Applied Convex Optimization, EE4530, 2015

Homework Set 2

Exercise 1. [2pt.]
Solve Exercise 2.12 of Boyd, Vandenberghe, CO.

Exercise 2. [1pt.]
Solve Exercise 3.2 of Boyd, Vandenberghe, CO.

Exercise 3. [2pt.]
Solve Exercise 3.16 of Boyd, Vandenberghe, CO.

Exercise 4. [2.5pt.]
(Matlab). Consider the LP,

$$\underset{\mathbf{x}}{\text{minimize}}\ \mathbf{c}^\mathsf{T}\mathbf{x}, \quad \text{subject to} \quad \mathbf{x} \geq 0, \mathbf{A}\mathbf{x} \leq \mathbf{b}.$$

(a) Suppose $\mathbf{x} \in \mathbf{R}^n$ and $\mathbf{A} \in \mathbf{R}^{n \times n}$. When can you find an analytical solution? What is it?

(b) Generate real-valued random instances of $\mathbf{c}$, $\mathbf{A}$ and $\mathbf{b}$, with $\mathrm{rank}(\mathbf{A}) < n$. Then, by using the Matlab command *linprog* and the *tic-toc* function, plot the computational time of solving these instances for $n = 10, 100, 1000, 10000$. Comment on the plot.

Exercise 5. [2.5pt.]
(Matlab) Consider the QP,

$$\underset{\mathbf{x}}{\text{maximize}}\ \|\mathbf{x}\|_2, \quad \text{subject to} \quad \|\mathbf{x}\|_2^2 \leq 1.$$

(a) Why is it not a convex problem?

(b) Find all the global maximizers of the problem and its unique maximum, analytically.

(c) Show that the problem does not have a solution if we substitute $\|\mathbf{x}\|_2^2 \leq 1$ with $\|\mathbf{x}\|_2^2 < 1$.

Consider the QP,

$$\underset{\mathbf{x}}{\text{minimize}}\ \mathbf{x}^\mathsf{T}\mathbf{Q}\mathbf{x} + \mathbf{c}^\mathsf{T}\mathbf{x}, \quad \text{subject to} \quad \mathbf{x} \geq 0, \mathbf{A}\mathbf{x} \leq \mathbf{b}.$$

for $\mathbf{Q} \in \mathbf{S}^n_{++}$, $\mathrm{rank}(\mathbf{A}) < n$. Generate real-valued random instances or it. Then, by using the Matlab command *quadprog* and the *tic-toc* function, plot the computational time of solving these instances for $n = 10, 100, 1000, 10000$. Comment on the plot.