Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Partial exam EE2S31 SIGNAL PROCESSING
Part 2: July 3, 2018

Closed book; two sides of one A4 of handwritten notes permitted

This exam consists of four questions (33 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (8 points)

Given is the stochastic process $X(t) = A|t|$ with random variable A uniformly distributed in the (continuous) interval $[2, 4]$.

(1 p) (a) Sketch three different realizations of process $X(t)$.

3 p) (b) Calculate the cumulative distribution function (CDF) $F_X(t)(x)$, as well as the probability density function (PDF) $f_X(t)(x)$ for $t \neq 0$.

(1 p) (c) Calculate the expected value $E[X(t)]$.

(2 p) (d) Calculate the autocorrelation function $R(t, \tau)$.

(1 p) (e) Argue whether or not process $X(t)$ is stationary, and, whether or not process $X(t)$ is wide sense stationary (WSS).

Question 2 (8 points)

For this question you might want to make use of Table 1, included at the end of this exam.

Given is the time continuous WSS procecss $X(t)$ with autocorrelation function

$$R_X(\tau) = e^{-|\tau|100} + 4$$

and expected value $E[X(t)] = 2$. Process $X(t)$ is sampled with $f_s = 100$ samples per second, leading to the process $X[n]$.

(1 p) (a) Give the autocorrelation function $R_X[k]$ and expected value $E[X[n]]$ of the sampled process $X[n]$.

Given is a system with impulse response $h[n]$ and the abovementioned sampled process $X[n]$ as input. The output is denoted by $Y[n]$.

(1 p) (b) Assuming that $h[n]$ is of the form $h[n] = a^n u[n]$, with $u[n]$ the unit-step function, $|a| \leq 1$, and $E[Y[n]] = 8$, calculate constant a.

For the remaining part of this question, assume $R_X[k]$ is given by $R_X[k] = \left(\frac{1}{2}\right)^{|k|}$.

(2 p) (c) Give the magnitude response $|H(\phi)|$ that would completely decorrelate process $X[n]$.

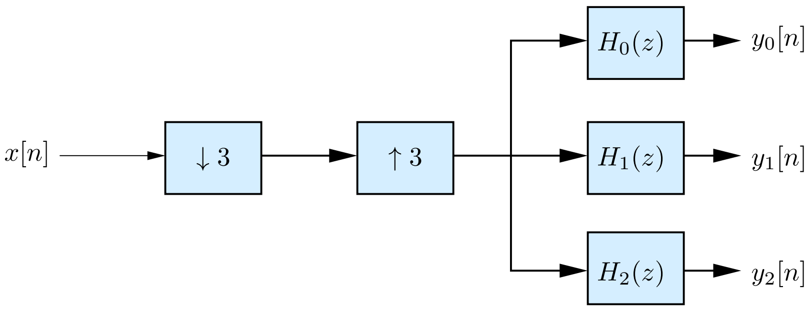
Now assume $h[n] = \delta[n-3]$.

(2 p) (d) Calculate the cross-correlation $R_{XY}[k]$ between input and output.

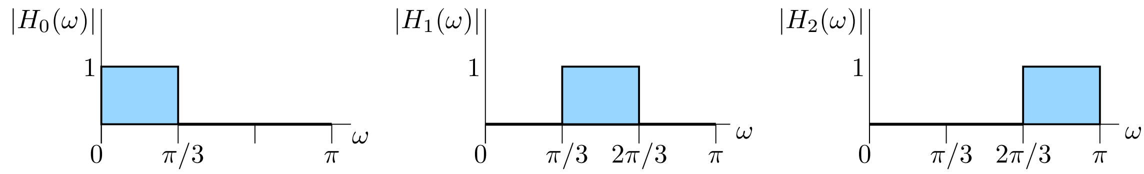
(2 p) (e) Calculate the autocorrelation $R_Y[k]$ of the output.

Question 3 (6 points)

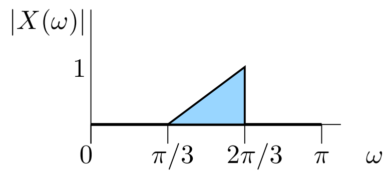
Consider the following multirate filter structure:



in which $H_0(z)$, $H_1(z)$ and $H_2(z)$ are a lowpass, bandpass, and highpas filter, respectively, as follows:



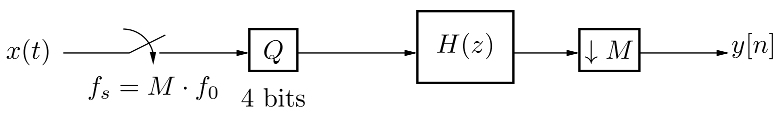
The real-valued input signal $x[n]$ has the following amplitude spectrum:



Draw the amplitude spectra of the outputs $y_0[n]$, $y_1[n]$, $y_2[n]$.

Question 4 (11 points)

An oversampling AD converter operates at $M = 256$ times the desired sampling rate $f_0 = 100$ kHz, en quantizes samples at 4 bits. Digitally, the sample rate is reduced by a factor $M = 256$. See figure:



The filter is given by $H(z) = 1 + z^{-1} + \cdots + z^{-(M-1)}$.

a Draw the amplitude spectrum of $H(z)$ (clearly mark the frequency axis).

In this plot, what is the location (value of ω) of the first zero of the transfer function?

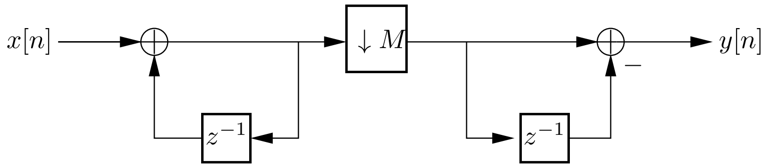
b What is the role of this filter?

If it was an ideal filter, what would be its specification?

c What is the effect of this filter on the quantization noise?

d Assume that the quantization noise is white. How many bits accuracy do you expect at the output? (Why?)

e An efficient implementation of the filter, in combination with the downsampler, is as follows:



Prove that this implementation indeed results in the desired response.

f Give an advantage and a disadvantage of this filter, compared to a more general Mth order FIR filter.

g To obtain a better filter response, we now use two of such filters $H(z)$ in cascade. Draw a block diagram of an *efficient* implementation (in combination with the downsampler).

Discrete Time function	Discrete Time Fourier Transform
$\delta[n] = \delta_n$	1
1	$\delta(\phi)$
$\delta[n-n_0] = \delta_{n-n_0}$	$e^{-j2\pi\phi n_0}$
$u[n]$	$\frac{1}{1-e^{-j2\pi\phi}} + \frac{1}{2}\sum_{k=-\infty}^{\infty}\delta(\phi+k)$
$e^{j2\pi\phi_0 n}$	$\sum_{k=-\infty}^{\infty}\delta(\phi-\phi_0-k)$
$\cos 2\pi\phi_0 n$	$\frac{1}{2}\delta(\phi-\phi_0)+\frac{1}{2}\delta(\phi+\phi_0)$
$\sin 2\pi\phi_0 n$	$\frac{1}{2j}\delta(\phi-\phi_0)-\frac{1}{2j}\delta(\phi+\phi_0)$
$a^n u[n]$	$\frac{1}{1-ae^{-j2\pi\phi}}$
$a^{\lvert n\rvert}$	$\frac{1-a^2}{1+a^2-2a\cos 2\pi\phi}$
g_{n-n_0}	$G(\phi)e^{-j2\pi\phi n_0}$
$g_n e^{j2\pi\phi_0 n}$	$G(\phi-\phi_0)$
g_{-n}	$G^*(\phi)$
$\sum_{k=-\infty}^{\infty} h_k g_{n-k}$	$G(\phi)H(\phi)$
$g_n h_n$	$\int_{-1/2}^{1/2} H(\phi')G(\phi-\phi')\,d\phi'$

Table 1. Discrete-Time Fourier transform pairs and properties. (from Signal Processing Supplement of "probability and stochastic processes"by Yates and Goodman).