# Chapter 1

# Continuous–time Signals

## 1.1 Basic Problems

**1.1** Notice that $0.5[x(t) + x(-t)]$, the even component of $x(t)$, is discontinuous at $t = 0$, it is 1 at $t = 0$ but $0.5$ at $t \pm \epsilon$ for $\epsilon \to 0$. Likewise the odd component of $x(t)$, or $0.5[x(t) - x(-t)]$, must be zero at $t = 0$ so that when added to the even component one gets $x(t)$.
$z(t)$ equals $x(t)$. See Fig. 1.

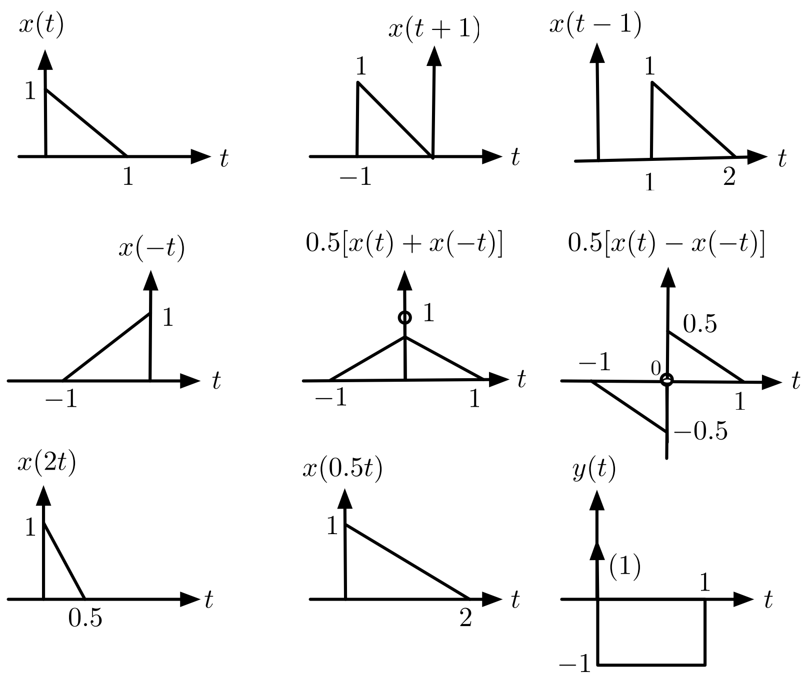


Figure 1.1: Problem 1

**1.2** (a) If $x(t) = t$ for $0 \leq t \leq 1$, then $x(t+1)$ is $x(t)$ advanced by $1$, i.e., shifted to the left by $1$ so that $x(0) = 0$ occurs at $t = -1$ and $x(1) = 1$ occurs at $t = 0$.

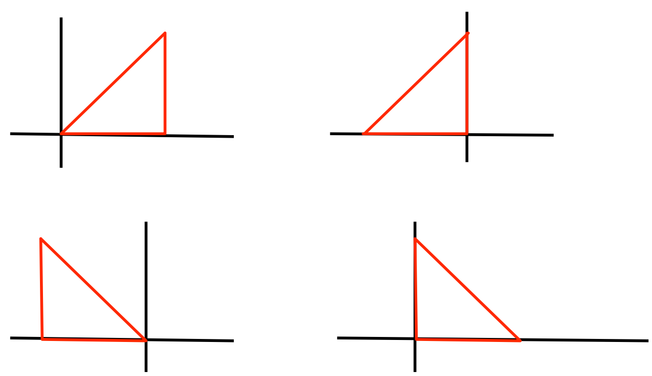

Figure 1.2: Problem 2: Original signal $x(t)$, shifted versions $x(t+1)$, $x(-t)$ and $x(-t+1)$.

The signal $x(-t)$ is the reversal of $x(t)$ and $x(-t+1)$ would be $x(-t)$ advanced to the right by $1$. Indeed,

$$\begin{array}{cc} t & x(-t+1) \\ 1 & x(0) \\ 0 & x(1) \\ -1 & x(2) \end{array}$$

The sum $y(t) = x(t+1) + x(-t+1)$ is such that at $t = 0$ it is $y(0) = 2$; $y(t) = x(t+1)$ for $t < 0$; and $y(t) = x(-t+1)$ for $t > 0$. Thus,

$$\begin{array}{ll} y(t) = x(t+1) = t+1 & 0 \leq t+1 < 1 \quad \text{or} \quad -1 \leq t < 0 \\ y(0) = 2 & \\ y(t) = x(-t+1) = -t+1 & 0 \leq -t+1 < 1 \quad \text{or} \quad 0 < t \leq 1 \end{array}$$

or

$$y(t) = \begin{cases} t+1 & -1 \leq t < 0 \\ 2 & t = 0 \\ -t+1 & 0 < t \leq 1 \end{cases}$$

(b) Except for the discontinuity at $t = 0$, $y(t)$ looks like the even triangle signal $\Lambda(t)$, their integrals are

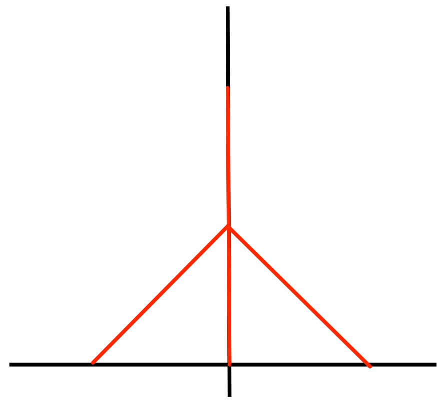

Figure 1.3: Problem 2: Triangular signal $y(t)$ with discontinuity at the origin.

identical as the discontinuity of $y(t)$ does not add any area.

**1.3** (a) We have that

i. $x(t)$ is causal because it is zero for $t < 0$. It is neither even nor odd.

ii. Yes, the even component of $x(t)$ is

$$\begin{aligned} x_e(t) &= 0.5[x(-t) + x(t)] \\ &= 0.5[e^t u(-t) + e^{-t} u(t)] = 0.5e^{-|t|} \end{aligned}$$

(b) $x(t) = \cos(t) + j\sin(t)$ is a complex signal, $x_e(t) = 0.5[e^{jt} + e^{-jt}] = \cos(t)$ so $x_o(t) = j\sin(t)$.

(c) The product of the even signal $x(t)$ with the sine, which is odd, gives an odd signal and because of this symmetry the integral is zero.

(d) Yes, because $x(t) + x(-t) = 2x_e(t)$, i.e., twice the even component of $x(t)$, and multiplied by the sine it is an odd function.

**1.4** The signal $x(t) = t[u(t) - u(t-1)]$ so that its reflection is

$$v(t) = x(-t) = -t[u(-t) - u(-t-1)]$$

and delaying $v(t)$ by 2 is

$$\begin{aligned} y(t) &= v(t-2) = -(t-2)[u(-(t-2)) - u(-(t-2)-1)] \\ &= (-t+2)[u(-t+2) - u(-t+1)] = (2-t)[u(t-1) - u(t-2)] \end{aligned}$$

On the other hand, the delaying of $x(t)$ by 2 gives

$$w(t) = x(t-2) = (t-2)[u(t-2) - u(t-3)]$$

which when reflected gives

$$z(t) = w(-t) = (-t-2)[u(-t-2) - u(-t-3)]$$

Comparing $y(t)$ and $z(t)$ we can see that these operations do not commute, that the order in which these operations are done cannot be changed, so that $y(t) \neq z(t)$ as shown in Fig. 1.4.

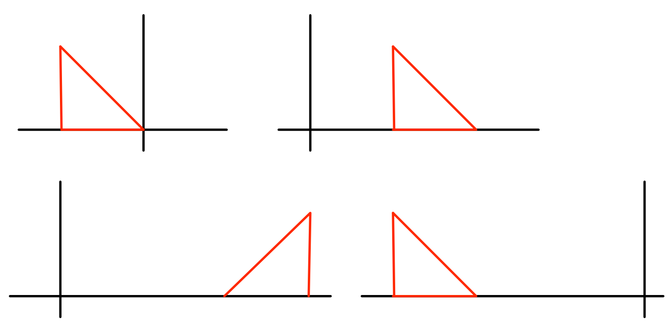

Figure 1.4: Problem 4: Reflection and delaying do not commute, $y(t) \neq z(t)$.

**1.6** (a) Using $\Omega_0 = 2\pi f_0 = 2\pi/T_0$ for

i. $\cos(2\pi t)$: $\Omega_0 = 2\pi$ rad/sec, $f_0 = 1$ Hz and $T_0 = 1$ sec.

ii. $\sin(t - \pi/4)$: $\Omega_0 = 1$ rad/sec, $f_0 = 1/(2\pi)$ Hz and $T_0 = 2\pi$ sec.

iii. $\tan(\pi t) = \sin(\pi t)/\cos(\pi t)$: $\Omega_0 = \pi$ rad/sec, $f_0 = 1/2$ Hz and $T_0 = 2$ sec.

(b) The fundamental period of $\sin(t)$ is $T_0 = 2\pi$, and $T_1 = 2\pi/3$ is the fundamental period of $\sin(3t)$, $T_1/T_0 = 1/3$ so $3T_1 = T_0 = 2\pi$ is the fundamental period of $z(t)$.

(c) i. $y(t)$ is periodic of fundamental period $T_0 = 1$.

ii. $w(t) = x(2t)$ is $x(t)$ compressed by a factor of 2 so its fundamental period is $T_0/2 = 1/2$, the fundamental period of $z(t)$.

iii. $v(t)$ has same fundamental period as $x(t)$, $T_0 = 1$, indeed $v(t + kT_0) = 1/x(t + kT_0) = 1/x(t)$.

(d) i. $x(t) = 2\cos(t)$, $\Omega_0 = 2\pi f_0 = 1$ so $f_0 = 1/(2\pi)$

ii. $y(t) = 3\cos(2\pi t + \pi/4)$, $\Omega_0 = 2\pi f_0 = 2\pi$ so $f_0 = 1$

iii. $c(t) = 1/\cos(t)$, of fundamental period $T_0 = 2\pi$, so $f_0 = 1/(2\pi)$.

(e) $z_e(t)$ is periodic of fundamental period $T_0$, indeed

$$\begin{aligned} z_e(t + T_0) &= 0.5[z(t + T_0) + z(-t - T_0))] \\ &= 0.5[z(t) + z(-t)] \end{aligned}$$

Same for $z_o(t)$ since $z_o(t) = z(t) - z_e(t)$.

**1.8** (a) $x(t)$ is a causal decaying exponential with energy

$$E_x = \int_0^\infty e^{-2t} dt = \frac{1}{2}$$

and zero power as

$$P_x = \lim_{T\to\infty} \frac{E_x}{2T} = 0$$

(b)

$$E_z = \int_{-\infty}^\infty e^{-2|t|} dt = 2 \underbrace{\int_0^\infty e^{-2t} dt}_{E_{z_1}}$$

(c) i. If $y(t) = \text{sign}[x_1(t)]$, it has the same fundamental period as $x_1(t)$, i.e., $T_0 = 1$ and $y(t)$ is a train of pulses so its energy is infinite, while

$$P_y = \int_0^1 1 \, dt = 1$$

ii. Since $x_2(t) = \cos(2\pi t - \pi/2) = \cos(2\pi(t - 1/4)) = x_1(t - 1/4)$, the energy and power of $x_2(t)$ coincide with those of $x_1(t)$.

(d) $v(t) = x_1(t) + x_2(t)$ is periodic of fundamental period $T_0 = 2\pi$, and its power is

$$P_v = \frac{1}{2\pi} \int_0^{2\pi} (\cos(t) + \cos(2t))^2 dt = \frac{1}{2\pi} \int_0^{2\pi} (\cos^2(t) + \cos^2(2t) + 2\cos(t)\cos(2t))dt$$

Using

$$\cos^2(\theta) = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$
$$\cos(\theta)\cos(\phi) = \frac{1}{2}(\cos(\theta + \phi) + \cos(\theta - \phi))$$

we have

$$\begin{aligned} P_v &= \underbrace{\frac{1}{2\pi}\int_0^{2\pi} \cos^2(t)dt}_{P_{x_1}} + \underbrace{\frac{1}{2\pi}\int_0^{2\pi} \cos^2(2t)dt}_{P_{x_2}} + \underbrace{\frac{1}{2\pi}\int_0^{2\pi} 2\cos(t)\cos(2t))dt}_{0} \\ &= \frac{1}{2} + \frac{1}{2} + 0 = 1 \end{aligned}$$

(e) Power of $x(t)$

$$\begin{aligned} P_x &= \frac{1}{T_0}\int_0^{T_0} x^2(t)dt \\ &= \int_0^1 \cos^2(2\pi t)dt \\ &= \int_0^1 (1/2 + \cos^2(4\pi t)dt = 0.5 + 0 = 0.5 \end{aligned}$$

Power of $f(t)$

$$
\begin{aligned}
P_f &= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} y^2(t)dt \\
&= \lim_{N\to\infty} \frac{1}{2(NT_0)}\int_{0}^{NT_0} y^2(t)dt \\
&= \frac{1}{2T_0}\int_{0}^{T_0} y^2(t)dt = 0.5P_s
\end{aligned}
$$

**1.11** (a) Yes, expressing $e^{j2\pi t} = \cos(2\pi t) + j\sin(2\pi t)$, periodic of fundamental period $T_0 = 1$, then the integral is the area under the cosine and sine in one or more periods (which is zero) when $k \neq 0$ and integer. If $k = 0$, the integral is also zero.

(b) Yes, whether $t_0 = 0$ (first equation) or a value different from zero, the two integrals are equal as the area under a period is the same. In the case $x(t) = \cos(2\pi t)$, both integrals are zero.

(c) It is not true, $\cos(2\pi t)\delta(t-1) = \cos(2\pi)\delta(t-1) = \delta(t-1)$.

(d) It is true, considering $x(t)$ the product of $\cos(t)$ and $u(t)$ its derivative is

$$\begin{aligned} \frac{dx(t)}{dt} &= \frac{d\cos(t)}{dt}u(t) + \cos(t)\frac{du(t)}{dt} \\ &= -\sin(t)u(t) + \cos(0)\delta(t) \end{aligned}$$

(e) Yes,

$$\begin{aligned} \int_{-\infty}^{\infty} \left[e^{-t}u(t)\right] \delta(t-2)d\tau &= \int_{0}^{\infty} \left[e^{-2}\right] \delta(t-2)d\tau \\ &= e^{-2} \end{aligned}$$

(f) Yes,

$$\begin{aligned} \frac{dx(t)}{dt} &= 0.5[e^t u(t) + e^t\delta(t)] + 0.5[-e^{-t}u(t) + e^{-t}\delta(t)] \\ &= 0.5[e^t - e^{-t}]u(t) + \delta(t) = \sinh(t)u(t) + \delta(t) \end{aligned}$$

(g) The even component $x_e(t)$ is a periodic full-wave rectified signal of amplitude $1/2$ and fundamental period $T_1 = \pi$.
Power of $x(t)$

$$P_x = 0.5\left[\frac{1}{\pi}\int_0^{\pi} x^2(t)dt\right]$$

Power of $x_e(t)$

$$P_{x_e} = \frac{1}{\pi}\int_0^{\pi}(0.5x(t))^2 dt = 0.5P_x$$

**1.12** (a) See Fig. 12a
$x(t) = |t| \underbrace{[u(t+2) - u(t-2)]}_{p(t)}$ Derivative

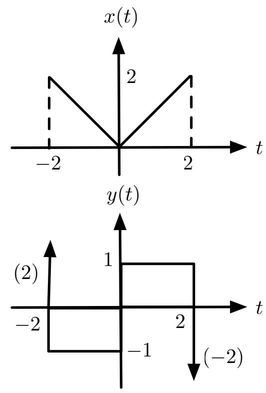


Figure 1.7: Problem 12

$$y(t) = \frac{dx(t)}{dt} = 2\delta(t+2) - u(t+2) + 2u(t) - u(t-2) - 2\delta(t-2)$$

(b) Integral

$$\int_{-\infty}^{t} y(t')dt' = \begin{cases} 0 & t < -2 \\ -t & -2 \leq t < 0 \\ t & 0 \leq t < 2 \\ 0 & t \geq 2 \end{cases}$$

which equals $x(t)$.

(c) Yes, because $x(t)$ is an even function of $t$.

**1.13** (a) The signal $x(t)$ is

$$x(t) = \begin{cases} 0 & t < -1 \\ t+1 & -1 \leq t \leq 0 \\ -1 & 0 < t \leq 1 \\ 0 & t > 1 \end{cases}$$

there are discontinuities at $t = 0$ and at $t = 1$. The derivative

$$\begin{aligned} y(t) &= \frac{dx(t)}{dt} \\ &= u(t+1) - u(t) - 2\delta(t) + \delta(t-1) \end{aligned}$$

indicating the discontinuities at $t = 0$, a decrease from 1 to $-1$, and at $t = 1$ an increase from $-1$ to 0.

(b) The integral

$$\begin{aligned} \int_{-\infty}^{t} y(\tau)d\tau &= \int_{-\infty}^{t} [u(\tau+1) - u(\tau) \\ & \quad -2\delta(\tau) + \delta(\tau-1)]d\tau = x(t) \end{aligned}$$

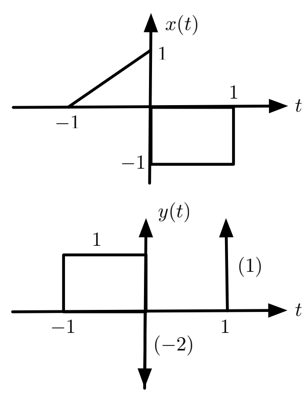


Figure 1.8: Problem 13

**1.16** (a) Because of the discontinuity of $x(t)$ at $t=0$ the even component of $x(t)$ is a triangle with $x_e(0)=1$, i.e.,

$$x_e(t)=\begin{cases} 0.5(1-t) & 0<t\leq 1\\ 0.5(1+t) & -1\leq t<0\\ 1 & t=0\end{cases}$$

while the odd component is

$$x_o(t)=\begin{cases} 0.5(1-t) & 0<t\leq 1\\ -0.5(1+t) & -1\leq t<0\\ 0 & t=0\end{cases}$$

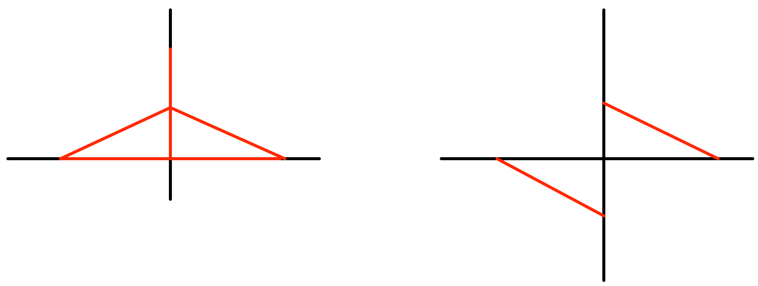

Figure 1.10: Problem 16: Even and odd decomposition of $x(t)$.

(b) The energy of $x(t)$ is

$$\begin{aligned}\int_{-\infty}^{\infty} x^2(t)dt &= \int_{-\infty}^{\infty}[x_e(t)+x_o(t)]^2 dt\\ &= \int_{-\infty}^{\infty} x_e^2(t)dt+\int_{-\infty}^{\infty} x_o^2(t)dt+2\int_{-\infty}^{\infty} x_e(t)x_o(t)dt\end{aligned}$$

where the last equation on the right is zero, given that the integrand is odd.
(c) The energy of $x(t)=1-t$, $0\leq t\leq 1$ and zero otherwise, is given by

$$\int_{-\infty}^{\infty} x^2(t)dt=\int_0^1(1-t)^2dt=t-t^2+\frac{t^3}{3}\,|_0^1\,=\frac{1}{3}$$

The energy of the even component is

$$\int_{-\infty}^{\infty} x_e^2(t)dt=0.25\int_{-1}^{0}(1+t)^2dt+0.25\int_0^1(1-t)^2dt=0.5\int_0^1(1-t)^2dt$$

where the discontinuity at $t=0$ does not change the above result. The energy of the odd component is

$$\int_{-\infty}^{\infty} x_o^2(t)dt=0.25\int_{-1}^{0}(1+t)^2dt+0.25\int_0^1(1-t)^2dt=0.5\int_0^1(1-t)^2dt$$

so that

$$E_x=E_{x_e}+E_{x_o}$$