# EE2S11
# Exercises Ch.5

# Problem 5.1

**5.1** A causal signal $x(t)$ having a Laplace transform with poles in the open-left s-plane (i.e., not including the $j\Omega$-axis) has a Fourier transform that can be found from its Laplace transform. Consider the following signals

$$x_1(t) = e^{-2t}u(t), \quad x_2(t) = r(t), \quad x_3(t) = x_1(t)x_2(t)$$

(a) Determine the Laplace transform of the above signals indicating their corresponding region of convergence.

(b) Determine for which of these signals you can find its Fourier transform from its Laplace transform. Explain.

(c) Give the Fourier transform of the signals that can be obtained from their Laplace transform.

**5.1** (a) The Laplace transforms are

$$x_1(t) = e^{-2t}u(t) \quad \Leftrightarrow \quad X_1(s) = \frac{1}{s+2} \quad \sigma > -2$$

$$x_2(t) = r(t) \quad \Leftrightarrow \quad X_2(s) = \frac{1}{s^2} \quad \sigma > 0$$

$$x_3(t) = te^{-2t}u(t) \quad \Leftrightarrow \quad X_3(s) = \frac{1}{(s+2)^2} \quad \sigma > -2$$

(b) The Laplace transforms of $x_1(t)$ and of $x_3(t)$ have regions of convergence containing the $j\Omega$-axis, and so we can find their Fourier transforms from their Laplace transforms by letting $s = j\Omega$

(c) The Fourier transforms of $x_1(t)$ and $x_3(t)$ are

$$X_1(\Omega) = \frac{1}{2+j\Omega}$$

$$X_3(\Omega) = \frac{1}{(2+j\Omega)^2}$$

# Problem 5.2

**5.2** There are signals whose Fourier transforms cannot be found directly by either the integral definition or the Laplace transform. For instance, the sinc signal

$$x(t) = \frac{\sin(t)}{t}$$

is one of them.

**(a)** Let $X(\Omega) = A[u(\Omega + \Omega_0) - u(\Omega - \Omega_0]$ be a possible Fourier transform of $x(t)$. Find the inverse Fourier transform of $X(\Omega)$ using the integral equation to determine the values of $A$ and $\Omega_0$.

**(b)** How could you use the duality property of the Fourier transform to obtain $X(\Omega)$? Explain.

# Problem 5.2 (cont'd)

**5.2** (a) In this case we are using the duality of the Fourier transforms so that the Fourier transform of the sinc is a pulse of magnitude $A$ and cut-off frequency $\Omega_0$ which we will need to determine.

The inverse Fourier transform is

$$\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} A[u(\Omega+\Omega_0) - u(\Omega-\Omega_0)]e^{j\Omega t}d\Omega \\ &= \frac{A}{2\pi}\int_{-\Omega_0}^{\Omega_0} e^{j\Omega t}d\Omega \\ &= \frac{A}{\pi t}\sin\Omega_0 t \end{aligned}$$

so that $A = \pi$ and $\Omega_0 = 1$, i.e.,

$$\frac{\sin(t)}{t} \quad \Leftrightarrow \quad \pi[u(\Omega+1) - u(\Omega-1)]$$

# Problem 5.2 (cont'd)

(b) The Fourier transform of $x_1(t) = u(t+0.5) - u(t-0.5)$ is

$$X_1(\Omega) = \left[\frac{1}{s}[e^{0.5s} - e^{-0.5s}]\right]_{s=j\Omega} = \frac{\sin(0.5\Omega)}{0.5\Omega}$$

Using the duality property we have:

$$x_1(t) = u(t+0.5) - u(t-0.5) \quad \Leftrightarrow \quad X_1(\Omega) = \frac{\sin(\Omega/2)}{\Omega/2}$$

$$X_1(t) = \frac{\sin(t/2)}{t/2} \quad \Leftrightarrow \quad 2\pi[u(\Omega+0.5) - u(\Omega-0.5)]$$

using the fact that $x_1(t)$ is even. Then using the scaling property

$$X_1(2t) = \frac{\sin(t)}{t} \quad \Leftrightarrow \quad \frac{2\pi}{2}[u((\Omega/2)+0.5) - u((\Omega/2)-0.5)]$$

$$\Leftrightarrow \quad \pi[u(\Omega+1) - u(\Omega-1)]$$

so $x(t) = X_1(2t) = \sin(t)/t$ is the inverse Fourier transform of $X(\Omega) = \pi[u(\Omega+1) - u(\Omega-1)]$

# Problem 5.6

**5.6** Consider a signal $x(t) = \cos(t), 0 \leq t \leq 1$,

(a) Find its Fourier transform $X(\Omega)$.

---

**5.6** (a) $x(t) = \cos(t)[u(t) - u(t-1)] = \cos(t)p(t)$, so

$$X(\Omega) = 0.5[P(\Omega+1) + P(\Omega-1)]$$

where

$$P(\Omega) = \frac{e^{-s/2}(e^{s/2} - e^{-s/2})}{s}|_{s=j\Omega} = 2e^{-j\Omega/2}\frac{\sin(\Omega/2)}{\Omega}$$