Digital Signal Processing - Lecture 1: Introduction

Signal Processing EE2S31

Delft University of Technology, The Netherlands

Course Organization

Course organization - DSP track

- Information
  - Website: general overview
  - Brightspace: more detailed information, quiz, forum
- Organization
  - DSP 1x a week($\pm$) on Monday, Tuesday, or Thursday
  - Exam comprises both tracks

Prerequisite: EE2S11 Signals and Systems

- Continuous-time vs discrete-time signals
- Linear time-invariant systems
- Fourier Transform, spectral representation
- Discrete-time Fourier transform

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Study materials - DSP track

- Theory
  - Lectures
  - Book (Proakis, Manolakis: Digital Signal Processing)
  - Collegerama videos
  - Important notes:
    1. Studying the slides is not sufficient; you need to read the book!
    2. Attending lectures is important; we solve exercises during lectures
- Practice
  - Brightspace Quiz (easy)
  - Exercises from book (more advanced)
  - Past exams on website

Lectures: Digital Signal Processing Track

Geethu Joseph

- Lectures 1-5

  Mid-term exam (Lectures 1-4)

Bori Hunyadi

- Lectures 6 - 8
- Exercise session

  Final exam (Lectures 5-8)

Exam

- The exam is conducted in two parts; both partial exams contain 50% of questions from each track
- The final grade is the average of the two partial exam results, rounded to half a digit
- The re-examination is conducted in one part (over all lecture material)
- The exams are closed-book, with one A4-size page (2 sides) of handwritten notes permitted

Introduction and Applications

What is a signal?

What is a signal?

Any measurable quantity that conveys information

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Introduction

What is a signal?

Any measurable quantity that conveys information

Examples

1. electrical: voltage output of amplifier
2. mechanical: acceleration of a car
3. acoustic: air pressure measured by a microphone
4. biological: body temperature
5. image and video: intensities of each pixel

Classification of Signals

1. Continuous-time vs discrete-time
2. Unquantized (continuous-amplitude) vs quantized (discrete amplitude)

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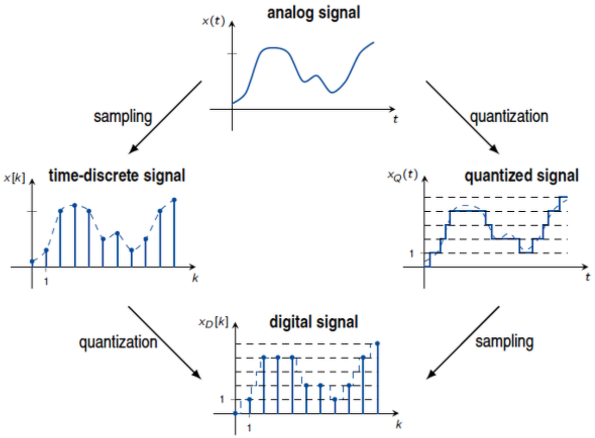
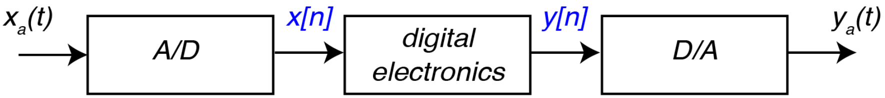
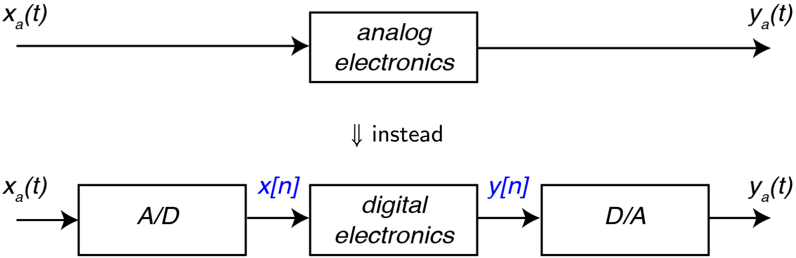


Digital Signal Processing

Processing of analog signals employing discrete-time operations implemented on digital hardware



Analog vs Digital Signal Processing



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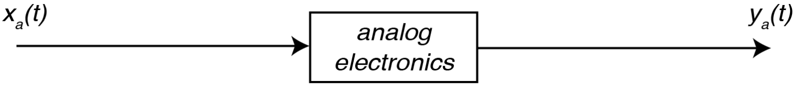

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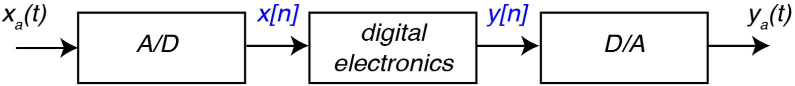
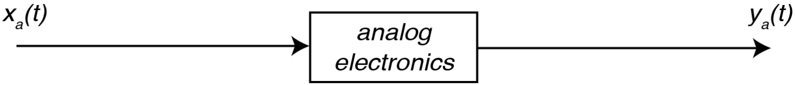


Pro:

- accuracy
- flexibility
- ease of data storage

Analog vs Digital Signal Processing



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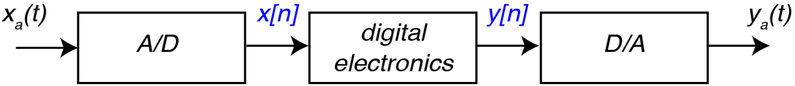


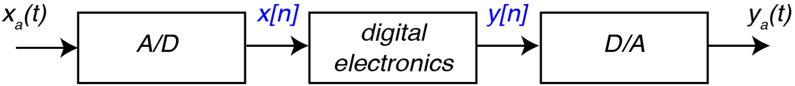
Pro:

- accuracy
- flexibility
- ease of data storage

Cons:

- extra complexity
- limited bandwidth
- quantization effects

Digital signal processing



sampling
quantization

filtering
spectral analysis

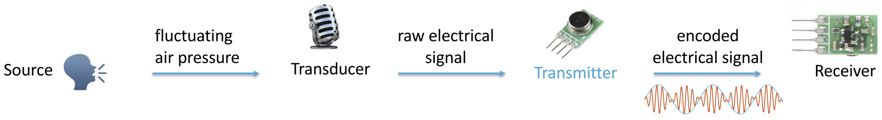
reconstruction

DSP applications

- Digital communication
- Audio signal processing
- Speech signal processing
- Image Processing
- Medical applications

DSP applications (1)

Mobile communication:

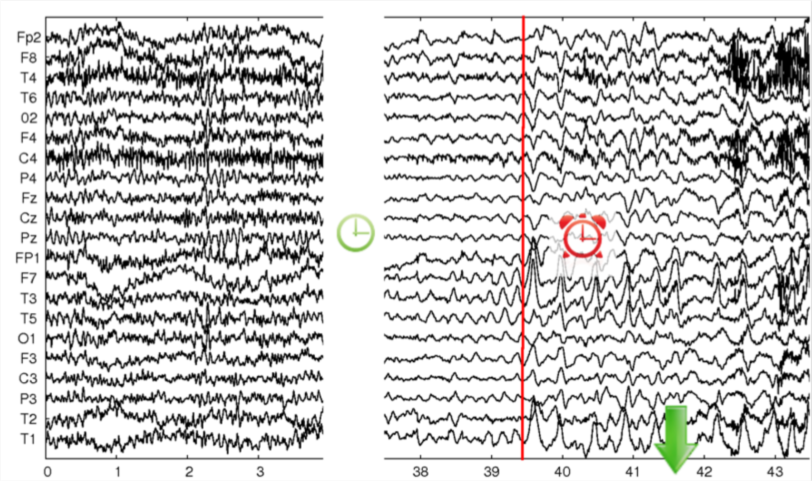


Signal Processing steps:

- Noise filtering
- A/D conversion
- Modulation
- Amplification

DSP Applications (2)

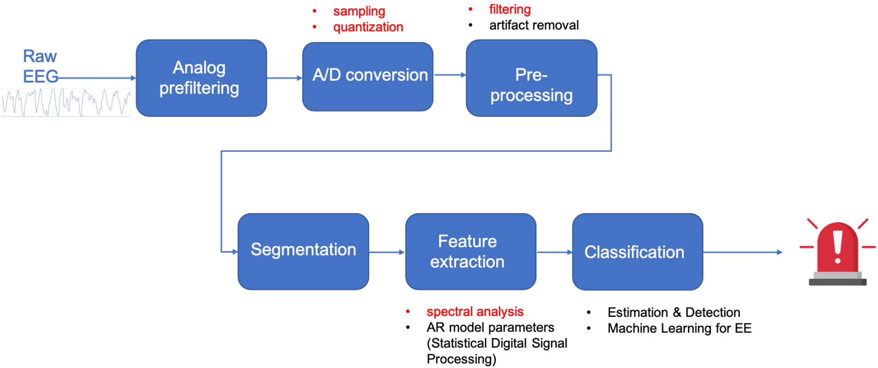
EEG processing for epileptic seizure detection:



abnormally synchronous activity

DSP Applications (3):

Seizure detection pipeline:



You will learn during this course

You can learn in the Master of EE, Signals&Systems

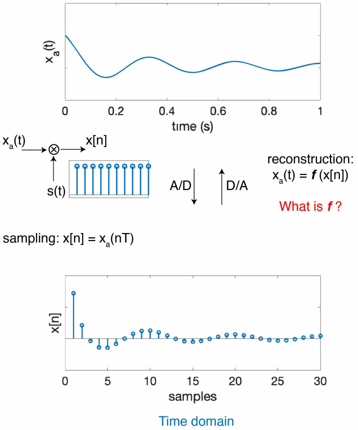
This Course

- Sampling and reconstruction
  - Non-ideal sampling and reconstruction
  - Sampling in the frequency domain: DFT
  - DFT basics
  - Spectral analysis and filtering using DFT
  - Efficient implementation of DFT: FFT
- Quantization and effects
  - Quantization, coding, sigma-delta
  - Round-off effects and filter structures
- Multirate signal processing

Recap: Ideal sampling and reconstruction

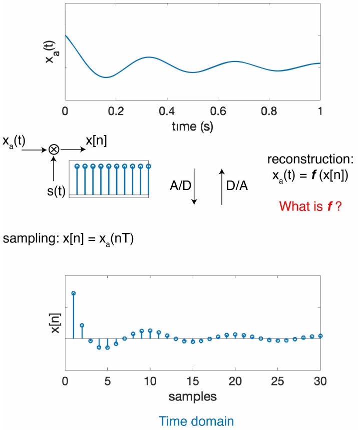
Reference: Chapter 6.1 of the textbook

Ideal sampling and reconstruction



Under which conditions can we reconstruct $x_a(t)$?

Ideal sampling and reconstruction



Under which conditions can we reconstruct $x_a(t)$?

To answer this question, we will investigate the form of the digital signal in the frequency domain.

Recap: Fourier Transform in continuous and discrete time

FT

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft} \mathrm{d}t$$

Inverse FT

$$x_a(t) = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi Ft} \mathrm{d}F$$

$F\,[Hz]$: frequency
$\Omega\,[radians/s]$: angular frequency
$\Omega = 2\pi F$

Recap: Fourier Transform in continuous and discrete time

FT

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft} \mathrm{d}t$$

DTFT

$$X(f) = \sum_{n=-\infty}^{\infty} x[n] e^{-j2\pi fn}$$

Inverse FT

$$x_a(t) = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi Ft} \mathrm{d}F$$

Inverse DTFT

$$x[n] = \int_{-1/2}^{1/2} X(f) e^{j2\pi fn} \mathrm{d}f$$

$F\,[Hz]$: frequency
$\Omega\,[radians/s]$: angular frequency
$\Omega = 2\pi F$

$f\,[cycles/sample]$: normalized frequency
$\omega\,[rad/sample]$: normalized angular frequency
$\omega = 2\pi f$

Recap: Fourier Transform in continuous and discrete time

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$f\,[cycles/sample]$: normalized frequency
$\omega\,[rad/sample]$: normalized angular frequency
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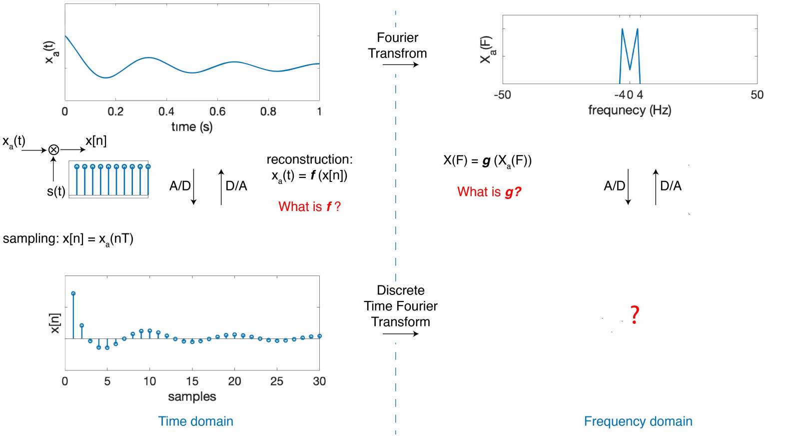
$$\Omega = \omega / T \qquad F = f \cdot F_s$$

Ideal sampling and reconstruction



Can we express the DTFT of the sampled signal using the FT of the analog signal?

DTFT of the sampled signal Vs the FT of the analog signal

- Recall the relation between the sampled and analog signals

$$x[n] = x_a(nT)$$

DTFT of the sampled signal Vs the FT of the analog signal

- Recall the relation between the sampled and analog signals

$$x[n] = x_a(nT)$$

- Expressing them in using inverse (DT)FT,

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} X(f)e^{j2\pi fn}\mathrm{d}f = \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft}\mathrm{d}F|_{t=nT}$$
$$= \int_{-\infty}^{\infty} X_a(F)e^{j2\pi FTn}\mathrm{d}F$$

DTFT of the sampled signal Vs the FT of the analog signal

- Recall the relation between the sampled and analog signals

$$x[n] = x_a(nT)$$

- Expressing them in using inverse (DT)FT,

$$\begin{aligned}\int_{-\frac{1}{2}}^{\frac{1}{2}} X(f)e^{j2\pi fn}\mathrm{d}f &= \int_{-\infty}^{\infty} X_a(F)e^{j2\pi Ft}\mathrm{d}F|_{t=nT} \\ &= \int_{-\infty}^{\infty} X_a(F)e^{j2\pi FTn}\mathrm{d}F\end{aligned}$$

- We try to find a function g rewrite

$$\int_{-\infty}^{\infty} X_a(F)e^{j2\pi FTn}\mathrm{d}F = \int_{-\frac{1}{2}}^{\frac{1}{2}} g(X_a(f))e^{j2\pi fn}\mathrm{d}f$$

$$\Longrightarrow X(f) = g(X_a(f))$$

DTFT of digital vs FT of analog signal

Our goal

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} g(X_a(f)) e^{j2\pi fn} \mathrm{d}f \implies X(f) = g(X_a(f))$$

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1. Divide the infinite interval to $F_s = 1/T$ long intervals

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-\infty}^{\infty} X_a(F) e^{j2\pi F/F_s n} \mathrm{d}F = \sum_{k=-\infty}^{\infty} \int_{kF_s - \frac{F_s}{2}}^{kF_s + \frac{F_s}{2}} X_a(F) e^{j2\pi F/F_s n} \mathrm{d}F$$

DTFT of digital vs FT of analog signal

Our goal

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} g(X_a(f)) e^{j2\pi fn} \mathrm{d}f \implies X(f) = g(X_a(f))$$

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2. Change of variables to match the limits of integrals $f \to F/F_s - k$

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \sum_{k=-\infty}^{\infty} \int_{-1/2}^{1/2} X_a(Fsf + kFs) e^{j2\pi(f+k)n} F_s \mathrm{d}f$$

DTFT of digital vs FT of analog signal

Our goal

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} g(X_a(f)) e^{j2\pi fn} \mathrm{d}f \implies X(f) = g(X_a(f))$$

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3. Exchange sum and integration and note that $e^{j2\pi (f+k)n} = e^{j2\pi fn}$ is periodic

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} \left[F_s \sum_{k=-\infty}^{\infty} X_a((f-k)Fs) \right] e^{j2\pi fn} \mathrm{d}f$$

DTFT of digital vs FT of analog signal

Our goal

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} g(X_a(f)) e^{j2\pi fn} \mathrm{d}f \implies X(f) = g(X_a(f))$$

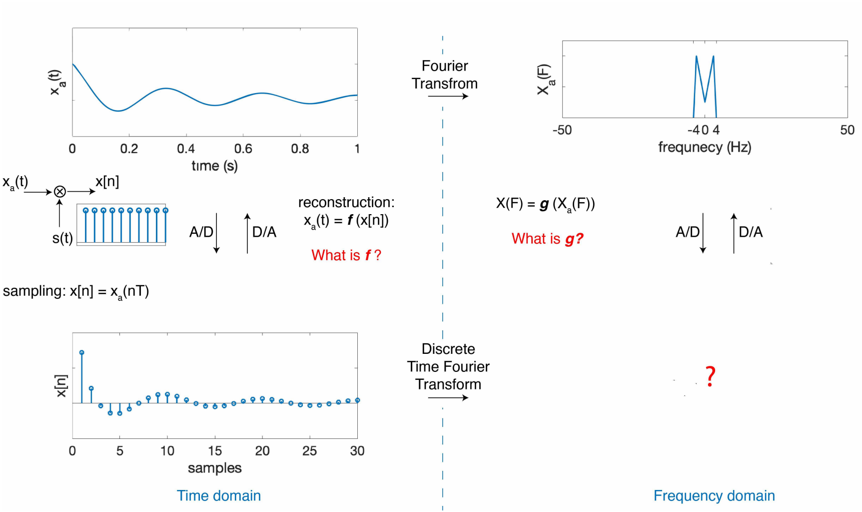
We proved that

$$\int_{-\infty}^{\infty} X_a(F) e^{j2\pi FTn} \mathrm{d}F = \int_{-1/2}^{1/2} \left[F_s \sum_{k=-\infty}^{\infty} X_a((f-k)Fs) \right] e^{j2\pi fn} \mathrm{d}f$$

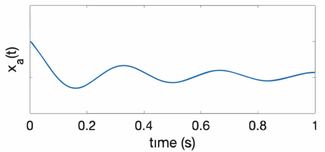
$$\implies X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)Fs) = \frac{1}{F_s} X(F - kF_s)$$

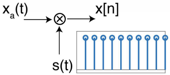
as $F = fF_s$

Ideal sampling and reconstruction



Ideal sampling and reconstruction



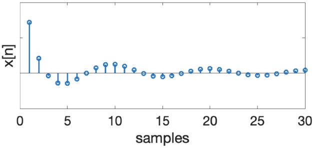


A/D ↓ ↑ D/A

reconstruction:
$x_a(t) = \boldsymbol{f}(x[n])$

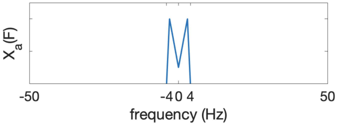
What is ***f*** ?

sampling: $x[n] = x_a(nT)$



Time domain

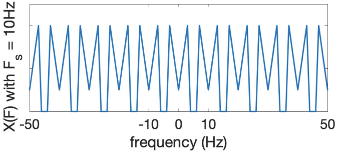
Fourier Transfrom →



$$X(F) = F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

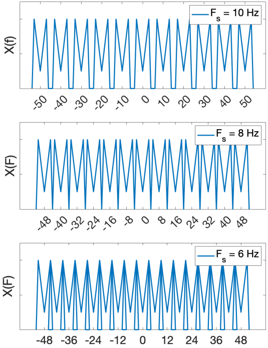
A/D ↓ ↑ D/A

Discrete Time Fourier Transform →



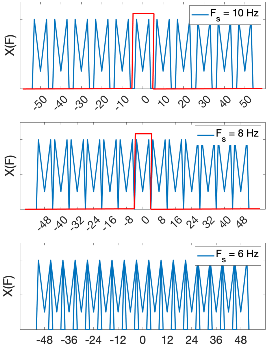
Frequency domain

Spectrum of the sampled signal



$$X(f) = F_s \sum_{k=-\infty}^{\infty} X_a((f-k)Fs)$$

Spectrum of the sampled signal



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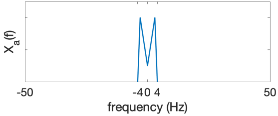
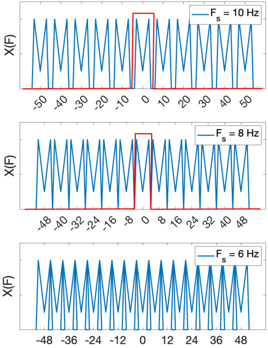


$$X_a(F) = \begin{cases} \frac{1}{F_s} X(F = fF_s) & \text{if} |F| < F_s/2 \\ 0 & \text{otherwise} \end{cases}$$

Spectrum of the sampled signal



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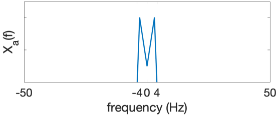

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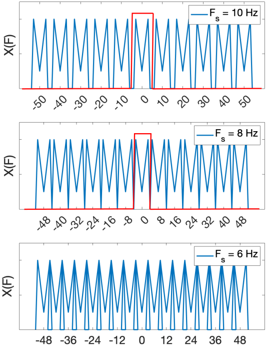
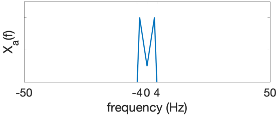
Sampling theorem

If the signal is bandlimited, it is possible to reconstruct the original signal from the samples, provided that the sampling rate is at least twice the highest frequency contained in the signal (i.e., the Nyquist rate).

Ideal reconstruction in frequency domain





Define an ideal low-pass filter $G(f)$:

$$G(F) = \begin{cases} F_s^{-1}, & \text{if } |F| \leq \frac{F_s}{2} \\ 0, & \text{otherwise} \end{cases}$$

Apply it $G(F)$ to $X(F)$:

$$X_a(F) = G(F)X(F)$$

Ideal reconstruction in time domain

- In the frequency domain, we have

$$X_a(F) = G(F)X(F),$$

where $G(F) = \begin{cases} F_s^{-1}, & \text{if } |F| \leq \frac{F_s}{2} \\ 0, & \text{otherwise} \end{cases}$

- In the time domain, we have

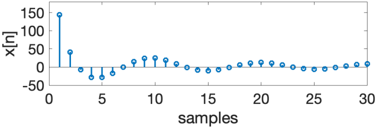
$$x_a(t) = g(t) * x(t) = \sum_{n=-\infty}^{\infty} x[n]g(t - nT),$$

where the interpolator is

$$g(t) = \text{inverse DFTF}(G(F)) = \frac{sin(\pi t/T)}{\pi t/T}$$

The ideal interpolator

Sampled signal

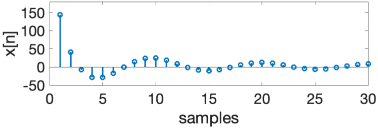


$$x[n]$$

Terms in the convolution operation

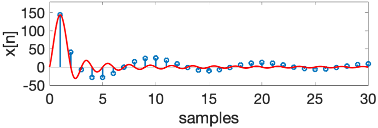
The ideal interpolator

Sampled signal



$$x[n]$$

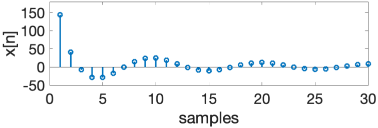
Terms in the convolution operation



$$x[1]\frac{sin(\pi/T(t-1T))}{(\pi/T)(t-1T)}$$

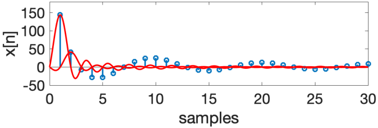
The ideal interpolator

Sampled signal



$$x[n]$$

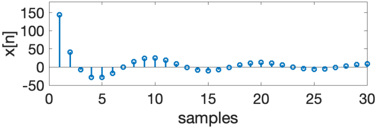
Terms in the convolution operation



$$x[2]\frac{sin(\pi/T(t-2T))}{(\pi/T(t-2T)}$$

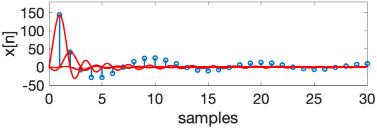
The ideal interpolator

Sampled signal



$$x[n]$$

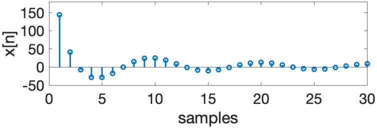
Terms in the convolution operation

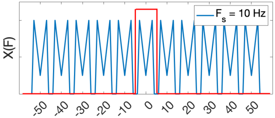


$$x[3]\frac{sin(\pi/T(t-3T))}{(\pi/T)(t-3T)}$$

The ideal interpolator

Sampled signal

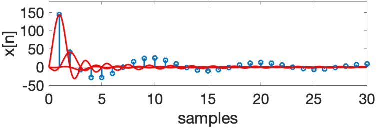




$$G(f) = \begin{cases} F_s^{-1}, & \text{if } |F| \leq= \frac{F_s}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$x[n]$$

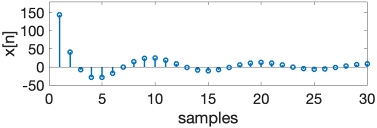
Terms in the convolution operation



$$x[3]\frac{sin(\pi/T(t-3T))}{(\pi/T)(t-3T)}$$

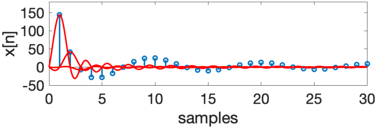
The ideal interpolator

Sampled signal



$$x[n]$$

Terms in the convolution operation



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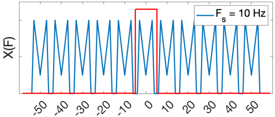
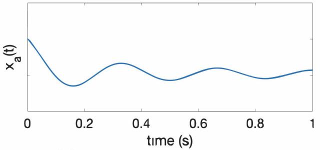
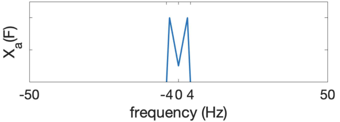


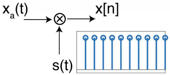
$$G(f) = \begin{cases} F_s^{-1}, & \text{if } |F| \leq= \frac{F_s}{2} \\ 0, & \text{otherwise} \end{cases}$$

Ideal sampling and reconstruction



Fourier Transfrom →





A/D ↓ ↑ D/A

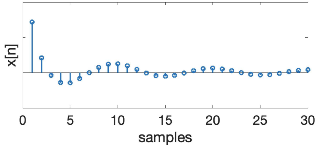
reconstruction:
$x_a(t) = \boldsymbol{f}(x[n])$

What is $\boldsymbol{f}$?

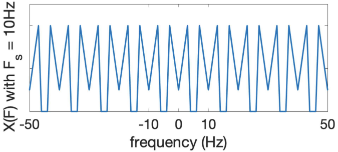
$$X(F) = F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

A/D ↓ ↑ D/A

sampling: $x[n] = x_a(nT)$



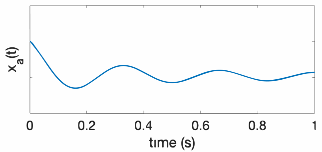
Discrete Time Fourier Transform →



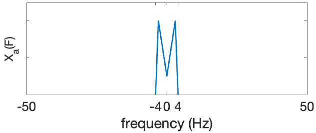
Time domain

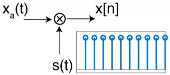
Frequency domain

Ideal sampling and reconstruction



Fourier Transfrom →





A/D ↓ ↑ D/A

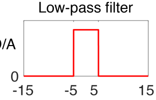
reconstruction:

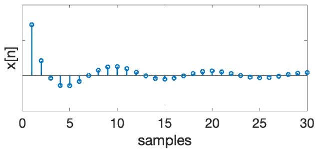
$$x_a(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(2\pi / T_s(t - nT_s))}{\pi / T_s(t - nT_s)}$$

sampling: $x[n] = x_a(nT)$

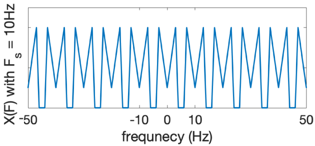
$$X(F) = F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

A/D ↓ ↑ D/A





Discrete Time Fourier Transform →



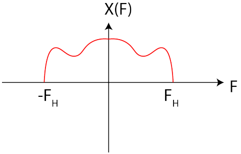
Time domain

Frequency domain

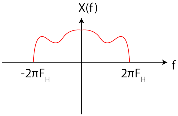
Quiz

Go to www.kahoot.it

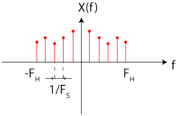
Quiz - question 1



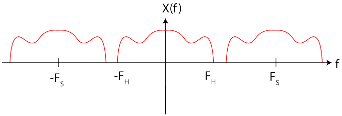
The spectrum of a continuous-time signal is depicted above. Which one of the figures below represents the spectrum of the sampled version of the signal?



a.

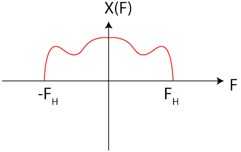


b.



c.

Quiz - question 1



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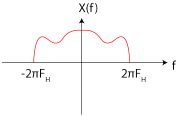
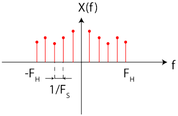
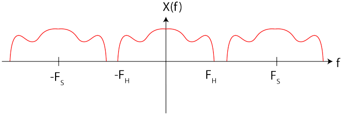


a.



b.



c.

Answer c

Quiz - question 2

Aliasing occurs when we

a. oversample a signal, i.e. with a sampling rate $F_s \gg 2F_H$
b. sample an aperiodic signal
c. sample below the Nyquist rate

Quiz - question 2

Aliasing occurs when we

a. oversample a signal, i.e. with a sampling rate $F_s \gg 2F_H$
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c. sample below the Nyquist rate

Answer c

Quiz - question 3

The reconstruction of an analog signal from its samples can happen using

a. Highpass filter in the frequency domain
b. Convolution with a sinc function in the time domain
c. The inverse Fourier transform

Quiz - question 3

The reconstruction of an analog signal from its samples can happen using

a. Highpass filter in the frequency domain
b. Convolution with a sinc function in the time domain
c. The inverse Fourier transform

Answer b

Quiz - question 4

What is the Nyquist rate for the analog signal

$$x_a(t) = 3\cos(50\pi t) + 10 sin(300\pi t) + cos(100\pi t)?$$

a. 300 Hz
b. 600 Hz
c. 100 Hz

Quiz - question 4

What is the Nyquist rate for the analog signal

$$x_a(t) = 3\cos(50\pi t) + 10 sin(300\pi t) + cos(100\pi t)?$$

a. 300 Hz
b. 600 Hz
c. 100 Hz

Answer a

Quiz - question 5

If a signal has a maximum frequency between 1000 Hz and 4000 Hz, which of the below is the most appropriate sampling rate?

a. 10000 Hz
b. 2000 Hz
c. 9000 Hz

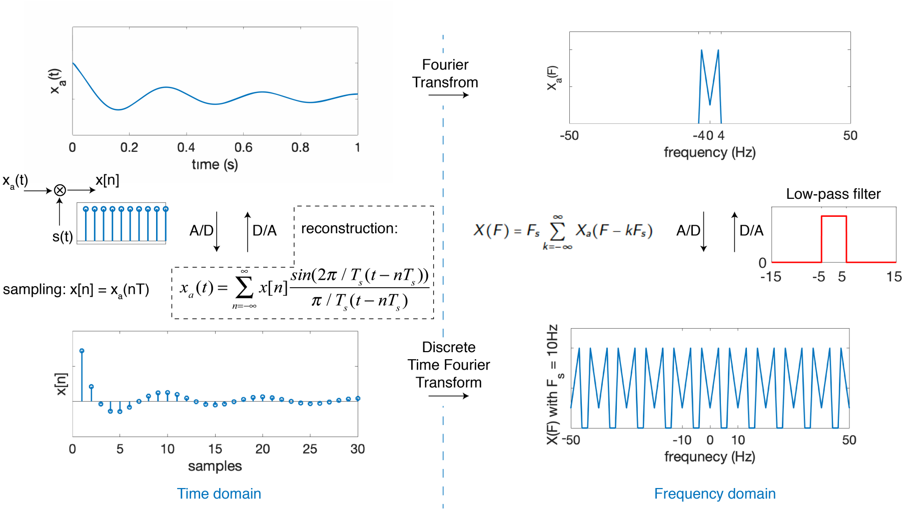
Quiz - question 5

If a signal has a maximum frequency between 1000 Hz and 4000 Hz, which of the below is the most appropriate sampling rate?

a. 10000 Hz
b. 2000 Hz
c. 9000 Hz

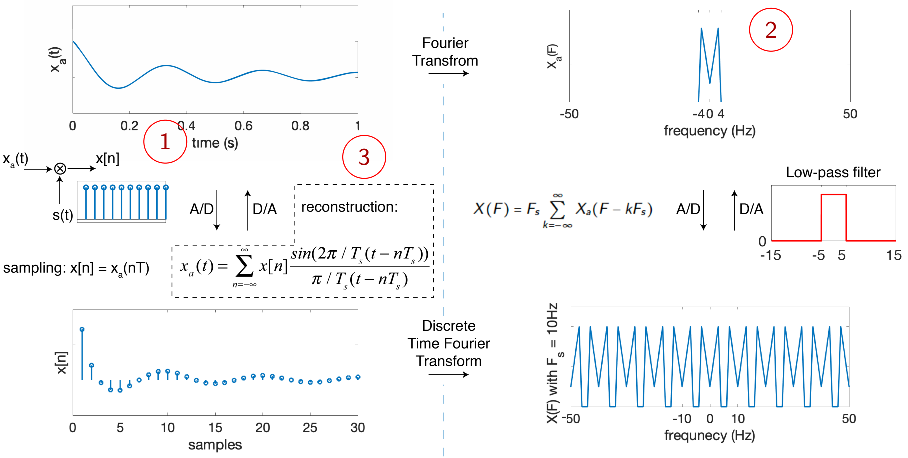
Answer c

Summary So Far



Nyquist Rate = Twice the maximum frequency

Non-ideal sampling and reconstruction



Time domain

Frequency domain

Sampling and reconstruction in practice

1. Delta pulse train for sampling: non-zero duration in practice
2. Signals often are non-low pass, non-bandlimited
    - How to sample non-bandlimited signals?
    - How to sample bandpass signals?
3. Sinc interpolation in practice is not possible: infinite length

Next Lecture: Non-ideal Cases

1. Delta pulse train for sampling: non-zero duration in practice
2. Signals often are non-low pass, non-bandlimited
   - How to sample non-bandlimited signals?
   - How to sample bandpass signals?
3. Sinc interpolation in practice is not possible: infinite length

Solve the following exercises from the book: 6.1, 6.2, 6.3, 6.4, 6.5 (solutions available on BrightSpace)