Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science

Partial exam EE2S11 Signals and Systems
Part 2: 1 February 2019, 13:30–15:30

Closed book; two sides of handwritten notes permitted

This exam consists of five questions (40 points)

Question 1 (12 points)

a) Given the signal $x[n] = [\cdots, 0, \boxed{0}, 1, 2, 0, \cdots]$.

Determine $y[n] = x[n] * x[-n]$ using the convolution sum (in time-domain).

b) Also determine $y[n]$ in a) via the z-transform (do you obtain the same result?).

c) Given $x[n] = (n-1)\, u[n]$. Determine $H(z)$, also specify the ROC.

d) Given $X(z) = \dfrac{4z}{(z-1)(z+0.25)}$, ROC $= \{|z| > 1\}$.

Determine $x[n]$ using the inverse z-transform.

e) Let $x[n] = u[n+2] - u[n-3]$. Determine the DTFT $X(e^{j\omega})$.

f) Suppose the DTFT of a signal $x[n]$ is $X(e^{j\omega})$. What is the DTFT of $\cos(3n) \cdot x[n]$?

Question 2 (6 points)

A causal system is specified by the transfer function

$$H(z) = \frac{z^2+1}{(z-0.9)(z+0.9)} \,.$$

a) Determine all poles and zeros of the system and draw a pole-zero plot.

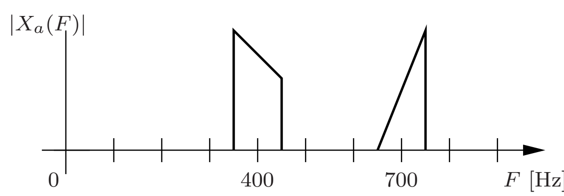
b) What is the ROC?

c) Is this a stable system?

d) Sketch the amplitude spectrum $|H(e^{j\omega})|$, also indicate values on the frequency axis.

Question 3 (7 points)

A real-valued continuous-time signal $x_a(t)$ has frequency components around 400 Hz and 700 Hz, as shown in the figure (the bands are 100 Hz wide).



a) At which frequency should we at least sample to avoid loss of information or distortion?

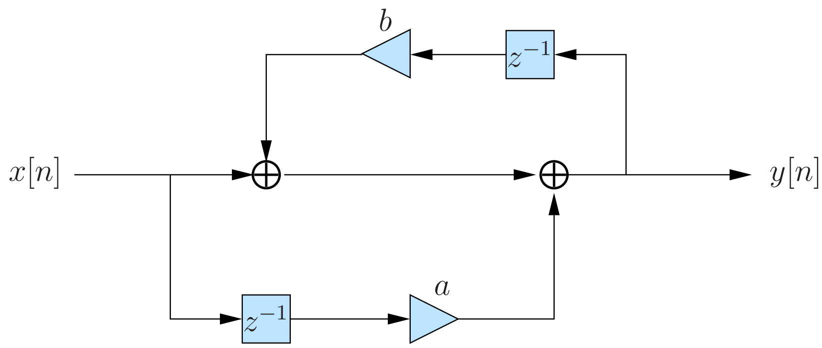
b) The signal is sampled at $F_s = 1000$ Hz, resulting in $x[n]$, no filtering is applied. Draw the amplitude spectrum of $x[n]$, also indicate the frequency axis for ω and relate it to the corresponding frequencies in Hz.

c) Is it possible to reconstruct the original signal $x_a(t)$ from the sampled signal? (Motivate your answer.)

d) Discuss what happens if $F_s = 1100$ Hz.

Question 4 (5 points)

a) Determine the transfer function $H(z)$ of the following realization:



b) Is this a minimal realization? (Why?)

c) Draw the "Direct form no. II" realization and also specify the coefficients.

Question 5 (10 points)

Design a first-order digital lowpass filter $H(z)$ satisfying the following specifications:

Passband frequency: $\omega_p = 0.3\pi$,
Damping outside the passband: at least 10 dB

Use the bilinear transform and base your design on an analog Butterworth filter.

a) What is the passband frequency in the analog frequency domain?

b) What is the generic form of $|H_a(j\Omega)|^2$ of a first-order analog Butterworth filter? What is the corresponding $H_a(s)$?

c) Determine $|H_a(j\Omega)|^2$ that meets the specifications.

d) What is the corresponding analog filter $H_a(s)$ that meets the specifications?

e) What is the desired digital filter $H(z)$?

f) Demonstrate (verify) that the resulting $H(z)$ meets the specifications.