Partial exam EE2S11 Signals and Systems
Part 2: 2 February 2018, 13:30–15:30

Closed book; two sides of handwritten notes permitted

This exam consists of five questions (40 points)

Question 1 (10 points)

a) Given the signals $x[n] = \delta[n+1] - 2\delta[n-2]$ and $h[n] = [\cdots, 0, \boxed{1}, 3, 2, 0, \cdots]$.
Determine $y[n] = x[n] * h[n]$ using the convolution sum (in time-domain).

b) Given $x[n] = -(\frac{1}{2})^{-n-1}u[-n-1]$. Determine $X(z)$ and also specify the ROC.

c) Given $X(z) = \dfrac{4z^{-1}}{(1-z^{-1})(1+0.25z^{-1})}$, ROC $= \{|z| < .25\}$.
Determine $x[n]$ using the inverse z-transform.

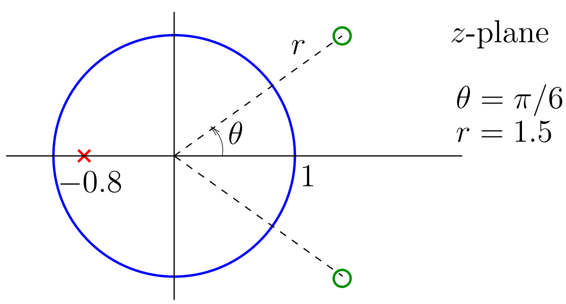
d) Given $x[n] = a^n u[n]$ with $|a| < 1$. Use the z-transform to determine $E = \sum_{n=0}^{\infty}(x[n])^2$.

e) Suppose the DTFT of a signal $x[n]$ is $X(e^{j\omega})$. What is the DTFT of $x[n-3]$?

f) Let $h[n]$ be the impulse response of an ideal low-pass filter with cut-off frequency at 0.4π. Let the impulse response of a new filter be $h_1[n] = (-1)^n h[n]$.
Determine the frequency response $H_1(e^{j\omega})$ in terms of $H(e^{j\omega})$, and give a sketch of it.

Question 2 (8 points)

A causal system $H(z)$ has the following pole-zero diagram:



a) What does the fact that $H(z)$ is a causal system tell you on the ROC of $H(z)$?

b) Specify $H(z)$. (Assume a gain such that $H(z) = 1$ for $z = 1$.)

c) Is this a stable system?

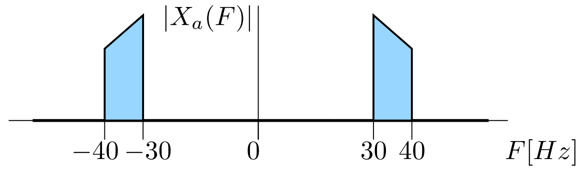
d) Sketch the amplitude spectrum $|H(e^{j\omega})|$, also indicate values on the frequency axis.

e) Give a pole-zero diagram of the inverse system, $G(z) = [H(z)]^{-1}$. Is this a causal stable system?

Question 3 (5 points)

A continuous-time signal $x_a(t)$ has frequencies in the range 30 to 40 Hz. The signal is sampled with period T_s so that we obtain a discrete-time signal $x[n] = x_a(nT_s)$.

The amplitude spectrum $|X_a(F)|$ of $x_a(t)$ is as follows (with F in hertz, using $\Omega = 2\pi F$):



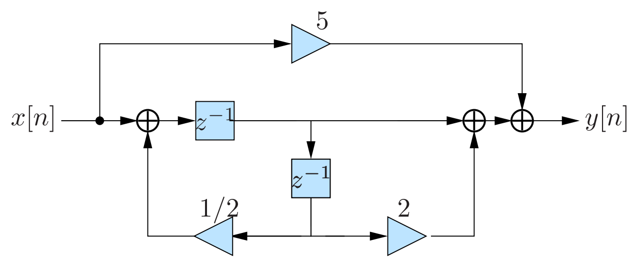
a) What is the Nyquist frequency at which $x_a(t)$ should be sampled to avoid aliasing?

b) We sample at a rate of 40 Hz. Give a drawing of the amplitude spectrum $|X(\omega)|$ of the signal $x[n]$. Also indicate the frequency axis for ω and relate it to the corresponding frequencies in Hz.

c) Is it possible to reconstruct $x_a(t)$ from $x[n]$? If not, why not? If yes, indicate how this could be done. (Assume ideal D/A converters and ideal low-pass filters.)

Question 4 (6 points)

a) Determine the transfer function $H(z)$ of the following realization:



b) Is this a minimal realization?

c) Draw the "Direct form no. II" realization and also specify the coefficients.

Question 5 (11 points)

We would like to design a *digital high*-pass filter with the following specifications:

Pass-band: starting at 5.0 kHz	Ripple in the pass-band : ≤ 0.5 dB
Stop-band: below 3.0 kHz	Stop-band damping: ≥ 30 dB
Sample rate: 20 kHz	

The digital filter is designed by applying the bilinear transform to an analog transfer function.

a) What are the pass-band and stop-band frequencies (in rad) in the digital time-domain?

b) What are the filter specifications in the analog time-domain?

c) Give a template expression for the amplitude response of an n-th order analog low-pass Butterworth filter.

d) What frequency transformation is needed to transform this into an analog high-pass filter? What is the resulting template expression for the analog high-pass filter, $|G(j\Omega)|^2$?

e) Use the design specifications to compute the unknown parameters of $|G(j\Omega)|^2$.

f) It is known that the poles of a low-pass Butterworth filter are all located on a (semi-)circle in the complex s-plane. What can you say about the poles and zeros of the high-pass filter, as a result of the lowpass-to-highpass transformation?

g) Is the bilinear transform suitable for transforming a high-pass analog filter? (Why?)

h) What can you say on the location of the zeros of the digital filter, resulting after the bilinear transformation?