TU Delft
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET 4386 Estimation and Detection

ASSIGNMENT

Spectrum Sensing for Cognitive Radios

1 Context

The concept of cognitive radio is to exploit the under utilized spectral resources by reusing unused spectrum in an opportunistic manner. A cognitive radio system generally involves primary users of the spectrum, who are incumbent licensees, and secondary users who seek to opportunistically use the spectrum when the primary users are idle. The cognitive radios must sense the spectrum to detect whether it is available or not. This exercise consists of two parts: (a) derive and implement a suitable detector; and (b) study the performance of the detector.

In a group of 2 students, make a short report (4-5 pages; pdf file) containing the required MATLAB scripts, plots, and answers.

System model

We will consider a very simple cognitive radio system. The primary user signal is an orthogonal frequency division multiplexing (OFDM) signal. OFDM is used by many of the current and also future wireless communications systems. A discrete time OFDM symbol block is first constructed by applying the inverse Fast Fourier Transform (IFFT) to N_d data symbols, and the output of the IFFT is given by

$$x[n] = \frac{1}{\sqrt{N_d}} \sum_{k=0}^{N_d-1} s[k] e^{j2\pi n \frac{k}{N_d}}, \; n = 0, 1, \ldots, N_d - 1$$

where N_d is the number of data symbols in one OFDM symbol block, and the $s[k]$'s denote the data symbols. The last N_c symbols of $x[n]$ are copied

and appended to the beginning of it as a cyclic prefix (CP), which can be used to deal with inter-symbol interference (ISI) at the receiver side. As a result, the length of the transmitted OFDM symbol $x[n]$ is extended to $N_d + N_c$. Many such OFDM symbols are transmitted one after the other. A secondary user must sense the spectrum to detect whether the OFDM signals of the primary user are present or not. If the primary user is not active, the secondary user can take the chance to make use of the available spectrum. In the presence of a primary user, the received OFDM signal which is corrupted by an additive white Gaussian noise (AWGN) channel at the secondary user can simply be modeled as $y[n] = x[n] + w[n]$, where $w[n]$ stands for the AWGN noise.

The goal now is to detect the presence of the primary user in Matlab. The dataset provided to you corresponds to the transmission of $K + 1$ consecutive independent OFDM symbol blocks $\mathbf{x} = [\mathbf{x}_0^T, \mathbf{x}_1^T, \ldots, \mathbf{x}_K^T]^T$ over an AWGN channel, resulting into the output $\mathbf{y} = \mathbf{x} + \mathbf{w}$. The noise vector $\mathbf{w}$ is a zero-mean complex Gaussian random process,where $E[\mathbf{w}\mathbf{w}^H] = \sigma_w^2 \mathbf{I}_{(K+1)(Nd+Nc)}$. This variance σ_w^2 is chosen according to the signal-to-noise ratio (SNR), which is defined as the ratio of the signal energy over the noise energy expressed in dB, i.e., $\text{SNR} = 10 \log \frac{\sigma_s^2}{\sigma_w^2}$, where $\sigma_s^2 = 1$. The dataset consists of 1000 realizations of the output signal $\mathbf{y}$ for 13 different SNR values.

2 Assignment

The secondary users want to detect whether there is a primary user present or not. Hence, we wish to decide between the following two hypotheses:

$$\begin{aligned} \mathcal{H}_0 : &\quad \mathbf{y} = \mathbf{w} \\ \mathcal{H}_1 : &\quad \mathbf{y} = \mathbf{x} + \mathbf{w} \end{aligned}$$

where $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I})$, and $\mathbf{w}$ is independent from $\mathbf{x}$. Further assume that $\mathbf{x}$ is zero-mean Gaussian with covariance matrix $E[\mathbf{x}\mathbf{x}^H] = \sigma_s^2 \mathbf{I}_{(K+1)(Nd+Nc)}$.

1. **(4pts)** Derive and implement a detector of your choice. Motivate the reason for selecting the detector that you have implemented. Use $P_{FA} = 0.05$ and 1000 test statistics, to plot the numerical P_D and theoretical P_D vs. SNR.

2. **(4pts)** How will your detector behave, if we have an inaccurate $\tilde{\sigma}_w^2$ instead of σ_w^2. Assume that $\tilde{\sigma}_w^2$ is 1 dB larger than σ_w^2. Compare it

with previous results. *Hint:* The structure of the OFDM signal due to the insertion of the CP could be exploited.

3. **(2pts)** Report writing and research.