TU Delft
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET 4386 Estimation and Detection

ASSIGNMENT

Clock Synchronization for Sensor Networks

1 Context

Clock synchronization among different sensor nodes each having its own autonomous clock oscillator is a key component of a wireless sensor network (WSN). A WSN enables coordinated functions such as distributed data sampling, data fusion, time-based channel sharing and scheduling, sleep and wake-up coordination, and other time-based tasks. These tasks demand a common time frame for the entire network. Individual clocks in a WSN drift from each other due to imperfections in the oscillator, aging, and other environmental variations, and it is essential to determine these drifts.

This exercise consists of two parts: (a) derive and implement a suitable estimator of the clock drift; and (b) study the performance of the estimator. In a group of 2 students, make a short report (4-5 pages; pdf file) containing the required MATLAB scripts, plots, and answers.

System model

We will consider a simple sensor network consisting of two sensor nodes, namely, *node 1* and *node 2*. The distance between *node 1* and *node 2* is denoted by d. Let t_i be the local time at the ith node and t be the true time. We approximate the relation between the local time and the true time using a first order affine clock model

$$t_i = t + \phi_i \quad \Leftrightarrow \quad t = t_i - \phi_i,\ i = 1, 2, \tag{1}$$

where $\phi_i \in \mathbb{R}$ is the clock-offset (also referred to as the time offset). Assume *node 1* has a relatively stable clock, i.e., $\phi_1 = 0$.

In order to synchronize *node 2* with *node 1*, we need to estimate ϕ_2. This is typically done by transmitting messages from *node 1* to *node 2*. The transmission and reception time-stamps are recorded during the forward link (i.e., *node 1* to *node 2*). The time-stamp recorded at the 1st node when the kth iteration message departs is denoted by t_k, and on arrival of the corresponding message, the 2nd node records the time-stamp r_k.

The time-of-flight for a line-of-sight (LOS) transmission from the 1st node to the 2nd node can be defined as $\tau = c^{-1}d$, where $c = 300$ m/s denotes the speed of the acoustic wave in a medium. Using (**??**), τ can be written in terms of the time-stamps recorded using respective local clocks of the 1st and 2nd node as

$$\tau = [(r_k + \epsilon_{2,k}) - \phi_2] - [(t_k + \epsilon_{1,k}) - \phi_1], \tag{2}$$

where $\epsilon_{1,k} \sim \mathcal{N}(0, 0.5\sigma^2)$ and $\epsilon_{2,k} \sim \mathcal{N}(0, 0.5\sigma^2)$ denote the measurement error on the time-stamps.

2 Assignment

You are given $K = 10$ noisy time stamps in the self explanatory data file `ClockSync.mat`, i.e., you are given $r_k + \epsilon_{2,k}$ and $t_k + \epsilon_{1,k}$. This means, *node 1* transmits $K = 10$ messages to *node 2*. Further, 10000 independent realizations of this experiment are available for six different noise variance (i.e., σ^2) values. You will have to answer the following questions:

1. **(4pts)** To begin with, assume that τ is not known. Can you derive a joint estimator for the range τ and clock offset ϕ_2. Compute the Cramér-Rao bound. What are the observations that can be made (e.g., identifiability, number of equations and number of unknowns, rank of the Fisher information matrix)?

2. **(4pts)** Now assume that τ is available and known. Derive an estimator for the clock offset ϕ_2 and the Cramér-Rao bound for this problem. Also compute the *mean square error* of the developed estimator, and is the estimator *efficient*? Plot (numerical and theoretical) mean square error vs. noise variance, and Cramér-Rao bound vs. noise variance, and explain these figures?

3. **(2pts)** Report writing and research.