Delft University of Technology
Faculty of Electrical Engineering, Mathematics, and Computer Science
Section Circuits and Systems

Exam EE2S11 Signals and Systems
Resit on complete course: 23 July 2019, 13:30–16:30

Closed book; one double-sided A4 page of handwritten notes permitted

This exam consists of seven questions (40 points). Answer in Dutch or English. Make clear in your answer how you reach the final result; the road to the answer is very important. Write your name and student number on each sheet.

Question 1 (5 points)

a) The one-sided Laplace transform of a causal signal $f(t)$ is given by

$$F(s) = \frac{5s + 13}{s(s^2 + 4s + 13)}, \qquad \mathrm{Re}(s) > 0.$$

Determine $f(t)$.

b) The one-sided Laplace transform of a causal signal $g(t)$ is given by

$$G(s) = \frac{s}{(s^2 + 9)(s + 2)}, \qquad \mathrm{Re}(s) > 0.$$

Determine $g(t)$.

Question 2 (7 points)

A signal $x(t)$ satisfies the differential equation

$$m\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{b^2}{4m}x(t) = 0 \qquad \text{for } t > 0,$$

where $m > 0$ and $b > 0$ are constants and the initial conditions are given by $x(0) = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t}(0) = v_0$.

a) Use the one-sided Laplace transform to determine the signal $x(t)$ for $t > 0$.

A signal $y(t)$ satisfies the differential equation

$$t\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mathrm{d}y}{\mathrm{d}t} + ty(t) = 0 \qquad \text{for } t > 0,$$

with $y(0) = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}t}(0) = 0$.

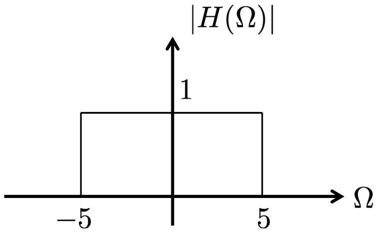
b) Assuming that $y(t)$ has a Laplace transform, show that it is given by

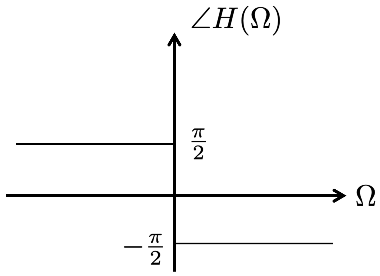
$$Y(s) = \frac{A}{\sqrt{s^2 + 1}},$$

where A is a constant.

c) Determine the constant A.

Question 3 (6 points)





Consider a continous-time LTI system of which the frequency response is shown above.

a) Calculate the impulse response of the system.

Assume that the input of the filter is a periodic signal $x(t)$ having a Fourier series representation

$$x(t) = \sum_{k=1}^{\infty} \frac{2}{k^2} \cos(2kt).$$

b) What can you say about the continuity and differentiability of the signal $x(t)$?

c) Indicate how the series converges to $x(t)$ (point-wise, in norm, etc). Motivate your answer.

d) Determine the steady-state response of the system.

Question 4 (5 points)

For a causal LTI system the response to an input signal $x = [\cdots, 0, \boxed{1}, 3, 3, 1, 0, \cdots]$ is given by $y = [\cdots, 0, \boxed{1}, 4, 6, 4, 1, 0, \cdots]$.

a) How are $x[n]$, $y[n]$ and the impulse response $h[n]$ of the system related? (Give a generic equation.)

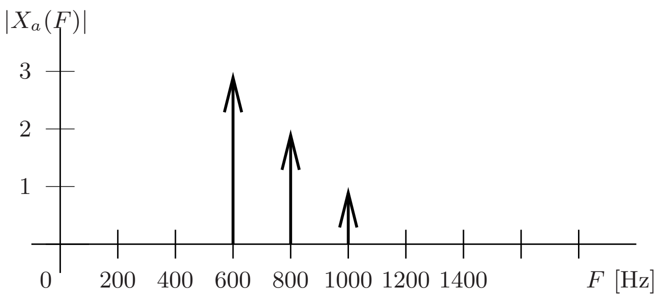
b) Determine the impulse response $h[n]$ of the system. (Hint: first determine the filter length.)

c) Determine the z-transform $X(z)$ and $Y(z)$ of $x[n]$ and $y[n]$ (also specify the ROC).

d) Compute $H(z)$ and verify your answer under b).

Question 5 (5 points)

The real-valued continuous-time signal $x_a(t)$ has frequency components as indicated below; the spectrum is real-valued.

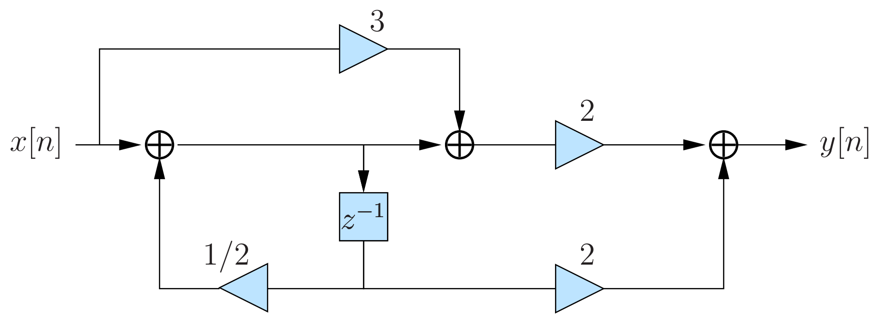


a) What is the (minimal) sampling frequency required to avoid aliasing?

b) The signal is sampled at $F_s = 1000$ Hz, resulting in $x[n]$, there is no filtering. What frequency components are present in the sampled signal?

c) Draw the amplitude spectrum of $x[n]$; clearly indicate the frequencies and amplitudes.

Question 6 (6 points)



a) Determine the transfer function $H(z)$ of the causal system shown above.

b) Determine its impulse response $h[n]$.

c) Is this a minimal system? (why)

d) Is this a stable system? (why)

e) Draw the "Direct form II" realization.

Question 7 (6 points)

We would like to design a *first-order* digital lowpass filter with the following specifications:

- Passband: until 7.2 kHz
- Damping outside the passband: at least 10 dB
- Sample rate: 48 kHz

The digital filter will be designed by applying the bilinear transform to an analog transfer function. We will use a Butterworth filter. The expression for the frequency response of a prototype n-th order low-pass Butterworth filter is

$$|H_a(\Omega)|^2 = \frac{1}{1 + \epsilon^2 (\Omega/\Omega_p)^{2n}}.$$

a) What is the passband frequency (in rad) in the digital time domain?

b) What is the passband frequency of the analog lowpass filter?

c) What is the frequency response $|H_a(\Omega)|^2$ for the analog filter that satisfies the specifications?

d) What is the transfer function $H_a(s)$ for that analog filter?

e) What is $H(z)$?

f) Demonstrate (verify) that the design of $H(z)$ satisfies the specifications.

Note: if you get stuck at some point, then make a reasonable assumption so you can continue with the rest of the questions.