

Partial exam EE2S11 Signals and Systems Part 2: 3 February 2017, 13:30–15:30

Closed book; two sides of handwritten notes permitted

This exam consists of five questions (40 points)

Question 1 (14 points)

- a) Given the signals $x[n] = u[n + 2] - u[n - 2]$, $h[n] = [\dots, 0, \boxed{1}, -2, 0, 0, \dots]$.

Determine $y[n] = x[n] * h[n]$ using the convolution sum (in time-domain).

- b) Given $x[n] = u[n] - (\frac{1}{2})^n u[n - 4]$. Determine $X(z)$ and also specify the ROC.

- c) Given $X(z) = \frac{4z^{-1}}{(1 - z^{-1})(1 + 0.25z^{-1})}$, ROC = $\{0.25 < |z| < 1\}$.

Determine $x[n]$ using the inverse z -transform.

- d) Given $x[n] = a^n u[n]$ with $|a| < 1$. Determine $y[n] = x[n] * x[-n]$. (Use the z -transform.)

- e) Determine, if it exists, the frequency response $H(e^{j\omega})$ for the system defined by the difference equation

$$y[n] = 1.6y[n - 1] - 0.64y[n - 2] + x[n] - x[n - 2]$$

- f) Given an LTI system with transfer function $H(z) = 1 - 2z^{-1}$.

Determine a (bounded) input signal $x[n]$ for which the output signal is equal to $y[n] = \delta[n] + \frac{1}{2}\delta[n - 1]$.

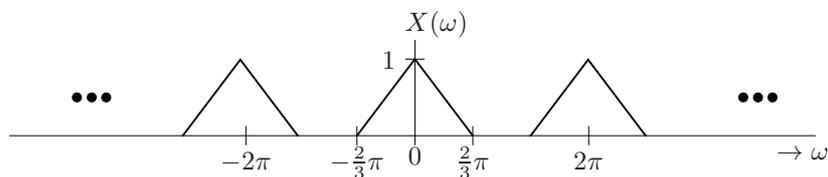
Question 2 (6 points)

The transfer function of a causal LTI system is given by $H(z) = \frac{z - 1}{z(z + 0.9)}$

- Determine all poles and zeros of the system and make a drawing in the complex z -plane.
- Specify the ROC.
- Is the system BIBO stable? (Why?)
- Draw, based on the poles and zeros of $H(z)$, the amplitude response. Is this a low-pass, high-pass or other kind of filter?

Question 3 (7 points)

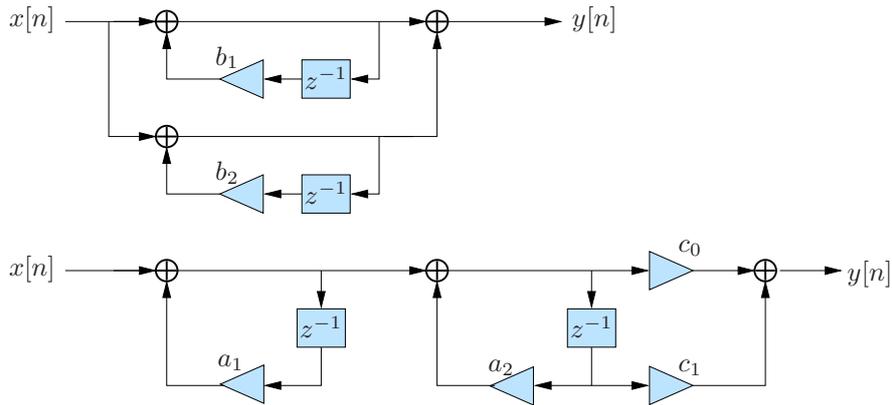
An analog signal $x_a(t)$ with Fourier transform $X_a(\Omega)$ is band-limited at 10 kHz. The signal is sampled without aliasing at a sampling frequency F_s , resulting in the discrete-time signal $x[n]$. The spectrum $X(\omega)$ of $x[n]$ is shown below:



- What is the relation between Ω and ω ?
- Which sampling frequency was used?
- What is the smallest frequency at which we can sample $x(t)$ without aliasing?
For this case, draw the resulting spectrum (also clearly mark the frequencies).
- $x_a(t)$ is reconstructed from $x[n]$ by means of an ideal D/A convertor and a low-pass filter.
Specify the pass-band and stop-band frequencies of the filter.

Question 4 (6 points)

Given the realizations:



- Determine a_1 , a_2 and c_0 , c_1 in terms of b_1 , b_2 such that both systems are equivalent.
- Is this a minimal realization?
- Draw the “direct form no. II” realization and also specify the coefficients.

Question 5 (7 points)

A “template” third-order Butterworth filter has the transfer function

$$H(s) = \frac{1}{s^3 + 2s^2 + 2s + 1}$$

The corresponding frequency response is $|H(j\Omega)|^2 = \frac{1}{1 + \Omega^6}$.

- Which frequency transform should we apply to the template to construct a low-pass Butterworth filter with a 3dB cut-off frequency of Ω_c ?
- What is the corresponding transfer function $G(s)$?

We now design an analog 3rd order low-pass Butterworth filter with a pass-band frequency of 3 rad/s, a stop-band frequency of 6 rad/s and a maximal damping in the pass-band of 0.5 dB.

- Give a suitable expression for the frequency response (squared-amplitude) of this filter and determine its parameters.
- For this filter, what is the minimal damping in the stop-band ?
- Which transform should be applied to $|H(j\Omega)|^2$ to obtain this filter?
Determine the corresponding transfer function.