7.9 Using the loss function

$$\alpha(\Omega) = 10\log_{10}\left(1 + \left[\frac{\Omega}{\Omega_{hp}}\right]^{2N}\right) \quad \Rightarrow \quad \Omega_{hp} = \frac{\Omega}{(10^{0.1\alpha(\Omega)} - 1)^{1/2N}}$$

which gives for $\Omega = 2000$

$$\Omega_{hp} = \frac{2000}{(10^{1.94} - 1)^{0.1}} = 1280.9 \text{ rad/sec.}$$

When $\alpha(\Omega_p) = \alpha_{max}$ we have

$$\Omega_p = \Omega_{hp}(10^{0.1\alpha_{max}} - 1)^{1/2N}$$

which gives after replacing the values on the right term $\Omega_p = 999.82$ (rad/sec).

7.11 The following script is used in finding the answers in this problem.

```
% Pr 7_11
clear all; clf
alphamax=0.5 ;alphamin=30; Wp=1500; Ws=3500;
% Butterworth
D=(10^(0.1*alphamin)-1)/(10^(0.1*alphamax)-1);
E=Ws/Wp;
N=ceil(log10(D)/(2*log10(E)))
%Whp=Wp/(10^(0.1*alphamax)-1)^(1/(2*N))
Whp=Ws/(10^(0.1*alphamin)-1)^(1/(2*N))
alpha_p=10*log10(1+(Wp/Whp)^(2*N))
alpha_s=10*log10(1+(Ws/Whp)^(2*N))
% Chebyshev
N=ceil(acosh(D^(0.5))/acosh(E))
eps=sqrt(10^(0.1*alphamax)-1)
Whp1=Wp*cosh(acosh(1/eps)/N)
alpha_p= 10*log10(1+(eps^2)*(cos(N*acos(1)))^2)
alpha_s=10*log10(1+(eps^2)*cosh(N*acosh(Ws/Wp))^2)
%% Check with MATLAB
[N1,Whp1]=buttord(Wp,Ws,alphamax,alphamin,'s')
[N2,Wn]=cheb1ord(Wp,Ws,alphamax,alphamin,'s')
```

The orders and the half-power frequency (Butterworth) and the passband frequency (Chebyshev) are verified using MATLAB functions *buttord* and *cheb1ord*.

(a) The lowpass Butterworth filter that satisfies the specifications is of minimum order $N_b = 6$, while the corresponding Chebyshev filter has minimum order $N_c = 4$. Typically for the same specifications $N_c < N_b$

(b) The half-power frequency of the designed Butterworth filter is $\Omega_{hp} = 1787.4$ (calculated so that the loss $\alpha(\Omega_p) = \alpha_{max}$). Another possible value for which $\alpha(\Omega_s) = \alpha_{min}$ is $\Omega_{hp} = 1968.4$.

To compute the half-power frequency of the Chebyshev filter we need the ripple factor ϵ which we find to be 0.3493. We obtain $\Omega_{hp} = 1639.7$.

(c) For the designed Butterworth filter with the first value of Ω_{hp} we get

$$\alpha(\Omega_p) = 0.5 \text{ dB}$$
$$\alpha(\Omega_s) = 35.023 \text{ dB}$$

and for the second half-power frequency

$$\alpha(\Omega_p) = 0.1635 \text{ dB}$$
$$\alpha(\Omega_s) = 30 \text{ dB}$$

The values of the loss function for the Chebyshev filter are

$$\alpha(\Omega_p) = 0.5 \text{ dB}$$
$$\alpha(\Omega_s) = 36.65 \text{ dB}$$

These values do not depend on the half–power frequency but rather on Ω_p which is the normalized frequency.

(d) The formulas for the order of the Butterworth and the Chebyshev filters depend on the ratio of Ω_p and Ω_s, so the orders of the filters do not change.

7.12 Since the dc loss is not zero, the normalized loss specifications are

$$\alpha_{max} = \alpha_1 - \alpha(0) = 0.5$$
$$\alpha_{min} = \alpha_2 - \alpha(0) = 30$$

with a dc loss of 20 dB. The following script is used to find the answers

```
% Pr 7_12
clear all; clf
alphamax=0.5 ;alphamin=30; Wp=1500; Ws=3500;
alpha0=20;
% Butterworth
K=10^(alpha0/20)
D=(10^(0.1*alphamin)-1)/(10^(0.1*alphamax)-1);
E=Ws/Wp;
N=ceil(log10(D)/(2*log10(E)))
Whp=Wp/(10^(0.1*alphamax)-1)^(1/(2*N))
alpha_p=10*log10(1+(Wp/Whp)^(2*N))
alpha_s=10*log10(1+(Ws/Whp)^(2*N))
% Chebyshev
N=ceil(acosh(D^(0.5))/acosh(E))
eps=sqrt(10^(0.1*alphamax)-1)
Whp1=Wp*cosh(acosh(1/eps)/N)
alpha_p= 10*log10(1+(eps^2)*(cos(N*acos(1)))^2)
alpha_s=10*log10(1+(eps^2)*cosh(N*acosh(Ws/Wp))^2)
alpha1=alpha0+10*log10(1+(eps^2)*cosh(N*acosh(0))^2);
Kc=10^(alpha1/20)
```

The following are the results

```
% Butterworth
K = 10 % dc gain
N =6 % minimum order
Whp =1.7874e+03 % half-power frequency
alpha_p = 0.5000 % loss at Wp
alpha_s =35.0228 % loss at Ws
% Chebyshev
N = 4 % min order
eps = 0.3493 % ripple factor
Whp1 = 1.6397e+03 % half-power frq
alpha_p = 0.5000 % loss at Wp
alpha_s = 36.6472 % loss at Ws
Kc =10.5925 % dc gain
```

Notice the computation of the dc gain in the Chebyshev filter. In this case the dc loss depends on the order of the filter and so it is not necessarily 0 dB, so to get the dc gain K_c we use

$$\alpha(0) = 10\log_{10}\left[\frac{K^2}{1+\epsilon^2 C_N^2(0)}\right] = 20\log_{10} K - 10\log_{10}(1+\epsilon^2 C_N^2(0))$$

as indicated in the script.

(d) The minimum orders of the filters depend on the ratio of the two frequencies and since it remains the same these do not change.