Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

ET 4386 Estimation and Detection

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Exercises -Estimation

Problem 1: We observe the data $x[n] = Ar^n + w[n]$ for $n = 0, ..., N-1$. The noise processs $w[n]$ is uncorrelated Gaussian with variance σ^2, and $r > 0$ is known.

Problem 1a: Assume that A is deterministic. Find the CRLB for A.

Problem 1b: Determine the MVU estimator for A and find its variance.

Problem 1c: Determine the BLUE estimator for A.

Problem 1d: What happens with the variance of the MVU estumator if $N \to \infty$?

Problem 1e: Now assume that A is a zero-mean random variable that is Gaussian distributed with variance σ_A^2, independent of $w[n]$. Calculate the MMSE estimator of A.

Problem 1f: Determine the minimum Bayesian MSE.

Problem 1g: Find the LMMSE estimator.

Problem 2: We observe the data $x[n]$ for $n = 0, ..., N-1$. Each sample has the conditional pdf

$$p(x[n]|\theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x[n] \leq \theta \\ 0, & \text{otherwise.} \end{cases}$$

Conditioned on θ, the observations are independent. The distribution of θ is $\theta \sim U(0, \beta)$.

Problem 2a: Find the MMSE estimator of θ.

Problem 2b: Consider the situation with little prior knowledge. For what value of β is the prior non-informative?

Problem 2c: How does the estimator change in the case of little prior knowledge.

Problem 2d: Determine the Bayesian ML estimator.

Problem 2e: Determine the MAP estimator.

Problem 3: We observe data $x[n] = x[n] + w[n]$ for $n = 0, ..., N-1$. Both $s[n]$ and $w[n]$ are zero-mean WSS processes, mutually uncorrelated. Their autocorrelation functions are given by $r_{ss}[k] = \sigma_s^2[k]$ and $r_{ww}[k] = \sigma^2[k]$

Problem 3a: Determine the LMMSE estimator of $\mathbf{s} = [s[0], s[1], ..., s[N-1]]^T$.

Problem 3b: Determine the minimum Bayesian MSE $Bmse(\hat{\theta}_i)$ for this linear estimator.

Problem 4: Given is the data model

$$x[n] = A + w[n]$$

for $n = 0, ..., N-1$. The unknown parameter A is assumed to have the distribution

$$p(A) = \begin{cases} \lambda \exp\left[-\lambda A\right], & A > 0 \\ 0, & A < 0, \end{cases}$$

with $\lambda > 0$ and $w[n]$ WGN independent of A with variance σ^2.

Problem 4a: Determine the MAP estimator of A.

Problem 4b: Determine the LMMSE estimator of A.

Problem 5: Given is the a posteriori pdf

$$p(\theta|x) = \begin{cases} \exp\left[-(\theta - x)\right], & \theta > x \\ 0, & \theta < x. \end{cases}$$

Problem 5a: Calculate the MMSE and MAP estimator of θ.

Problem 6: Given is the distribution

$$p(x[n]) = \begin{cases} \theta \exp\left[-\theta x[n]\right], & x[n] > 0 \\ 0, & x[n] < 0, \end{cases}$$

for $n = 0, ..., N-1$. The samples $x[n]$ are independent over time.

Problem 6a: Give the CRLB for $\hat{\theta}$.

Problem 6b: Does an unbiased estimator exist that achieves the CRL bound?

Problem 6c: Determine the MLE estimator.

The a priori distribution $p(\theta)$ is given by

$$p(\theta) = \begin{cases} \lambda \exp\left[-\lambda \theta\right], & \theta > 0 \\ 0, & \theta < 0. \end{cases}$$

Problem 6d: Calculate the MAP estimator.

Problem 6e: Calculate the MMSE estimator.