Partial exam EE2S11 Signals and Systems
Part 2: trial exam 14 January 2016 (2 hours)

Closed book; two sides of handwritten notes permitted.

This exam consists of five questions (45 points)

Question 1 (12 points)

Given the signals

$$x[n] = \begin{cases} 1, & 0 \le n \le 5, \\ 0, & \text{elsewhere} \end{cases} \qquad h[n] = [\cdots, 0, \boxed{1}, -2, 1, 0, \cdots]$$

a) Determine $y[n] = x[n] * h[n]$ using the convolution sum (in time-domain).

b) Determine the z-transforms $X(z)$ and $H(z)$, Also specify the regions of convergence (ROCs).

c) Determine $y[n] = x[n] * h[n]$ using the (inverse) z-transform.

d) Given

$$X(z) = \frac{1}{1 - 1\frac{1}{2}\, z^{-1} + \frac{1}{2}\, z^{-2}}\,, \qquad z \in \text{ROC}.$$

Determine $x[n]$ using the inverse z-transform if (d1) ROC: $|z| > 1$, (d2) ROC: $|z| < \frac{1}{2}$, (d3) ROC: $\frac{1}{2} < |z| < 1$.

e) Given $x[n] = (-1)^n u[n]$. Determine the DTFT $X(\omega)$.

f) Given $X(\omega) = \cos(\omega)$. Determine $x[n]$.

Question 2 (8 points)

The response of a causal LTI system is given by

$$H(z) = \frac{z+1}{(z+1)^2 + 1}, \quad z \in \text{ROC}$$

a) Determine all poles and zeros of the system, and draw them in the complex z-plane.

b) Specify the ROC.

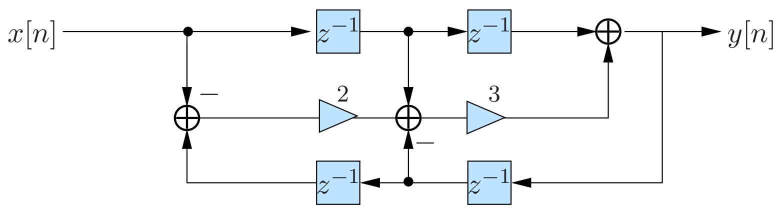
c) Is the system BIBO stable? (Why?)

d) Determine the frequency response of the system.

e) Determine the impulse response $h[n]$.

Question 3 (8 points)

Given the following system:



a) Determine the transfer functions $H(z)$ of the system.

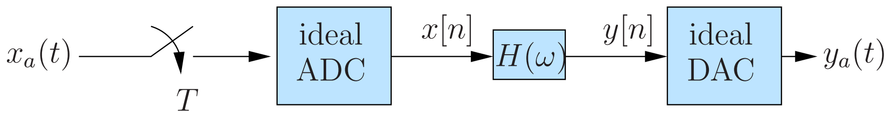
b) Is this a stable system? (Why?)

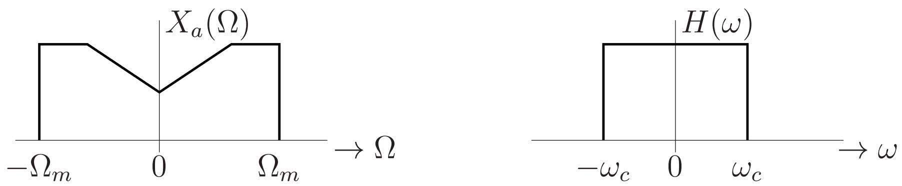
c) Is this a minimal realization? (Why?)

d) Draw the "Direct Form no. II" realization, and also specify the coefficients.

Question 4 (7 points)

A continuous-time signal $x_a(t)$ has a spectrum as indicated below. $x_a(t)$ is sampled with a frequency $1/T$ equal to the Nyquist rate, filtered by an ideal low-pass filter $H(\omega)$, and converted back to an analog signal $y_a(t)$. The cut-off frequency of $H(\omega)$ is $\omega_c = \Omega_m T/3$.





a) Give an expression for T.

b) Draw the spectrum corresponding to $x[n]$. Also mark the frequencies.

c) Draw the spectrum corresponding to $y[n]$. Also mark the frequencies.

d) Draw the spectrum corresponding to $y_a(t)$. Also mark the frequencies.

Question 5 (10 points)

We would like to design a digital low-pass filter with the following specifications:

– Ripple in the pass-band : ≤ 1 dB
– Pass-band: 4 kHz
– Stop-band damping: ≥ 40 dB
– Stop-band: starting at 6.0 kHz
– Sample rate: 24 kHz

The digital filter is designed by applying the bilinear transform to an analog transfer function.

a) What are the pass-band and stop-band frequencies in the digital time-domain?

b) What are the filter specifications in the analog time-domain?

c) Compute the required filter order for a Butterworth filter

d) Compute the required filter order for a Chebyshev filter
(Remark: $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$.)

e) Draw the frequency response of the resulting two digital filters after applying the bilinear transform. Also indicate the relation to the filter specifications.