Exam EE2S11 Signals and Systems
Resit on complete course: 25 juli 2016, 13:30–16:30

Closed book; one double-sided A4 page of handwritten notes permitted

This exam consists of seven questions (45 points)

Question 1 (4 points)

Which of the following statements are true? (Just provide the answers, no motivation necessary.)

a) If $x(t)$ is a causal signal, the Region of Convergence (ROC) of its Laplace transform lies to the right of a line through the rightmost singularity.

b) If $x(t)$ is an anti-causal signal, the ROC of its Laplace transform lies to the left of a line through the leftmost singularity.

c) If $x(t)$ is a noncausal signal and its Laplace transform exists, then its ROC is a strip in the s-plane parallel to the imaginary axis.

d) The ROC does not contain any singularities.

Question 2 (8 points)

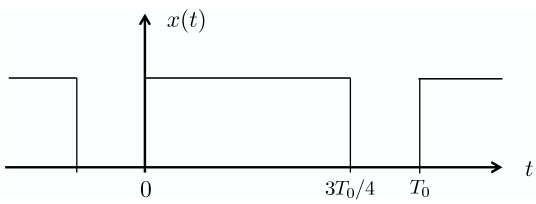
Determine the inverse Laplace transform of the following functions:

a) $X(s) = \dfrac{2s+1}{s^2+9}$, ROC: $\mathrm{Re}(s) > 0$

b) $Y(s) = \dfrac{s^2+2}{s^3-s}$, ROC: $\mathrm{Re}(s) > 1$

c) $Z(s) = \dfrac{2}{s^2(s-1)}$, ROC: $\mathrm{Re}(s) > 1$

Question 3 (8 points)



Consider the T_0-periodic signal $x(t)$ as depicted above. The Fourier series of $x(t)$ can be expressed as

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} \left(c_k \cos(k\Omega_0 t) + d_k \sin(k\Omega_0 t)\right), \qquad \Omega_0 = \frac{2\pi}{T_0},$$

where the coefficients c_k and d_k are given by

$$c_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\cos(k\Omega_0 t)dt, \quad k = 0, 1, 2, \cdots$$

$$d_k = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} x(t)\sin(k\Omega_0 t)dt, \quad k = 1, 2, \cdots$$

a) Compute the Fourier coefficients c_k and d_k.

b) The decay of the Fourier coefficients is $\mathcal{O}(1/k)$. Explain why this is the case.

c) What would be the effect on the Fourier coefficients c_k and d_k if we raise the signal $x(t)$ by a constant $c \in \mathbb{R}$? That is, if we consider the signal $x(t) + c$.

d) What would be the effect on the Fourier coefficients c_k and d_k if we shift the signal $x(t)$ by a constant $c \in \mathbb{R}$? That is, if we consider the signal $x(t - c)$.

e) Considering the shifted signal of item d), for what values of c do the coefficients c_k and/or d_k vanish? Motivate your answer.

Question 4 (10 points)

Determine the z-transform for the following discrete-time signals, also specify the ROC:

a) $x[n] = u[n] + \left(\frac{1}{2}\right)^n u[n-2]$,

b) $x[n] = 2\sin(\frac{\pi}{6}n)u[-n]$.

Determine the signals $x[n]$ corresponding to the z-transforms:

c) $X(z) = (1 - z^{-1})^2, \quad$ ROC: $z \neq 0$.

d) $X(z) = \dfrac{1.2}{(1 - z^{-1})(1 + 0.2z^{-1})}, \quad$ ROC: $0.2 < |z| < 1$.

Determine the frequency response $H(e^{j\omega})$ for the system defined by the difference equation:

e) $y[n] = 0.5y[n-1] + x[n] + x[n-1], \quad n \geq 0$.

Question 5 (4 points)

A continuous-time signal $x_a(t)$ has a Fourier transform $X_a(\Omega) = u(\Omega + 1) - u(\Omega - 1)$.

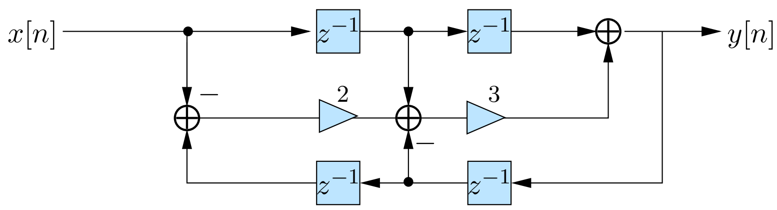
a) Determine $x_a(t)$.

b) What is the largest value of the sampling period T_s that would not cause aliasing when sampling $x_a(t)$?

c) We sample the signal at $T_s = \pi$. Draw the sampled signal $x[n]$ (also specify the values on the axes).

Question 6 (4 points)

Consider the following system realization:



a) Determine the transfer function $H(z)$.

b) Is this a stable system? (Why?)

c) Is this a minimal realization? (Why?)

Question 7 (7 points)

A second-order analog lowpass filter (Butterworth filter) has transfer function

$$G_a(s) = \frac{1}{s^2 + \sqrt{2}s + 1}\,.$$

The 3-dB cut-off frequency of this filter is $\Omega_c = 1$ rad/s.

Using the bilinear transform and the above filter as a template, we will now design a digital *high*-pass filter $H(z)$ with cut-off frequency $\omega_c' = \frac{2}{3}\pi$.

a) What should be the corresponding cut-off frequency in the analog frequency domain?

b) Which frequency transformation should be used?

c) What is $H_a(s)$?

d) What is $H(z)$?

e) Verify that the design meets the specifications.