat{\mathbf{b}}_q = (\mathbf{U}\mathbf{P}\mathbf{U}^T)^{-1} \mathbf{U}\mathbf{P} \tilde{\boldsymbol{\phi}}_q \tag{26}$$

where $\mathbf{P} \triangleq \mathbf{I}_N - \frac{1}{N} \mathbf{1}_N \mathbf{1}_N^T$ is the $N \times N$ orthogonal projection matrix. We note that the Vandermonde structure of $\mathbf{U}$ can also be exploited in the above LS solution [14] to save computations.

4. NUMERICAL RESULTS

To demonstrate the performance of the proposed method, we present the results of simulated experiments. The noise power σ_n^2 is adjusted to give the desired SNR defined as SNR $= -10\log_{10}\sigma_n^2$ [dB]. In each of the simulation examples we evaluated the RMSE on the estimated polynomial coefficients. We defined the RMSE on the estimation of the p-th coefficient b_p of the q-th signal as $\text{RMSE}_q(b_p) \triangleq \sqrt{\frac{1}{N_{exp}} \sum_{n=1}^{N_{exp}} (\hat{b}_{q,p,n} - b_{q,p})^2}$ where $\hat{b}_{q,p,n}$ is the estimate of b_p of the q-th signal at the n-th trial, and $N_{exp} = 100$ is the number of Monte-Carlo (MC) independent trials. For comparison, in each simulation we added the theoretical covariances of the estimates [15], and also compared the results with the associated CRLB [9, Section 2].

Unless otherwise stated, we use a $M = 10$ element ULA with half wavelength spacing. We consider two PPSs of order two ($P = 2$), that is, chirp signals, sampled with a sampling frequency of $f_s = 1/T_s = 50$ [Hz], and the number of samples is $N = 200$ [9]. The continuous time signals are given by $s_1(t) = e^{j2\pi\xi f_s t(0.9-0.2t/T)}$, $s_2(t) = e^{j2\pi\xi f_s t(0.1-0.8t/T)}$, where $\xi = 0.25$, and $T = N/f_s$.

We compare the RMSE of the estimated initial frequency (b_1) and the estimated frequency rate ($2b_2$) versus the SNR. We considered SNR values from -2[dB] to 3[dB] with a step of 1[dB]. The DOAs of the signals are $\theta_1 = -20$ [deg] and $\theta_2 = 5$ [deg]. The RMSE results are presented in Fig. 1. As can be seen, for low SNR the performance of the AMLE is superior. However, as the SNR increases, the RMSE of the proposed SEES method improves and approaches the CRLB. Also, observe that the theoretical variances are similar to those obtained by the CRLB.

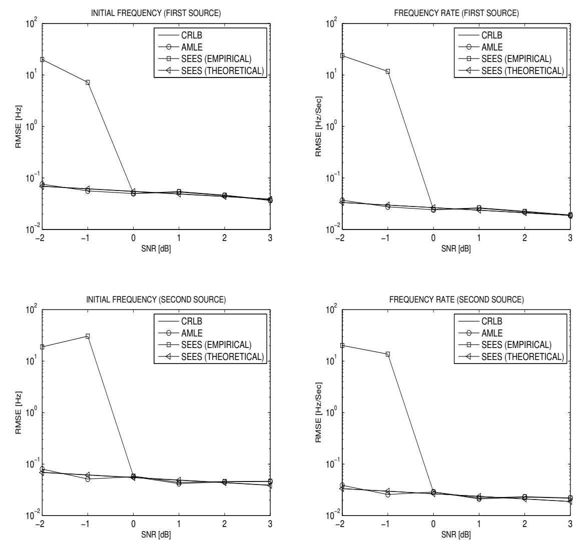

Fig. 1. RMSEs of the estimated initial frequency and the frequency rate, and the CRLB, versus the SNR for chirp signals.

We also compare the RMSE of the estimated initial frequency (b_1) and the estimated frequency rate ($2b_2$) versus the separation in the DOA of the two signals. We set the DOA of the first signal at 0 [deg]. The DOA of the second signal is varied from 2[deg] to 14[deg] with a step of 2[deg]. The SNR is 5[dB]. The RMSE results are presented in Fig. 2. Also added are the approximated closed form expressions for the theoretical RMSEs for two PPSs with closely spaced DOAs [15]. As can be seen, the proposed SEES estimator is inferior to the AMLE at low SNR, while both have similar performance at high SNR. The theoretical expressions for the RMSEs predict well the performance.

Finally, we compare the RMSE of the estimated polynomial coefficients for quadratic FM signal versus the SNR. We use a $M = 12$ element ULA with half wavelength spacing. The continuous time quadratic FM signals are [7, Eq. (41)] $s_1(t) = e^{j(2\pi\xi f_s(t-5t^2/T+7t^3/3T^2))}$, $s_2(t) = e^{j(2\pi\xi f_s(-t+3t^2/T-6t^3/3T^2))}$, where $\xi = 0.4$, and $T = N/f_s$, where $N = 256$, and $f_s = 1/T_s = 8192$ [Hz]. The DOAs of the signals are $\theta_1 = -30$ [deg] and $\theta_2 = -15$ [deg]. We varied the SNR from -4[dB] to 6[dB] with a step of 2[dB]. We only consider the SEES estimator and the AMLE, since the complexity of the AMLE is very high in this case. The RMSE results are presented in Fig. 3 for the first signal only since the results of the second signal are similar. As can be observed, the SEES estimator has similar performance at high SNR as predicted by the CRLB. Similar results are presented in [15] for fourth order PPSs.

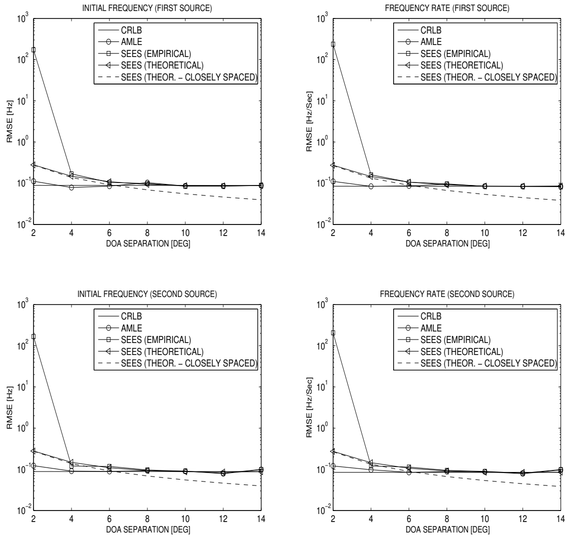

Fig. 2. RMSEs of the estimated initial frequency and the frequency rate, and the CRLB, versus the DOA separation.

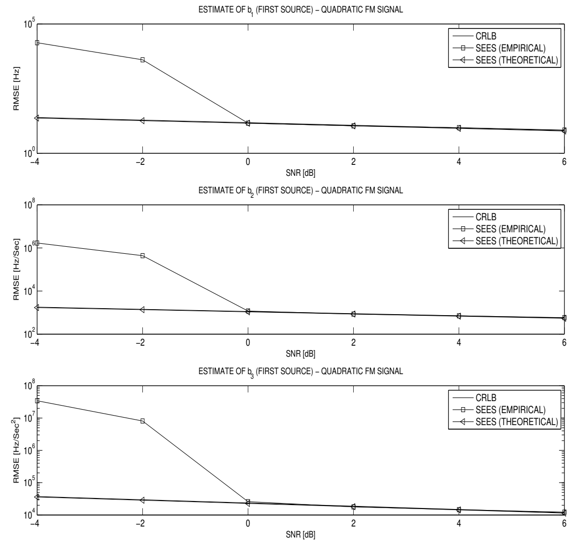

Fig. 3. RMSEs of the estimated initial frequency and the frequency rate, and the CRLB, versus the SNR for quadratic FM signals.

5. CONCLUSION

We proposed an approach for estimating polynomial phase signals observed by a sensor array assuming an unknown array manifold. First, the signals are separated using a blind source separation method, and then the coefficients of the polynomial phase of each signal are estimated using a LS method. The complexity of the algorithm increases linearly with respect to the polynomial order and to the number of samples. Simulations show that the estimator achieves the CRLB at moderate or high SNR.

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