Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science

EE2S11 SIGNALS AND SYSTEMS

Resit exam, 10 July 2023, 13:30–16:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (31 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each answer sheet.

Question 1 (4 points)

Determine the one-sided Laplace transform of the following signals and specify their ROC as well. *Note:* $u(t)$ denotes the Heaviside unit step function.

(a) $f(t) = \delta(2t)$

(b) $g(t) = t\,u(t)$

(c) $m(t) = (t-1)u(t-1)$

(d) $n(t) = (t-1)u(t)$

Question 2 (4 points)

Find the inverse Laplace transform of the following signals:

(a) $F(s) = \dfrac{4s+20}{s^2+4s+13}, \quad \mathrm{Re}(s) > -2.$

(b) $G(s) = \dfrac{1}{(s^2+4)(s^2+9)}, \quad \mathrm{Re}(s) > 0.$

Question 3 (6 points)

Let $x(t)$ be a periodic signal with period $T_0 = 1$. A single period of $x(t)$ is denoted by $x_1(t)$ and is given by

$$x_1(t) = \begin{cases} \sin(2\pi t) & 0 \le t \le 0.5 \\ 0 & 0.5 < t \le 1. \end{cases}$$

(a) Determine the one-sided Laplace transform of $x_1(t)$ and its ROC.

(b) Determine the DC component of the periodic signal $x(t)$.

(c) Determine the Fourier coefficient X_1 of the periodic signal $x(t)$.

(d) Determine the Fourier coefficients X_k for k even and $k \ge 2$.

(e) Determine the Fourier coefficients X_k for k odd and $k \ge 2$.

Question 4 (6 points)

(a) Given the signal $x[n] = [\cdots, 0, \boxed{3}, 2, 1, 0, 0, \cdots]$, where the 'box' denotes the value for $n = 0$. Determine $r[n] = x[n] * x[-n]$ using the convolution sum.

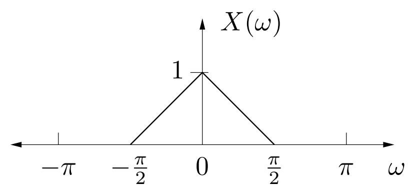
Determine the z-transform for the following discrete-time signal, and specify also the ROC:

(b) $x[n] = (2)^n \, u[-n]$,

Determine the signal $x[n]$ corresponding to the following z-transform:

(c) $X(z) = \dfrac{2z^2}{(z-1)(z-2)}, \quad \text{ROC} = \{1 < |z| < 2\}$.

The signal $x[n]$ is specified by its DTFT (here we assume that $X(\omega)$ is real-valued):



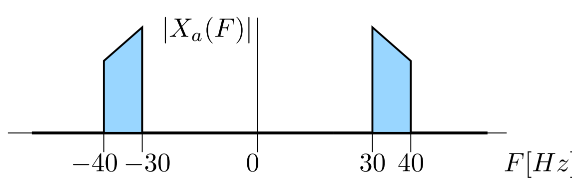
(d) Determine and draw the DTFT of $x_1[n] = x[n] \cos(\pi n/4)$.

Note: specify the DTFT in terms of $X(\omega)$.

Question 5 (3 points)

A continuous-time signal $x_a(t)$ has frequencies in the range 30 until 40 Hz. The signal is sampled with period T so that we obtain a time series $x[n] = x_a(nT)$.

The amplitude spectrum of $x_a(t)$ appears as follows:

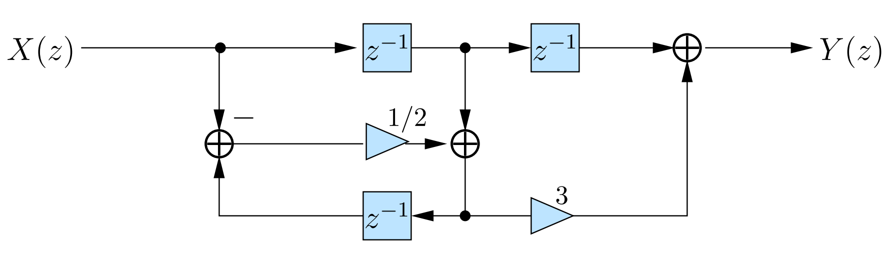


(a) What is the Nyquist frequency at which we would have to sample to avoid any aliasing?

(b) We sample the signal at 40 Hz. Make a drawing of the amplitude-spectrum $|X(\omega)|$ resulting from this sample frequency. Also mark the frequencies.

Question 6 (3 points)

Given the realization



(a) Determine the transfer function $H(z)$ corresponding to this realization.

(b) Is this a stable realization? (Why?)

(c) Is this a minimal realization? (Why?)

Question 7 (5 points)

Design an analog 3rd order low-pass Butterworth filter $H(\Omega)$ with a passband frequency of 2 rad/s, a stopband frequency of 4 rad/s and a maximal damping in the passband of 1 dB.

(a) What is the general expression for the frequency response (squared-amplitude) of a prototype Butterworth low-pass filter.

(b) Determine the unknown filter parameters such that the specifications are met.

(c) For this filter, what is the minimal damping in the stopband ?

(d) We wish to transform this filter to a *high*-pass filter $G(\Omega)$ with a passband frequency $\Omega'_p = 2$ rad/s. What transformation do we use, what is the resulting frequency response $|G(\Omega)|^2$, and what is the resulting stopband frequency?