Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

EE2S11 SIGNALS AND SYSTEMS

Part 2 exam, 27 January 2022, 13:30–15:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of five questions (35 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each answer sheet.

Question 1 (9 points)

(a) Let $x[n] = u[n]$, a unit step function, and let $h[n] = [\cdots, 0, \boxed{0}, 1, -1, 0, 0, \cdots]$, where the 'box' denotes the value for $n = 0$. Determine the convolution $y[n] = x[n] * h[n]$.

(b) Determine the z-transform for the following discrete-time signal, also specify the ROC:

$$x[n] = u[n] + \left(\tfrac{1}{2}\right)^n u[n-2]\,.$$

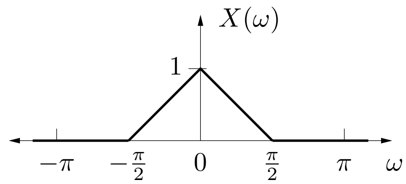
(c) Given the transfer function

$$H(z) = \frac{z^{-1}(1 - z^{-1})}{1 + 2\,z^{-1}}\,.$$

Assume the system is stable. Specify the ROC and determine $h[n]$.

(d) Determine the frequency response for $H(z)$ in (c).

(e) The signal $x[n]$ is given by its DTFT (assume that $X(\omega)$ is real-valued):



Determine and draw the DTFT of $y[n] = x[n]\cos(\frac{\pi}{4}n)$.

Question 2 (7 points)

Given is the difference equation of a causal system:

$$y[n] = x[n] + x[n-1] - 0.81\,y[n-2]\,.$$

(a) Determine the corresponding transfer function $H(z)$, also specify the ROC.

(b) Determine the poles and zeros of the transfer function (also those at $z = 0$ and $z = \infty$) and draw the corresponding pole-zero plot.

(c) Based on the pole-zero plot, give a sketch of the amplitude spectrum $|H(e^{j\omega})|$.

(d) Is $H(z)$ a stable transfer function? (Why?)

Question 3 (6 points)

A continuous-time signal $x_a(t)$ has a Fourier transform $X_a(\Omega) = \delta(\Omega + 1) + \delta(\Omega - 1)$.

(a) Determine $x_a(t)$.

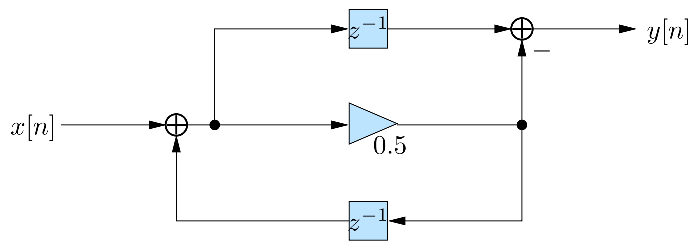
(b) What is the largest value of the sampling period T_s that would not cause aliasing when sampling $x_a(t)$?

(c) We sample the signal at $T_s = \pi$. Draw the sampled signal $x[n]$ (also specify the values on the axes).

(d) Determine and draw the corresponding spectrum $X(\omega)$ (also specify the values on the axes).

Question 4 (6 points)

Given the realization of a causal system:



(a) Determine the transfer function $H(z)$ of this realization.

(b) Is this a minimal realization? (Why?)

(c) Draw the "direct form no. 2" realization.

Question 5 (7 points)

A "template" third-order Butterworth filter has the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}.$$

The corresponding amplitude response is $|H(j\Omega)|^2 = \dfrac{1}{1 + \Omega^6}$.

(a) Which frequency transform should we apply to the template to construct a *high*-pass Butterworth filter with a 3dB cut-off frequency of Ω_c?

(b) What is the corresponding transfer function $G(s)$?

We aim to design an analog 3rd order high-pass Butterworth filter $G(s)$ with a pass-band frequency of 6 rad/s, a stop-band frequency of 3 rad/s and a maximal damping in the pass-band of 0.5 dB.

(c) Give a suitable expression for the amplitude response $|G(j\Omega)|^2$ of this filter and determine its parameters.

(d) For this filter, what is the minimal damping in the stop-band ?

(e) Which transform should be applied to the template $|H(j\Omega)|^2$ to obtain this filter?

Using this, determine the transfer function $G(s)$ of the high-pass filter.