TU Delft
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET 4386 Estimation and Detection

ASSIGNMENT

Signal Detection

1 Context

The detection of signals plays an important role in many signal processing related applications. In this assignment we will investigate the detection of the presence of a partially known signal $As[n]$ in noisy realizations.

In a group of 2 students, make a short report (4-5 pages; pdf file) containing the required MATLAB scripts, plots, and answers.

System model

In this assignment we observe sequences of noisy samples with length $N = 10$. For each of these observed sequences, we wish to detect whether the signal $As[n]$ with $n \in \{0, ..., N-1\}$ is present. This can be fornulated using the following two hypotheses

$$\begin{aligned} \mathcal{H}_0 : &\quad x[n] = w[n] \quad n = 0, 1, ..., N-1 \\ \mathcal{H}_1 : &\quad x[n] = As[n] + w[n] \quad n = 0, 1, ..., N-1, \end{aligned}$$

where $w[n] \sim \mathcal{N}(0, \sigma_w^2)$, and $w[n]$ is IID and independent from $s[n]$. In the first part of this assignment we assume that both A and $s[n]$ are known. The necessary matlab variables are stored in the matlab file *Data.mat*. The observed data is stored in variable x as a matrix structure. This is a 3-dimensional variable, where the third dimension corresponds to different values of A that have been used to generate the data. The values of A that have been used are also given in *Data.mat*. Given a specific index for the

third dimension (i.e., related to a certain value for A), each row in x is one sequence of N observations $x[n]$. In total there are $M = 10000$ realization, each of length N.

2 Assignment

In the data set both $s[n]$ and the different values for A are given. Initially assume that these are indeed both known.

1. **(3pts)** Derive and implement a detector of your choice. Motivate the reason for selecting the detector that you have implemented.

2. **(2pts)** Use several values for P_{FA}, e.g., $P_{FA} \in \{10^{-1}, 10^{-2}, ..., 10^{-5}\}$ and the M realizations, to plot the numerical P_D and theoretical P_D versus the SNR

$$\text{SNR} = 10 \log_{10} \left(\frac{\sum_{n=0}^{N-1} A^2 s^2[n]}{\sigma_w^2} \right).$$

   To know whether $As[n]$ is actually present in a sequence, the matlab variable *mask* can be used. This variable is also given in the file *Data.mat* and indicates (with a "1") whether the signal was actually present.

3. **(3pts)** Now consider the case where $s[n]$ is still known, but the exact value and sign of A is unknown. How does the detection performance change?

4. **(2pts)** Report writing and research.