TU Delft
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET 4386 Estimation and Detection

ASSIGNMENT

Accelerometer Calibration

1 Context

An Inertial Measurement Unit (IMU) comprises of motion and rotational sensors to estimate the position, orientation, and velocity of moving objects. A typical IMU generally consists of three orthogonal, gyroscopes, three orthogonal accelerometers, and three orthogonal magnetometers. See for example the IMU MPU-9250, which is shown in Figure 1.The accuracy of the inertial systems are highly dependent on the calibration of the IMU to remove the systematic errors. Calibration is the process of comparing the instruments output with a known reference information and determining the coefficients that force the output to agree with the reference information over a range of output values [1].

In this project, you will apply various estimators, e.g., the Maximum Likelihood Estimator (MLE) and performance bounds, e.g., the Cramer-Rao Lower Bound (CRLB) for the challenge of calibrating the acceleormeter. In a group of 2 students, make a short report (4-5 pages) to answer the questions with plots and analysis. The report should be supplemented with the code in the Appendix.

1.1 Accelerometer Sensor Model

The accelerometer measures the linear acceleration of the system in the inertial reference frame. The signal in noise model of the measured output of the accelerometer can be written as [1]

$$\mathbf{y}_k = \boldsymbol{\mu}_k + \mathbf{v}_k, \tag{1}$$

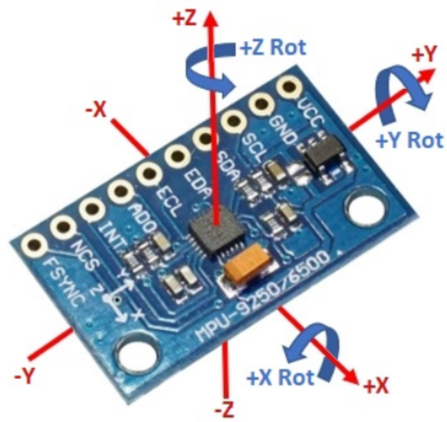

Figure 1: MPU-9250 IMU.

where $\mathbf{y}_k$ is the output at the k-th measurement, and $\mathbf{v}_k$ is zero-mean white Gaussian noise with variance σ^2; the signal model $\boldsymbol{\mu}_k = [\mu_{kx}\ \mu_{ky}\ \mu_{kz}]^{\mathrm{T}}$ is defined as

$$\boldsymbol{\mu}_k = \boldsymbol{\mu}(\boldsymbol{\theta}, \mathbf{u}_k) = \mathbf{K}\mathbf{T}^{-1}\mathbf{u}_k + \mathbf{b}, \tag{2}$$

where $\mathbf{u}_k = g[-\sin(\phi)\ \ \cos(\phi)\sin(\varphi)\ \ \cos(\phi)\cos(\varphi)]^{\mathrm{T}}$ is the input force at the k-th measurement (ϕ, φ, and g are pitch, roll, and the magnitude of the gravity vector, respectively). Also, $\mathbf{K}$, $\mathbf{T}$, and $\mathbf{b}$, denote the scale factor matrix, the misalignment angles matrix, and the bias vector, respectively, which are defined as follows

$$\mathbf{K} \triangleq \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}, \tag{3}$$

where k_i is the unknown scaling of the i-th accelerometer,

$$\mathbf{T} \triangleq \begin{bmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ 0 & 1 & -\alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix}, \tag{4}$$

and

$$\mathbf{b} \triangleq [b_x \ \ b_y \ \ b_z]^{\mathrm{T}}, \tag{5}$$

where b_i is the unknown bias of the i-th accelerometer output, and $[\cdot]^{\mathrm{T}}$ denotes the transpose operation. The unknown calibration parameters of the accelerator are populated in the vector as

$$\boldsymbol{\theta} \triangleq \begin{bmatrix} k_x & k_y & k_z & \alpha_{yz} & \alpha_{zy} & \alpha_{zx} & b_x & b_y & b_z \end{bmatrix}^{\mathrm{T}}. \tag{6}$$

To estimate the unknown calibration parameters in (6), the IMU is required to be rotated in $M \geq 9$ different orientations [1]. Each time the IMU is placed in a new orientation, the pitch (ϕ) and roll (φ) rotational angles appear in the input force $\mathbf{u}_k = g[-\sin(\phi) \;\; \cos(\phi)\sin(\varphi) \;\; \cos(\phi)\cos(\varphi)]^{\mathrm{T}}$ to the system.

1.2 Measurement Model

Collecting all the measurement outputs and signal models into bigger vectors, the total measurement output $\tilde{\mathbf{y}}$, and the signal model $\tilde{\boldsymbol{\mu}}$ are defined as:

$$\begin{aligned} \tilde{\mathbf{y}} &\triangleq [\mathbf{y}_1^{\mathrm{T}} \;\; \mathbf{y}_2^{\mathrm{T}} \;\; \cdots \;\; \mathbf{y}_{MN}^{\mathrm{T}}]^{\mathrm{T}} \\ \tilde{\boldsymbol{\mu}} &\triangleq [\boldsymbol{\mu}_1^{\mathrm{T}} \;\; \boldsymbol{\mu}_2^{\mathrm{T}} \;\; \cdots \;\; \boldsymbol{\mu}_{MN}^{\mathrm{T}}]^{\mathrm{T}}, \end{aligned} \tag{7}$$

where N is the number of samples which are taken in each rotation, and M is the number of orientations that the IMU is placed in. Based on the signal in noise model (1), the output signal, $\mathbf{y}_k$, has a Gaussian distribution as

$$\mathbf{y}_k \sim N(\boldsymbol{\mu}_k, \sigma^2 \mathbf{I}_3). \tag{8}$$

Hence, the probability density function of the measurement vector $\tilde{\mathbf{y}}$ is

$$p(\tilde{\mathbf{y}}; \boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{MN}{2}} |\mathbf{C}|^{\frac{1}{2}}} \exp\left(- \frac{\sum_{k=1}^{MN} \|\mathbf{y}_k - \boldsymbol{\mu}_k(\boldsymbol{\theta})\|^2}{2\sigma^2} \right), \tag{9}$$

where

$$\mathbf{C} = \sigma^2 \mathbf{I}_{3MN}. \tag{10}$$

In (10), $\mathbf{C}$ is the covariance matrix of $\tilde{\mathbf{y}}$, $\mathbf{I}$ is the identity matrix with the subscripted dimension, $|\cdot|$ denotes the determinant, and $\|\cdot\|$ refers to the Euclidean length of a vector.

2 Assignment

You are given $K = 500$ noisy measurements data set for accelerometer calibration in file Measurements.mat. For each data set, the IMU was rotated into $M = 25$ known orientations, and in each orientation $N = 30$ samples were taken. The noise variance is $\sigma^2 = 0.024$ $[m/s^2]$ and $g = 9.80665$ $[m/s^2]$. In this assignment, we consider that the vector of rotational angles, i.e.,

$$\boldsymbol{\eta} \triangleq [\phi_0 \quad \varphi_0 \quad \phi_1 \quad \varphi_1 \quad \cdots \quad \phi_{M-1} \quad \varphi_{M-1}]^{\mathrm{T}} \tag{11}$$

is known and fixed for all $K = 500$ measurements. This vector is given in file Eta.mat. You can also find the vector of true parameter $\boldsymbol{\theta}$ in file Theta.mat.

You are required to answer the following questions:

1. By using the given noisy measurements data set and the accelerator measurement model in (1), first estimate the nine calibration parameter $\boldsymbol{\theta}$ in (6) and then plot the square-root of the mean square errors (MSE) of the nine estimated calibration parameter $\boldsymbol{\theta}$ versus the number of samples $N \in \{1, 2, \cdots, 30\}$ (Monte Carlo simulation). Is the MLE derived in [1] the optimal estimator for the system model in (1)? Please comment (3pts).

2. Obtain the average value and standard deviation of the estimated calibration parameters $\boldsymbol{\theta}$ in Question 1 for $N = 30$ and $M = 25$ (1pts).

3. Compare the results of the proposed estimator in Question 2 with a different arbitrary estimator, and report and comment the results (2pts).

4. Obtain and plot the CRLB versus the noise variance in the range of $\sigma^2 \in [0.01 \ 0.1]$ for $N = 30$ and $M = 25$ for the given true parameter vector $\boldsymbol{\theta}$ in file Theta.mat (2pts).

5. Report writing and research (2pts).

3 Corrections and Hints

- There are typos in equation (10) and (15) of [1]. The correction are as follows:

$$\mathbf{u}_k \triangleq \mathbf{R}_n^p \mathbf{g} = \mathbf{R}_n^p \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = \begin{bmatrix} -\mathrm{s}\phi \\ \mathrm{c}\phi\mathrm{s}\varphi \\ \mathrm{c}\phi\mathrm{c}\varphi \end{bmatrix} g, \tag{12}$$

where $\|\mathbf{u}_k\| = g$, and

$$p(\tilde{\mathbf{y}};\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{MN}{2}}|\mathbf{C}|^{\frac{1}{2}}}\exp\left(-\frac{\sum_{k=1}^{MN}\|\mathbf{y}_k - \boldsymbol{\mu}_k(\boldsymbol{\theta})\|^2}{2\sigma^2}\right), \tag{13}$$

where

$$\mathbf{C} \triangleq \sigma^2\mathbf{I}_{3MN}. \tag{14}$$

- Consider re-parametrization of $\mathbf{H}(\boldsymbol{\theta}_1)$ to solve the minimization problem in (22) of [1]. For more information refer to [1].

References

[1] G. Panahandeh, I. Skog, and M. Jansson, "Calibration of the accelerometer triad of an inertial measurement unit, maximum likelihood estimation and cramer-rao bound," in *IPIN*. IEEE, 2010, pp. 1–6.