Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Circuits and Systems Group

EE2S11 SIGNALS AND SYSTEMS

Midterm exam, 8 December 2021, 13:30–15:30

Closed book; two sides of one A4 page with handwritten notes permitted. Graphic calculators not permitted. Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

This exam has three questions (30 points).

Question 1 (8 points)

(a) Given the signal $f(t) = u(t^2 - 4t)$, where u is the Heaviside unit step function. The derivative of f is of the form

$$\frac{\mathrm{d}f}{\mathrm{d}t} = A\,\delta(t-\alpha) + B\,\delta(t-\beta),$$

where A, α, B, and β are constants with $\alpha > \beta$. Determine the constants A, α, B, and β.

(b) Given the signals $v(t) = u(-t)$ and $w(t) = p(t)$, where u is the Heaviside unit step function and p the standard rectangular pulse:

$$p(t) = \begin{cases} 1 & \text{for } 0 < t < 1, \\ 0 & \text{for } t < 0 \text{ and } t > 1. \end{cases}$$

Determine the signal $z(t) = v(t) * w(t)$ by directly evaluating the convolution integral.

Answer

(a) $\alpha = 4$, $\beta = 0$, $A = 1$, $B = -1$.

(b)

$$z(t) = \begin{cases} 1 & t < 0, \\ 1 - t & 0 < t < 1, \\ 0 & t > 1 \end{cases}$$

Question 2 (11 points)

(a) The one-sided Laplace transform of a causal signal $f(t)$ is given by

$$F(s) = \frac{s^3 + 3s^2 + s + 8}{s^2 + 4s}, \quad \mathrm{Re}(s) > 0.$$

Determine $f(t)$.

(b) The one-sided Laplace transform of the signal

$$f(t) = (1 - e^{-t})^3 u(t),$$

is of the form

$$F(s) = \frac{C}{p(s)}, \quad \mathrm{Re}(s) > 0,$$

where C is a constant and $p(s)$ a polynomial in s. Determine C and $p(s)$.

(c) The two-sided Laplace transform of a noncausal signal $y(t)$ is given by

$$Y(s) = \frac{1}{s^2}\left(e^{-s} - 1\right), \quad \mathrm{Re}(s) < 0.$$

Plot $y(t)$.

(d) Determine the two-sided Laplace transform of $g(t) = t^2, \quad -\infty < t < \infty$.

Answer

(a) $f(t) = \delta'(t) - \delta(t) + 2u(t) + 3e^{-4t}u(t)$.

(b) $C = 6$ and $p(s) = s(s+1)(s+2)(s+3)$.

(c) $y(t) = z(t)$, from Problem 1b.

(d) Does not exist.

Question 3 (11 points)

Given the periodic signal $x(t)$ with fundamental period $T_0 = \pi$ and

$$x(t) = \cos(t), \quad 0 < t < \pi.$$

(a) Determine the average value of this signal.

(b) Expand $x(t)$ in a Fourier sine series.

(c) The Fourier coefficients of $x(t)$ decay as $1/k$ for $k \to \infty$. Explain why.

(d) The signal $y(t)$ has a fundamental period $T_0 = 2\pi$ and is given by

$$y(t) = \mathrm{sign}(t),$$

on the interval $(-\pi, \pi)$. Use the Fourier series of $y(t)$ to show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots .$$

Answer

(a) Average is $X_0 = 0$.

(b)

$$x(t) = \frac{8}{\pi} \sum_{k=1}^{\infty} \frac{k}{4k^2 - 1} \sin(2kt)\,.$$

(c) $x(t)$ is discontinuous at the end points.

(d)

$$y(t) = \frac{4}{\pi} \sum_{k=1, k \text{ odd}}^{\infty} \frac{1}{k} \sin(kt)\,, \qquad -\pi < t < \pi\,.$$

Substitute $t = \pi/2$ to obtain the desired result.