Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

EE2S11 SIGNALS AND SYSTEMS

Resit exam, 11 July 2022, 13:30–16:30

Closed book; one A4 (two sides) of handwritten notes permitted. No other tools except a basic pocket calculator permitted.

This exam consists of seven questions (37 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each answer sheet.

Question 1 (6 points)

The behavior of a SISO LTI system is governed by the differential equation

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} + 2\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 2y(t) = x(t) + \frac{\mathrm{d}x(t)}{\mathrm{d}t},$$

where $x(t)$ is the input signal and $y(t)$ is the output signal.

(a) Determine the transfer function $H(s)$ and the impulse response $h(t)$ of the system.

(b) Determine the output signal $y(t)$ in case the input signal is given by

$$x(t) = (1 - t)e^{-t}u(t)$$

and the system is initially at rest.

(c) Determine $\lim_{t\to\infty} y(t)$ in case the input signal is given by

$$x(t) = 2u(t)$$

and the system is initially at rest.

Question 2 (6 points)

(a) The one-sided Laplace transform of a causal signal $x(t)$ is given by

$$X(s) = \frac{3s + 9}{s^2 + 2s + 10}, \qquad \mathrm{Re}(s) > -1.$$

Determine $x(t)$.

(b) Determine the inverse Laplace transform of the signal

$$Y(s) = \frac{2}{s^2(s-1)}, \quad \mathrm{Re}(s) > 1.$$

Question 3 (5 points)

Given the periodic signal $x(t)$ with fundamental period $T_0 = \pi$ and

$$x(t) = \cos(t), \quad 0 < t < \pi\,.$$

(a) Determine the average value of this signal.

(b) Expand $x(t)$ in a Fourier sine series.

(c) The Fourier coefficients of $x(t)$ decay as $1/k$ for $k \to \infty$. Explain why.

Question 4 (8 points)

(a) Given the signal $x[n] = [\cdots, 0, \boxed{1}, 2, 3, 0, 0, \cdots]$, where the 'box' denotes the value for $n = 0$. Determine $r[n] = x[n] * x[-n]$ using the convolution sum.

(b) Given an input signal $x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n]$, where $u[n]$ is the unit step function. Determine the z-transform $X(z)$, also specify the ROC.

(c) Let

$$X(z) = \frac{2z^2}{2z^2 + z - 1}, \qquad \text{ROC: } |z| > 1.$$

Compute the inverse z-transform $x[n]$.

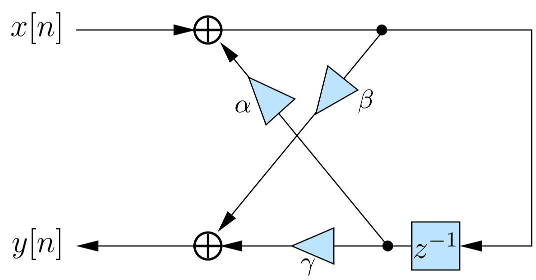
(d) Let $h[n] = [\cdots, 0, \boxed{1}, 2, 0, -2, -1, 0, \cdots]$.

Compute the DTFT $H(e^{j\omega})$ and subsequently determine the magnitude response $|H(e^{j\omega})|$.

(e) Compute all poles and zeros of $X(z)$ in (c) and draw a pole-zero plot.

Question 5 (4 points)

Consider this realization of a causal system:



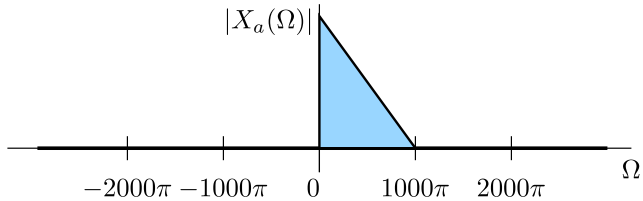
(a) Determine the transfer function $H(z)$.

(b) Under which conditions on the coefficients (α, β, γ) is this a stable system?

(c) Draw the "direct form no. II" realization, also specify the coefficients.

Question 6 (3 points)

A complex continuous-time signal $x_a(t)$ has amplitude spectrum as shown in the figure below. The signal is sampled with period $T = 2$ ms; the resulting signal is $x[n] = x_a(nT)$.



(a) Draw the amplitude spectrum $|X(\omega)|$ of $x[n]$. Also indicate values on the axes.

(b) Can we perfectly recover $x_a(t)$ from $x[n]$? (Why, or why not?)

Question 7 (5 points)

Consider an analog integrator with transfer function $H_a(s) = \frac{1}{s}$. We aim to implement this in the digital domain using the bilinear transformation.

(a) What is the resulting transfer function $H(z)$ if we apply the bilinear transformation?

(b) What is the corresponding impulse response $h[n]$? Is this indeed an integrator?

(c) Compute and draw the amplitude and phase response of both $H_a(s)$ and $H(z)$. Where do (don't) they match?

(d) More in general: what are two advantages of the filter design technique using the bilinear transformation?