Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Partial exam EE2S31 SIGNAL PROCESSING
Part 1: May 27th 2019

Closed book; two sides A4 of handwritten notes permitted

This exam consists of four questions (34 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (10 points)

Given is the probability density function (pdf) $f_X(x)$ of the random variable X. That is,

$$f_X(x) = \begin{cases} \frac{1}{\lambda}e^{-\frac{1}{\lambda}x} & \text{for} \quad x \geq 0, \\ 0 & \text{otherwise.} \end{cases}$$

(2 p) (a) In a particular situation it is known that $X \geq 1$. Give the conditional pdf $f_{X|X\geq 1}(x)$.

(2 p) (b) Show that the moment generating function of the pdf $f_{X|X\geq 1}(x)$ is given by

$$\phi_{X|X\geq 1}(s) = \frac{\frac{1}{\lambda}e^s}{\frac{1}{\lambda} - s}, \quad \text{for} \quad s \leq \frac{1}{\lambda}.$$

(2 p) (c) Calculate $E[X|X \geq 1]$.

Now we consider the situation where the parameter λ in the pdf $f_X(x)$ is unknown. To estimate λ we define an estimator $\hat{\lambda}$ using N independently identically distributed random variables X, say $X_1, X_2, ..., X_N$, each having thus the distribution $f_X(x)$ given above. The estimator is given by $\hat{\lambda} = \frac{\sum_{n=1}^{N} X_n}{N}$.
Hint for questions 1d and 1e:
The moment generating function of pdf $f_X(x)$ is given by $\phi_X(s) = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}-s}$.

(2 p) (d) Argue whether or not $\hat{\lambda}$ is an unbiased estimator for λ.

(2 p) (e) Argue whether or not $\hat{\lambda}$ is a consistent estimator for λ.

Question 2 (9 points)

Given a real analog input signal $x(t)$ with a spectrum that is nonzero only at frequencies $3kHz \leq |F| \leq 4kHz$ and $4.5kHz \leq |F| \leq 6kHz$.

(1 p) (a) What is the Nyquist sampling rate?

(1 p) (b) We know that the frequencies at and above 4.5kHz contain only noise, all useful information is found below. Therefore, we can pass $x(t)$ through a filter before sampling. What is the frequency response of the (ideal) filter?

(1 p) (c) After filtering, what is the minimum sampling rate that still enables the exact reconstruction of the signal?

(3 p) (d) How can we reconstruct $x(t)$ from the samples $x(nT_s)$ in the ideal case? Why and how does practical (non-ideal) reconstruction differ from this? What is the effect of non-ideal reconstruction in frequency domain? Explain!

(1 p) (e) Realising the analog prefilter in (b) with a sharp cut-off is costly. Give the frequency response of a non-ideal filter that can achieve the same desired effect for the given signal.

(2 p) (f) Let's assume that we cannot afford to use a prefilter at all before sampling. What is the minimum sampling rate, such that we can still reconstruct the useful part of the signal?

Question 3 (7 points)

Given is the joint probability density function (pdf)

$$f_{X,Y}(x,y) = \begin{cases} c(y+x) & \text{for} \quad -1 \leq x \leq y \leq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(2 p) (a) Calculate the value of constant c.

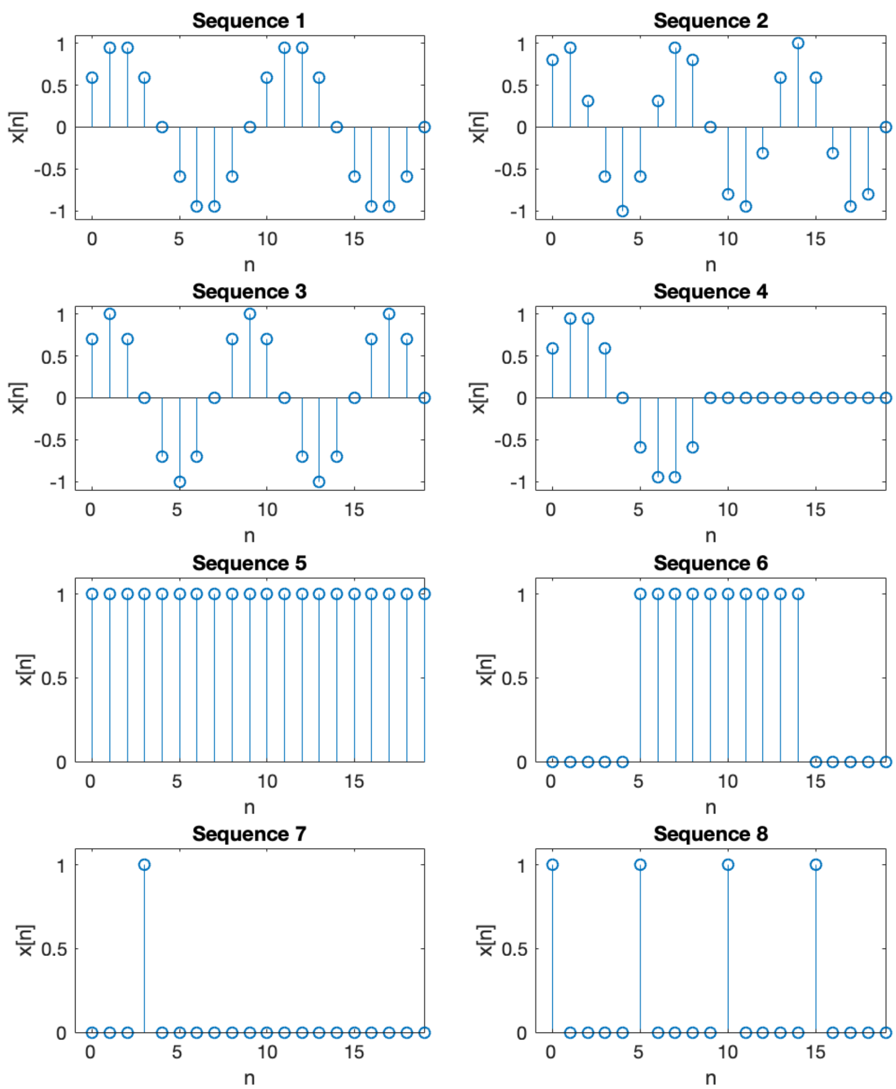
(3 p) (b) Calculate the marginal PDFs $f_X(x)$ and $f_Y(y)$ and argue whether or not random variables X and Y are independent.

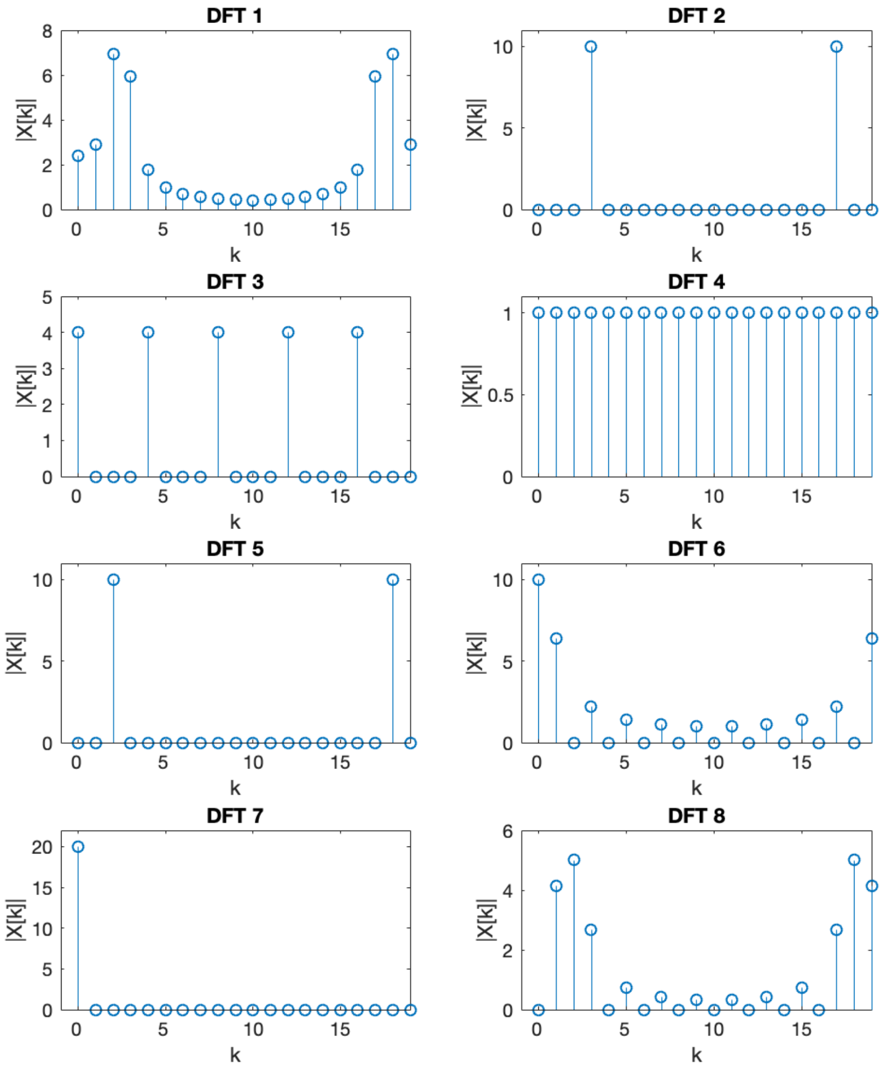
(2 p) (c) Calculate $E[Y|X]$.

Question 4 (8 points)

The figures below visualize discrete time sequences, and their corresponding DFTs in randomized order. Match the sequences with the DFTs by filling in the table below:

Sequence #	DFT #
1	
2	
3	
4	
5	
6	
7	
8	




DFT 1
DFT 2
DFT 3
DFT 4
DFT 5
DFT 6
DFT 7
DFT 8
|X[k]|
k