Exam EE2S11 Signals and Systems
Resit on complete course: 24 July 2018, 13:30–16:30

Closed book; one double-sided A4 page of handwritten notes permitted

This exam consists of seven questions (40 points)

Question 1 (4 points)

Given the signal $f(t) = \begin{cases} 0, & \text{for } t < 0, \\ 2, & \text{for } t = 0, \\ 4, & \text{for } 0 < t < 2, \\ 3, & \text{for } t = 2, \\ 2, & \text{for } t > 2. \end{cases}$

a) Express $f(t)$ in terms of two unit step functions.

b) Determine the Laplace transform of $f(t)$ including its ROC.

c) Determine the derivative $g(t) = \frac{\mathrm{d}f}{\mathrm{d}t}$.

d) Determine the Laplace transform of $g(t)$ including its ROC.

Question 2 (4 points)

Given the signal $f(t) = \begin{cases} 0, & \text{for } t < 0, \\ \sin(t), & \text{for } 0 \le t \le \pi, \\ 0, & \text{for } t > \pi. \end{cases}$

a) Determine the Laplace transform of $f(t)$ including its ROC.

Now consider a signal $g(t)$ that vanishes for $t < 0$ and is periodic with a period 2π for $t > 0$, that is, $g(t + 2\pi) = g(t)$ for $t > 0$. On the interval $0 \le t \le 2\pi$, the signal $g(t)$ is given by

$$g(t) = \begin{cases} \sin(t) & \text{for } 0 \le t \le \pi, \\ 0 & \text{for } \pi < t \le 2\pi. \end{cases}$$

b) Determine the Laplace transform of $g(t)$ including its ROC.

Question 3 (7 points)

Consider the 2π-periodic signal $x(t)$, of which one period is described by $x(t) = \begin{cases} 1, & 0 \le t < \pi, \\ 0, & \pi \le t < 2\pi. \end{cases}$

a) Sketch the signal $x(t)$ (show at least three periods of x).

The Fourier series representation of a 2π-periodic signal $x(t)$ is given by

$$x(t) = c_0 + 2\sum_{k=0}^{\infty} \left(c_k \cos(kt) + d_k \sin(kt)\right), \quad \text{where} \quad \begin{cases} c_k = \dfrac{1}{2\pi}\displaystyle\int_{t_0}^{t_0+2\pi} x(t)\cos(kt)dt, & k = 0, 1, 2, \cdots \\ d_k = \dfrac{1}{2\pi}\displaystyle\int_{t_0}^{t_0+2\pi} x(t)\sin(kt)dt, & k = 1, 2, \cdots \end{cases}$$

b) Show that the Fourier coefficients c_k and d_k of the signal $x(t)$ specified above are given by

$$c_k = \begin{cases} 0, & k \neq 0, \\ \frac{1}{2}, & k = 0, \end{cases} \qquad \text{and} \qquad d_k = \frac{1}{2\pi k}\left(1 - (-1)^k\right) \text{ for all } k.$$

We can also express the signal $x(t)$ into its complex Fourier series representation, given by

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jkt}, \qquad \text{where} \qquad X_k = \frac{1}{2\pi}\int_{t_0}^{t_0+2\pi} x(t)e^{-jkt}dt, \quad k \in \mathbb{Z}.$$

c) What is the relation between the coefficients c_k, d_k and the coefficients X_k? Motivate your answer.

d) The Fourier coefficients X_k in this example are purely imaginary, except for $k = 0$, for which X_k is real. What is the reason that X_0 is real valued and that X_k is imaginary for $k \neq 0$? Motivate your answer.

e) The magnitude and phase of the Fourier coefficients X_k in this example are even and odd symmetric, respectively. That is, $|X_k| = |X_{-k}|$ and $\angle X_k = -\angle X_{-k}$. What is the reason for these specific symmetry properties? Motivate your answer.

Question 4 (6 points)

a) Determine the z-transform for the following discrete-time signal, also specify the ROC:

$$x[n] = \frac{1}{2}\left(1 + (-1)^n\right) u[n].$$

b) Let $x[n] = \alpha^n u[n]$, where $\alpha < 1$. Determine the convolution $r[n] = x[n] * x[-n]$.

c) Assuming $x[n]$ is causal, determine the signal $x[n]$ corresponding to the z-transform:

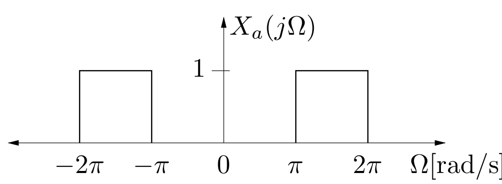
$$X(z) = \frac{z^{-2} + 10z^{-1}}{z^{-2} + 6z^{-1} + 8}.$$

d) Determine the signal $x[n]$ corresponding to the DTFT

$$X(e^{j\omega}) = e^{j\pi/4}\delta(\omega - 1) + e^{-j\pi/4}\delta(\omega + 1).$$

Question 5 (6 points)

The continuous-time signal $x_a(t)$ has a (real-valued) Fourier transform $X_a(j\Omega)$ as shown here:



a) Is this a bandlimited signal? What is its maximal frequency (in Hz)?

b) At what sampling period T_s should we sample this signal, avoiding aliasing?

We sample the signal at $T_s = \frac{1}{2}$ sec, and obtain the discrete-time signal $x[n]$. The DTFT spectrum of $x[n]$ is $X(e^{j\omega})$.

c) How is ω related to Ω?

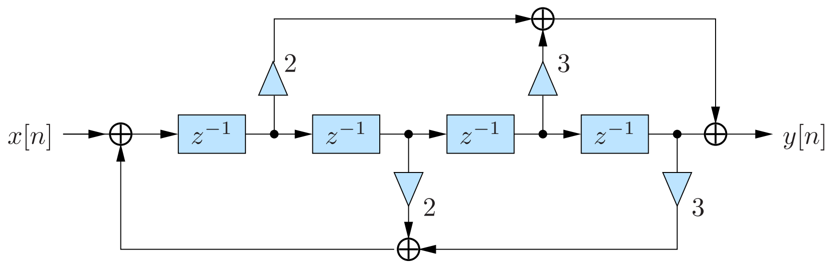
d) Draw $X(e^{j\omega})$; also indicate the values on both axes.

Consider now the signal $y_a(t) = (x_a(t))^2$.

e) Make a plot of $Y_a(j\Omega)$; also indicate the values on both axes.

Question 6 (6 points)

Consider the following system realization:



a) Determine the transfer function $H(z)$.

b) Determine all poles and zeros of $H(z)$.

c) Is this a stable system? (Why?)

d) Is this a minimal realization? (Why?)

Question 7 (7 points)

We would like to design a digital *high*pass filter with the following specifications:

– Passband: starting at 3.0 kHz; ripple in the passband: ≤ 1 dB

– Stopband: below 2.0 kHz; stopband damping: ≥ 40 dB

– Sample rate: 12 kHz

The digital filter will be designed by applying the bilinear transform on an analog transfer function. We will use a Chebyshev filter. The expression for the frequency response of a prototype n-th order low-pass Chebyshev filter is

$$|H(\Omega)|^2 = \frac{1}{1 + \epsilon^2 T_n^2(\Omega)} .$$

a) What are the passband and stopband frequencies (in rad) in the digital time domain?

b) What are the filter specifications in the analog time domain?

c) Which transformation will you use to transform a lowpass into a highpass filter? What are the specifications of the analog lowpass filter?

d) Compute the required filter order n for a Chebyshev filter.
(Remark: $\cosh^{-1}(x) = \ln(x + \sqrt{x^2 - 1})$.)

e) Suppose that the resulting Chebyshev lowpass filter has this form:

$$H(s) = \frac{B(s)}{A(s)} = \frac{1}{a_0 + a_1 s + \cdots + a_n s^n} .$$

How can you find the filter coefficients of the digital *high*pass filter?