Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

EE2S11 SIGNALS AND SYSTEMS

Midterm exam, 14 December 2022, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has five questions (18 points).

Question 1 (5 points)

A sliding window averager is a system with an input signal $x(t)$ and output signal

$$y(t) = \frac{1}{T_1 + T_2} \int_{\tau=t-T_1}^{t+T_2} x(\tau)\,\mathrm{d}\tau,$$

with $T_1 \geq 0$, $T_2 \geq 0$, and $T_1 + T_2 \neq 0$.

(a) Show that this system is linear and time invariant (LTI).

(b) Determine the transfer function $H(s)$ of this system.

(c) Determine the impulse response $h(t)$ of this system.

(d) For what value(s) of T_1 and T_2 is the system causal? Motivate your answer.

Answer

(a) Apply definitions.

(b)

$$H(s) = \frac{1}{T_1 + T_2} \frac{1}{s} \left(e^{sT_2} - e^{-sT_1} \right)$$

(c)

$$h(t) = \frac{1}{T_1 + T_2} [u(t + T_2) - u(t - T_1)]$$

(d) For $T_2 = 0$ and any $T_1 > 0$.

Question 2 (4 points)

Consider a SISO system with input signal $x(t)$ and output signal $y(t)$. The behavior of the system is described by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = x(t) \quad \text{for } t > 0^-$$

and the initial condition is $y(0^-) = 6$.

Let the input signal be given by $x(t) = \delta(t)$.

(a) Is the output signal $y(t)$ continuous at $t = 0$? Motivate your answer without computing $y(t)$ explicitly.

(b) Determine the output signal $y(t)$.

(c) Is the output signal equal to the impulse response $h(t)$ of the system? Motivate your answer.

Now let the input signal be given by $x(t) = 12e^{-2t}$ for $t > 0^-$.

(d) Is the output signal $y(t)$ continuous at $t = 0$? Motivate your answer without computing $y(t)$ explicitly.

(e) Determine the output signal $y(t)$.

Answer

(a) Right-hand side is a delta function. Left-hand side should have a delta function as well, which means that y must jump at $t = 0$. No, not continuous.

(b) $y(t) = 7e^{-3t}u(t)$. Note that $y(0^+) = 7$, $y(0^-) = 6$

(c) No. Initial condition does not vanish.

(d) Yes. If y is discontinuous at $t = 0$ then its derivative produces a delta function, but there is no delta function on the right-hand side.

(e) $y(t) = (12e^{-2t} - 6e^{-3t})u(t)$. Note that $y(0^+) = y(0^-) = 6$.

Question 3 (5 points)

The Laplace-domain signal

$$X(s) = \frac{s-1}{(s+1)^2(s-2)}$$

can be written in the form

$$X(s) = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s-2}.$$

(a) Determine A, B, and C.

Determine the corresponding time-domain signal $x(t)$ in case

(b) $\text{ROC}_x = \{s \in \mathbb{C}; \text{Re}(s) > 2\}$.

(c) $\text{ROC}_x = \{s \in \mathbb{C}; -1 < \text{Re}(s) < 2\}$.

(d) $\text{ROC}_x = \{s \in \mathbb{C}; \text{Re}(s) < -1\}$.

Answer

(a) $A = -1/9$, $B = 2/3$, $C = 1/9$.

(b)

$$x(t) = \left(-\frac{1}{9}e^{-t} + \frac{2}{3}te^{-t} + \frac{1}{9}e^{2t}\right)u(t)$$

(c)

$$x(t) = \left(-\frac{1}{9}e^{-t} + \frac{2}{3}te^{-t}\right)u(t) - \frac{1}{9}e^{2t}u(-t)$$

(d)

$$x(t) = \left(\frac{1}{9}e^{-t} - \frac{2}{3}te^{-t} - \frac{1}{9}e^{2t}\right)u(-t)$$

Question 4 (2 points)

The exponential Fourier series of a periodic signal $x(t)$ of fundamental period T_0 is given by

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{3}{4 + (k\pi)^2} e^{jk\pi t}.$$

(a) Determine the fundamental period T_0.

(b) Determine the average value (dc value) of $x(t)$.

(c) Is $x(t)$ even, odd, or neither? Motivate your answer.

(d) One of the frequency components of $x(t)$ can be expressed as $A\cos(3\pi t)$. Determine A.

Answer

(a) $T_0 = 2$

(b) Average value $= X_0 = 3/4$.

(c) $x(t)$ is even

(d) Relevant frequency component is obtained by adding the $k = -3$ and $k = 3$ terms in the Fourier expansion. $A = 6/(4 + 9\pi^2)$

Question 5 (2 points)

Suppose you have the Fourier series of two periodic signals $x(t)$ and $y(t)$ of fundamental periods T_1 and T_2, respectively. Let X_k and Y_k be the Fourier coefficients corresponding to $x(t)$ and $y(t)$.

(a) If $T_1 = T_2$, what are the Fourier coefficients Z_k of $z(t) = x(t) + y(t)$ in terms of X_k and Y_k?

(b) If $T_1 = 2T_2$, what are the Fourier coefficients Z_k of $z(t) = x(t) + y(t)$ in terms of X_k and Y_k?

Answer

(a) $Z_k = X_k + Y_k$

(b) $Z_k = X_k$, k odd, $Z_k = X_k + Y_{k/2}$, k even.