Delft University of Technology
Faculty of Electrical Engineering, Mathematics and Computer Science

EE2S11 SIGNALS AND SYSTEMS

Part 1, 14 December 2017, 13:30 - 15:30

Closed book; two A4-size pages with handwritten notes permitted. Graphic calculators not permitted.

This exam has four questions (35 points)

Question 1 (8 points)

Let

$$p(t) = \begin{cases} 1 & \text{for } 0 < t < 1, \\ 0 & \text{elsewhere} \end{cases}$$

and consider the signal $x(t) = p(t) * p(t) * p(t)$, where the asterisk denotes convolution.

a) Determine $x(t)$ for $t > 3$.

The signal

$$f(t) = \begin{cases} 0 & \text{for } t < 0, \\ \sin(t) & \text{for } 0 \le t < \pi, \\ \pi/2 & \text{for } t = \pi, \\ t & \text{for } t > \pi, \end{cases}$$

can be written as

$$f(t) = \sin(t)u(t) + g(t - \pi)u(t - \pi),$$

where $u(t)$ is the unit step function.

b) Determine the signal $g(t)$.

c) Determine the one-sided Laplace transform of $f(t)$.

Question 2 (9 points)

a) Find the one-sided Laplace transform (including its ROC) of the signal

$$x(t) = \sin(t)\cos(t)u(t),$$

where $u(t)$ is the Heaviside unit step function.

b) Find the one-sided Laplace transform (including its ROC) of the signal

$$y(t) = t^2 \sin(2t)u(t).$$

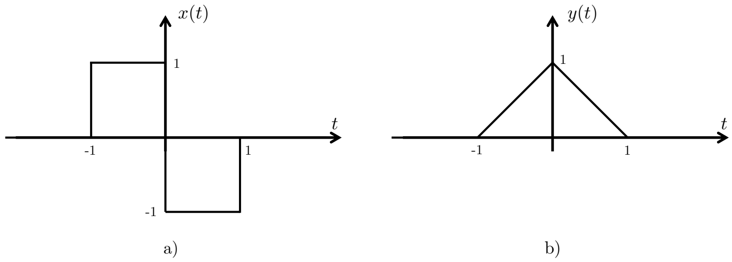
c) The one-sided Laplace transform of a causal signal $z(t)$ is given by

$$Z(s) = \frac{3s + 9}{s^2 + 2s + 10}, \qquad \text{Re}(s) > 0.$$

Determine $z(t)$.

Question 3 (12 points)

Consider the continuous-time signals $x(t)$ and $y(t)$ depicted below:



The Fourier transform of $x(t)$ is given by

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t}dt \tag{1}$$

and similarly for $y(t)$.

a) Without explicitly computing the Fourier transform, what can you say about the spectrum of $x(t)$ and $y(t)$. Is it discrete or continuous, is it real, imaginary of complex-valued, and do we have symmetry in the spectrum? Motivate your answer.

b) Compute the Fourier transform of $x(t)$ using (1).

c) Compute the Fourier transform of $x(t)$ using the Laplace transform.

d) What is the value $X(0)$ and what information does this value give you about $x(t)$?

Now assume that the shown signal $x(t)$ is the input to an LTI system of which the output is given by the signal $y(t)$.

e) Compute the Fourier transform $Y(\Omega)$ of $y(t)$.

f) What is the order of decay of both spectra? Explain which one decays fastest and why.

g) Give a sketch of both $|X(\Omega)|$ and $|Y(\Omega)|$.

h) Show that the frequency response of the system is given by

$$H(j\Omega) = \frac{1}{j\Omega}, \quad \Omega \in \mathbb{R} \setminus \{0\}.$$

Question 4 (6 points)

Consider a causal LTI system with transfer function

$$H(s) = \frac{s}{(s+1)^2+3}, \quad s \in \text{ROC}.$$

a) Determine the poles and zeros of the system and draw them in the complex s-plane. Is the system BIBO stable? Motivate your answer.

b) Give an expression for the frequency, magnitude and phase response of the system.

c) Sketch the magnitude and phase response and indicate the values of $|H(j\Omega)|$ and $\angle H(j\Omega)$ for the (angular) frequencies $\Omega = 0, \Omega = \pm\sqrt{3}$ and $\Omega = \pm\infty$.

d) How should we modify the transfer function if we want to have a faster decay at $\Omega = \pm\infty$? Motivate your answer.